Two-component model of superconductivity in a checkerboard background

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Introduction

There is experimental evidence that, in some cuprates, the break down of antiferromagnetic order with doping proceeds through the formation of inhomogeneous magnetic structures – static or fluctuating. We assume that such structures could be one of the keys to solving the puzzle of superconductivity in cuprates. We think so, in part, because energy scale related to these structures is of the same order as T_C.

One of the controversies surrounding the issue of magnetic modulation in cuprates is whether these modulations are one-dimensional stripe-like or two-dimensional checkerboard-like. The stripe interpretation is more popular at present. However, the possibility of checkerboard-like modulation is not experimentally excluded (see Refs. [2], [3]).

In particular, the experiment of Ref. [1] (reproduced in Fig. 1) shows that the polarization of the spin modulation harmonics in $La_{1.48}Nd_{0.4}Sr_{0.12}CuO_4$ are transversely polarized. This observation is consistent with either two stripe domains or with two-dimensional checkerboard of spin vortices shown in Fig. 2.

Our goal is to explore what kind of superconductivity can exist in such a two-dimensional background.

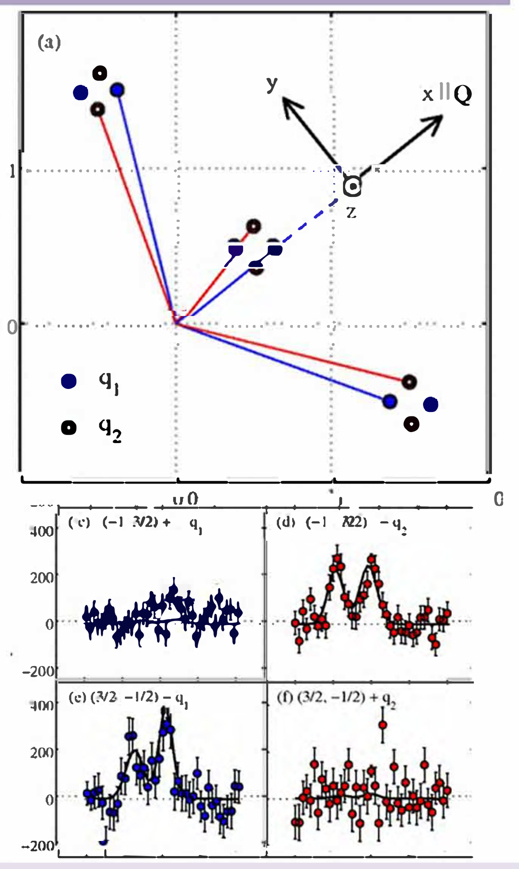

Fig. 1. Inelastic magnetic neutron scattering peaks indicating that spin-modulation harmonics have transverse polarization. from Ref.1.

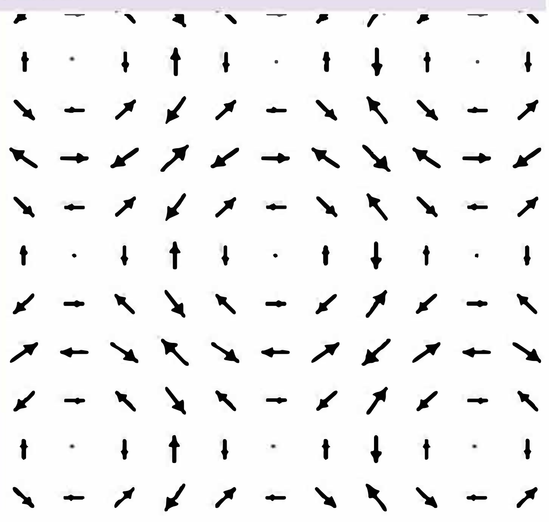
Fig. 2. Checkerboard of spin vortices possibly existing in cuprates. From. Ref.[2]

Previous results & basic model

A model of interacting fermions in the presence of a closely related texture called 'grid' (shown in Fig.3) was considered in Ref. [4]. A similar model for spin vortex checkerboard was considered in Ref. [5].

Here we focus on the grid background, because the model is simpler.

The Hamiltonian of this model is:

$$\mathcal{H} = \varepsilon_a \sum_i a_i^+ a_i + \varepsilon_b \sum_{i,j(i),\sigma}^{\eta_i=1} b^+_{ij\sigma} b_{ij\sigma} + g \sum_{i,j(i)}^{\eta_i=1} (b^+_{ij,-} b^+_{ij,+} a_i a_j + h.c.) \qquad (1)$$

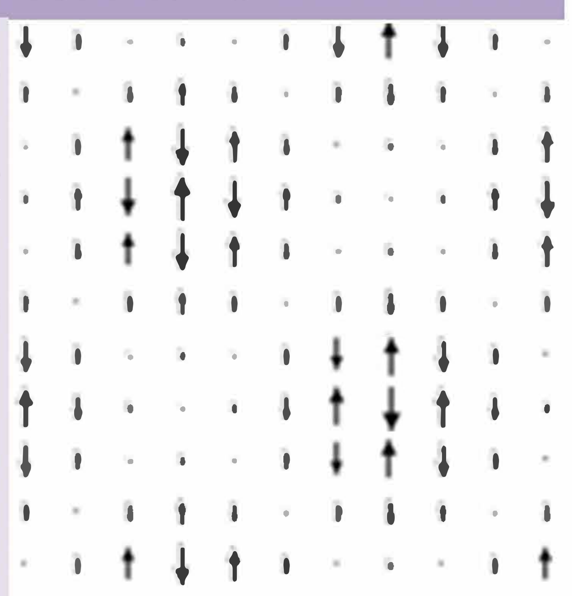
Fig. 3. Grid spin structure. From. Ref.[4]

where a and b are local fermionic annihilation operators, shown in Fig. 4.

In this model a mean-field phase transition accompanied by the emergence of two-fermion anomalous averages occurs. It was previously associated with the superconducting transition.

But that model (and analogous model for spin vortex background) have the following difficulty: they do not include any term changing the center-of-mass position of particles and, as a result, suffer from the lack of phase stiffness.

We can overcome this difficulty by considering a model that includes higher energy quantum states with the following Hamiltonian:

$$\mathcal{H}_{basic} = \varepsilon_a \sum_i a_i^+ a_i + \tilde{\varepsilon}_a \sum_i \tilde{a}_i^+ \tilde{a}_i + \varepsilon_b \sum_{i,j(i),\sigma}^{\eta_i=1} b^+_{ij\sigma} b_{ij\sigma} + U \sum_{i,j(i)}^{\eta_i=1} b^+_{ij+} b^+_{ij-} b_{ij-} b_{ij+} + t \sum_{i,j(i)} \left(b^+_{ij,\eta_i} a_i + b^+_{ij,-\eta_i} \tilde{a}_i + h.c. \right) \qquad (2)$$

where U is the on-site interaction energy between b-states, t is the hopping element, $\tilde{a}$ is an annihilation operator of the first excited state inside antiferromagnetic domain, and $\tilde{\varepsilon}_a$ is the energy of this state.

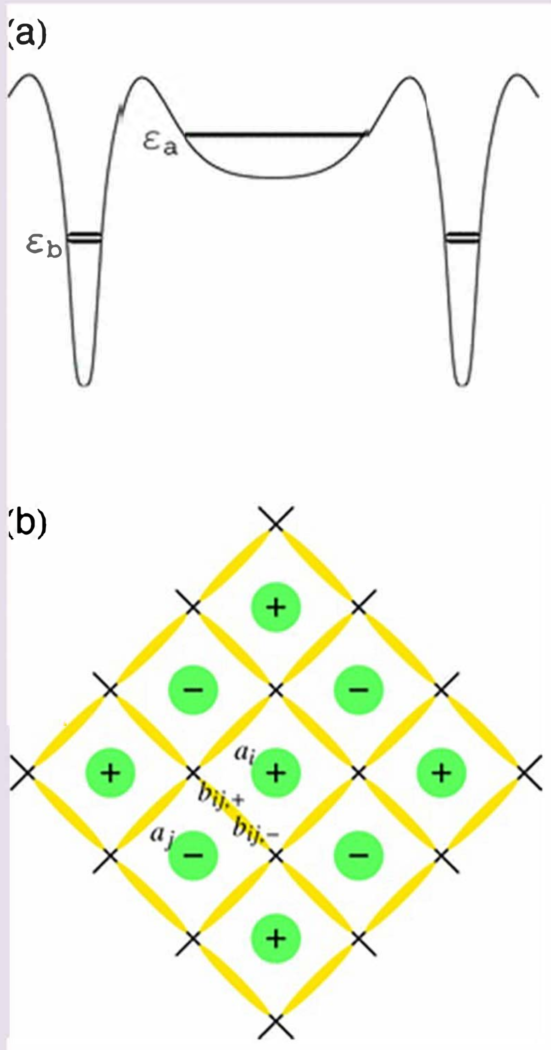

Fig. 4. (a) Model quantum states for grid checkerboard: one a-state with energy "a inside every AF domain", and two degenerate states with energy "b inside every stripe element". (b) Two-dimensional scheme of a-states and b-states. Each circle represents the center of an a-state, while each ellipsoid extended along the stripe boundaries represents the location of two b-states.

Boson-fermion regime

We can now make a few reasonable assumptions about Hamiltonian (2):

- U is sufficiently large and negative.
- t is small.
- ε_b and $\tilde{\varepsilon}_a$ are positive and of the order of $|U|$, so we can consider single b-states and excited a-states as virtual.

This way, we obtain effective Hamiltonian:

$$\mathcal{H}_{eff} = \varepsilon_a \sum_i a_i^+ a_i + (2\varepsilon_b + U) \sum_{i,j(i)}^{\eta_i=1} d^+_{ij} d_{ij} + g \sum_{i,j(i)}^{\eta_i=1} (d^+_{ij} a_i a_j + h.c.) + \text{terms of order } t^4 \qquad (3)$$

where d is the operator of annihilation of a pair of b-states.

This is a boson-fermion model, which is somewhat similar to the model of Ref.[6]. It explains the possible origin of the interaction term g in Hamiltonian 1.

Importantly, Hamiltonian (3) has very small but non-vanishing hopping, which is responsible for the phase stiffness in the superconducting phase.

Static checkerboard suppresses superconducting phase stiffness

An important question here is how the temperature of the mean-field transition for Hamiltonians (1) or (3) is related to the temperature of the superconducting transition.

In Ref. [7], it was pointed out that, if the phase stiffness is low, then phase fluctuations can significantly reduce the superconducting critical temperature in comparison with the mean-field critical temperature.

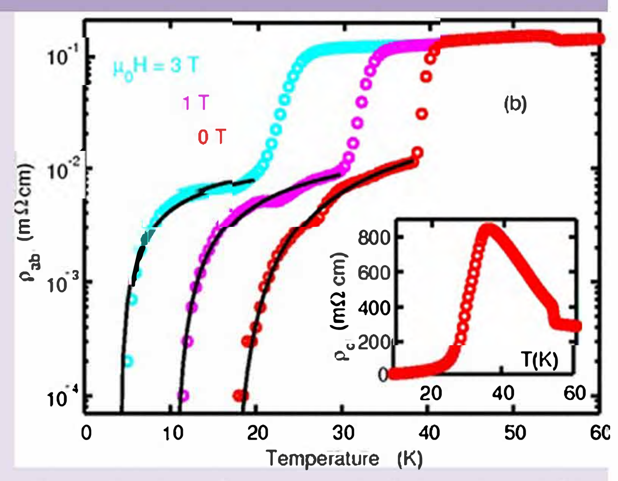

Fig. 5. In-plane resistivity in 1/8-doped LBCO. From Ref.[8].

In Hamiltonian (3), there are two reasons for the small phase stiffness:

1. Hopping elements are small.
2. Occupied d-states obstruct the hopping for a-states and *vice versa*.

The second reason may play the key role in the suppression superconductivity in lanthanum cuprates around 1/8 doping. At this doping level the d-states are fully occupied in the normal state. As a result, mean-field transition is accompanied by a resistance drop without the onset of superconductivity, which is consistent with the results of Ref.[8], shown in Fig.5.

References:

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