Inflation and reheating in the early Universe
Lecture #1
Introduction: present Universe and Hot Big Bang Theory

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“Quantum fields:
from gravity and cosmology
to physics of condensed matter”
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Standard Model: Major Problems

Gauge fields (interactions): γ, $W^{\pm}$, Z, g

Three generations of matter: $L = \binom{\nu_L}{e_L}$, e_R; $Q = \binom{u_L}{d_L}$, d_R, u_R

- Describes
  - all experiments dealing with electroweak and strong interactions
- Does not describe (PHENO) (THEORY)
  - Neutrino oscillations
  - Dark matter (Ω_{DM})
  - Baryon asymmetry (Ω_B)
  - Inflationary stage
  - Reheating
  - Dark energy (Ω_Λ)
  - Strong CP-problem
  - Gauge hierarchy
  - Quantum gravity

Must explain all above ???

Problems in astrophysics. . . (?)

- Origin of extragalactic magnetic fields
- First stars and reionization of the Universe
- Mechanism of SuperNovae explosion
- Sources of Ultra-high energy cosmic rays (EeV-scale)
- Extremely low IR extragalactic background
- Too old White Dwarfs
- Origin of Fast Radio Bursts
- Origin of ICECUBE neutrinos (PeV-scale)
- Black hole physics
- . . .
- Helioseismology vs helioemissivity
- Origin of the heat at the Earth

New Physics and New Cosmology may be

either responsible for

or testable there

Experimenal data in Cosmology and Astrophysics

- Each experiment may be unique (unrepeatable):
  – observe only one Universe
  – (so far) registered only one SN explosion
  – might observe only one magnetic monopole (?)
  – can study only one star
  – (so far) can study only one planet
  ...
- we register photons, neutrinos, gravitational waves, electrons, positrons, protons, nuclei,
  but only photons, neutrinos and gravitational waves can point at the source
- Can not directly check the model of sources
- Can not directly check the media in between

Outline

"Natural" units in particle physics

$$\hbar = c = k_{\rm B} = 1$$

measured in GeV: energy E, mass M, temperature T

$$m_p = 0.938\ \text{GeV}, \quad 1\ \text{K} = 8.6 \times 10^{-14}\ \text{GeV}$$

measured in GeV^{-1}: time t, length L

$$1\ \text{s} = 1.5 \times 10^{24}\ \text{GeV}^{-1}, \quad 1\ \text{cm} = 5.1 \times 10^{13}\ \text{GeV}^{-1}$$

Gravity (General Relativity): $V(r) = -G\frac{m_1 m_2}{r}$ $\qquad [G] = M^{-2}$

$$M_{\rm Pl} = 1.2 \times 10^{19}\ \text{GeV} = 22\ \mu\text{g} \qquad G \equiv \frac{1}{M_{\rm Pl}^2}$$

“Natural” units in cosmology

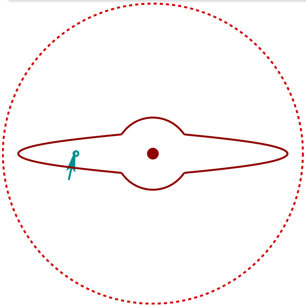
$$1\ \mathrm{Mpc} = 3.1 \times 10^{24}\ \mathrm{cm}$$

$1\ \mathrm{AU} = 1.5 \times 10^{13}$ cm — mean Earth-to-Sun distance

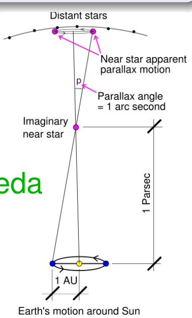
$1\ \mathrm{ly} = 0.95 \times 10^{18}$ cm — distance light travels in one year

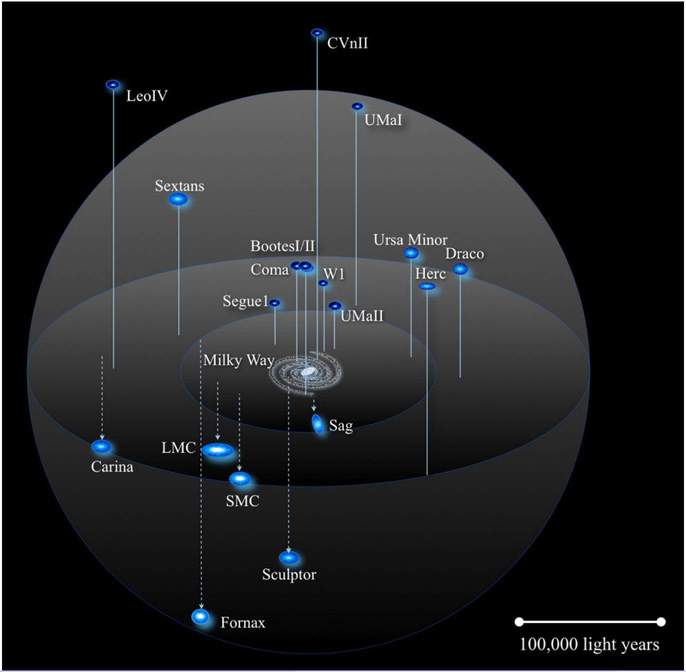
$1\ \mathrm{yr} = 3.16 \times 10^{7}$ s

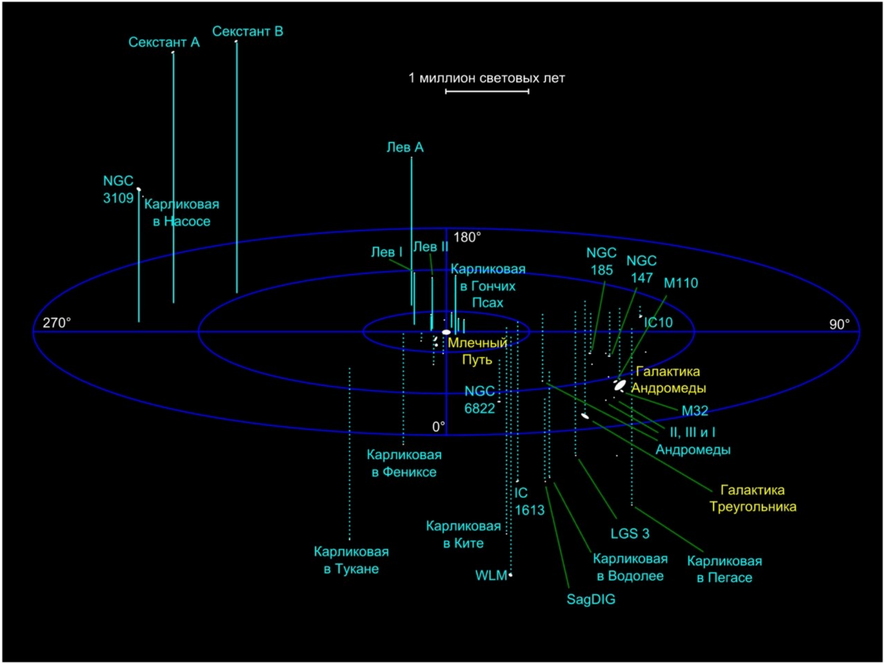
$1\ \mathrm{pc} = 3.3\ \mathrm{ly} = 3.1 \times 10^{18}$ cm — distance to object which has a parallax angle of one arcsec

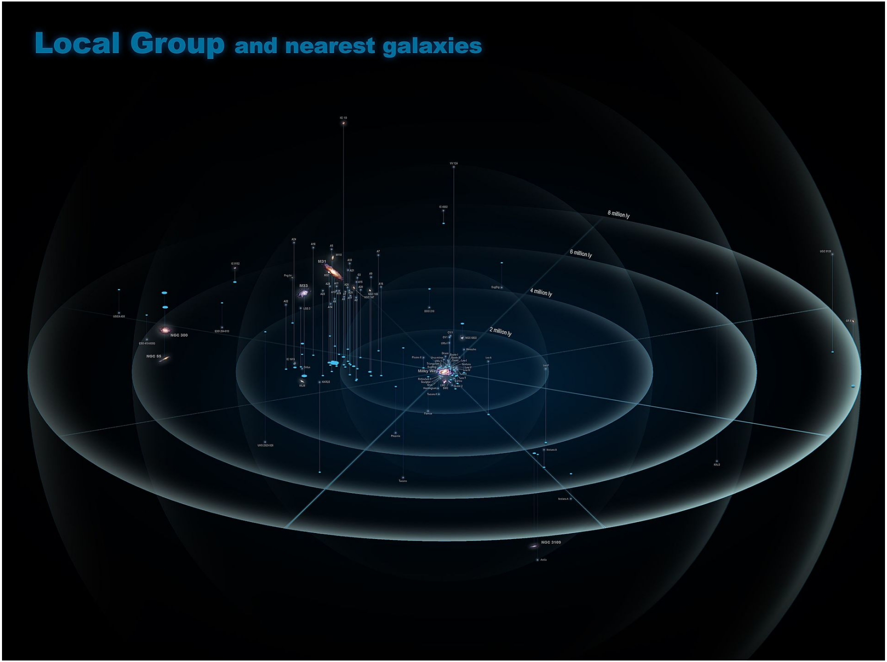


100 AU — Solar system size
1.3 pc — nearest-to-Sun stars
1 kpc — size of dwarf galaxies
50 kpc — distance to dwarves
0.8 Mpc — distance to Andromeda
1-3 Mpc — size of clusters
15 Mpc — distance to Virgo




CVnII
LeoIV
UMaI
Sextans
BootesI/II
Ursa Minor
Draco
Coma
W1
Herc
Segue1
UMaII
Milky Way
Sag
LMC
Carina
SMC
Sculptor
Fornax
100,000 light years

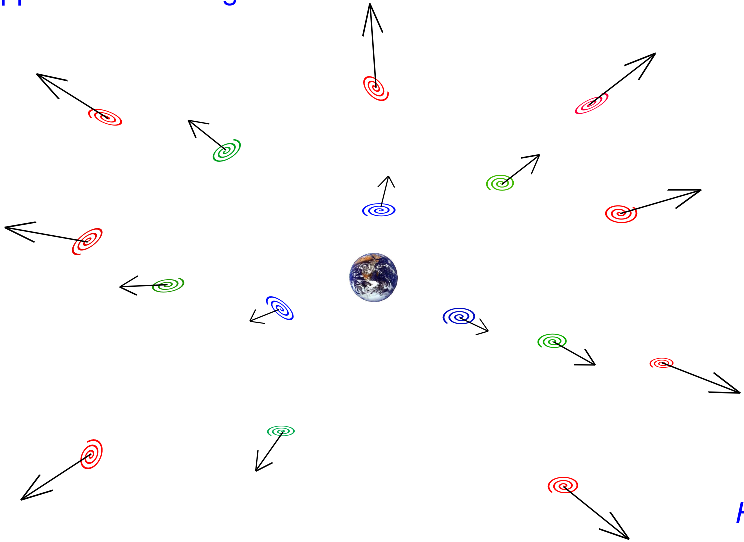

Секстант A
Секстант B
1 миллион световых лет
Лев A
NGC 3109
Карликовая в Насосе
180°
Лев I
Лев II
Карликовая в Гончих Псах
NGC 185
NGC 147
M110
IC10
270°
90°
Млечный Путь
Галактика Андромеды
NGC 6822
M32
II, III и I Андромеды
0°
Карликовая в Фениксе
Галактика Треугольника
IC 1613
LGS 3
Карликовая в Ките
Карликовая в Тукане
Карликовая в Водолее
Карликовая в Пегасе
WLM
SagDIG


Local Group and nearest galaxies
8 million ly
6 million ly
4 million ly
2 million ly
M31
M33
NGC 300
NGC 55
Milky Way
NGC 3109

Outline

Universe is expanding

Doppler redshift of light



$L \propto a(t)$

$n \propto a^{-3}(t)$

$H(t) = \frac{\dot{a}(t)}{a(t)}$

Hubble parameter

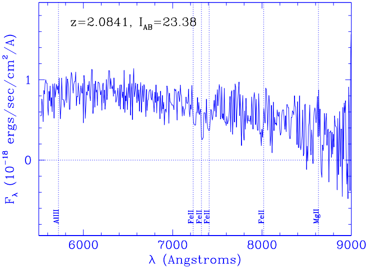
Hubble Law

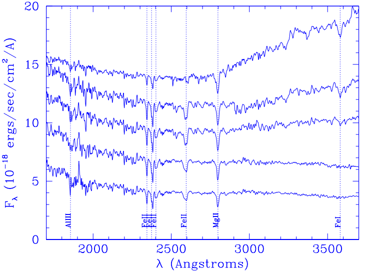
$H(t_0)\, r = v_r$

Expansion: redshift z $\qquad \lambda_{\text{abs.}}/\lambda_{\text{em.}} \equiv 1+z$

$z \ll 1$ Hubble law : $z = H_0 r$

$$H_0 = h \cdot 100\,\frac{\text{km}}{\text{s}\cdot\text{Mpc}}\,, \qquad h \approx 0.68$$



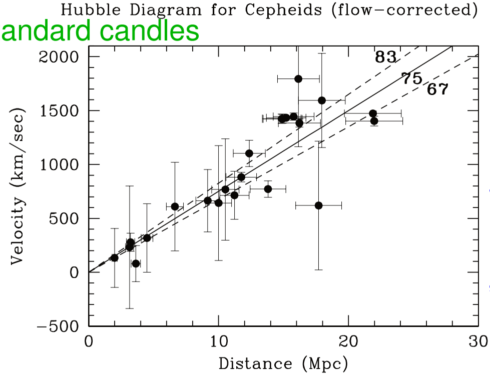


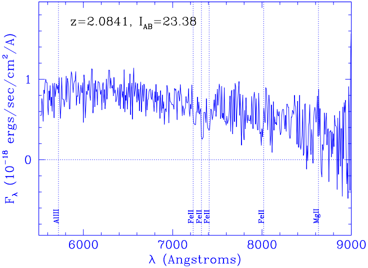
Expansion: redshift z $\qquad \lambda_{\text{abs.}}/\lambda_{\text{em.}} \equiv 1 + z$

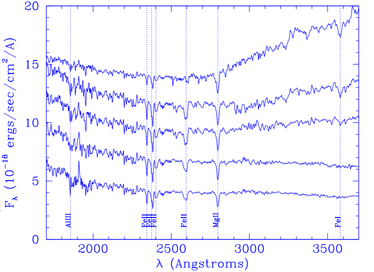
$z \ll 1$ Hubble law : $z = H_0 r$

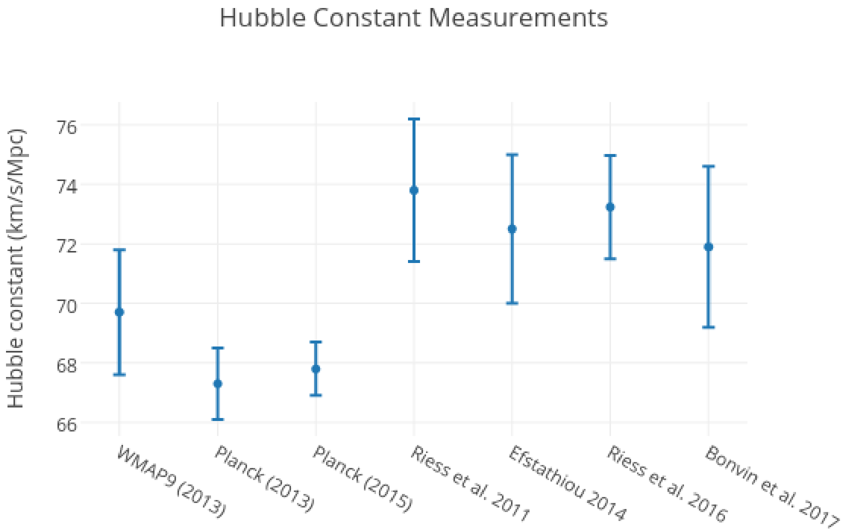
$$H_0 = h \cdot 100\,\frac{\text{km}}{\text{s}\cdot\text{Mpc}}\,, \qquad h \approx 0.68$$

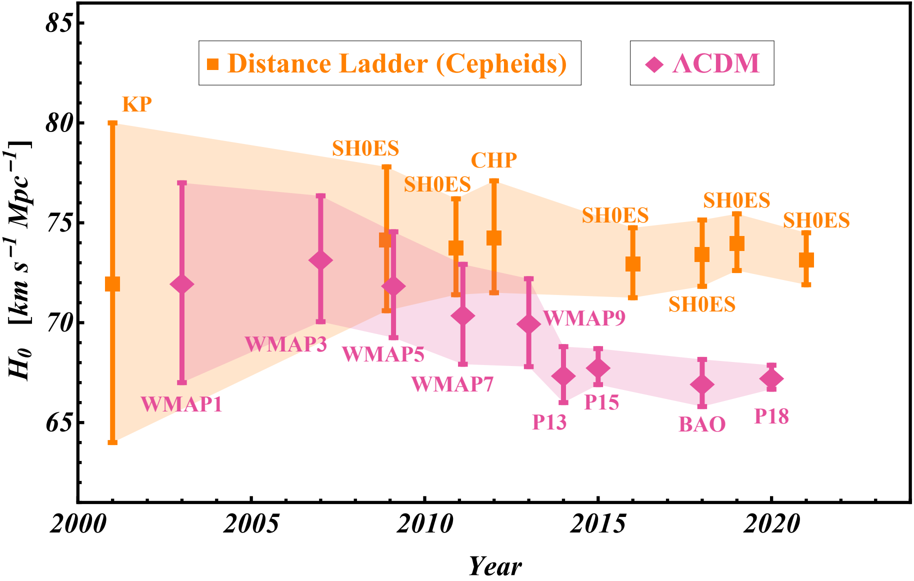
standard candles



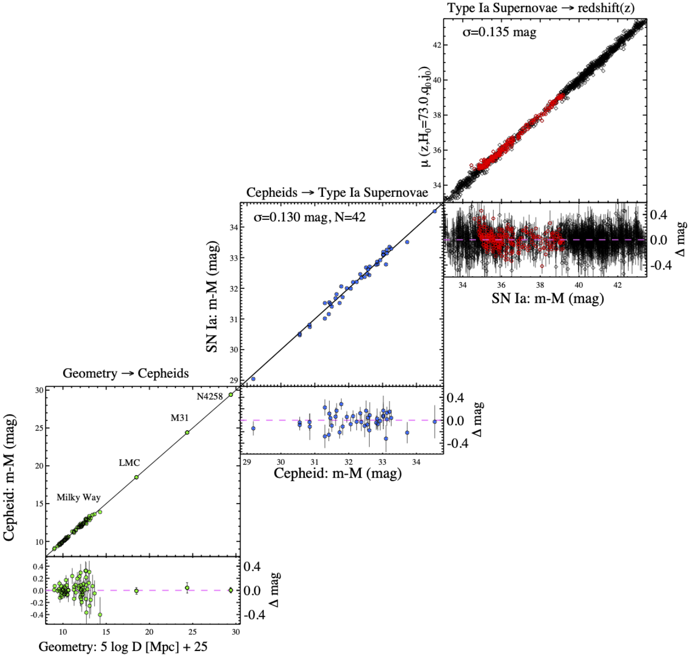





Hubble Constant Measurements
Hubble constant (km/s/Mpc)
76
74
72
70
68
66
WMAP9 (2013)
Planck (2013)
Planck (2015)
Riess et al. 2011
Efstathiou 2014
Riess et al. 2016
Bonvin et al. 2017

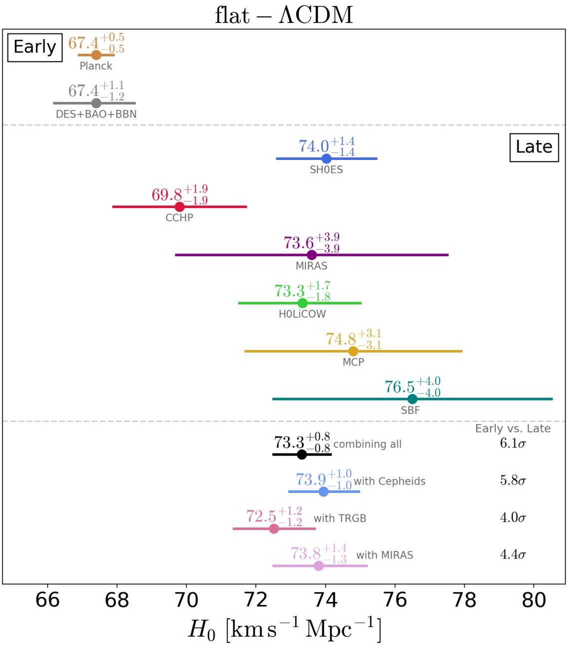

Distance Ladder (Cepheids)
ΛCDM
KP
SH0ES
CHP
WMAP1
WMAP3
WMAP5
WMAP7
WMAP9
P13
P15
BAO
P18
85
80
75
70
65
2000
2005
2010
2015
2020
Year
H_0 [$km\ s^{-1}\ Mpc^{-1}$]

2105.05208

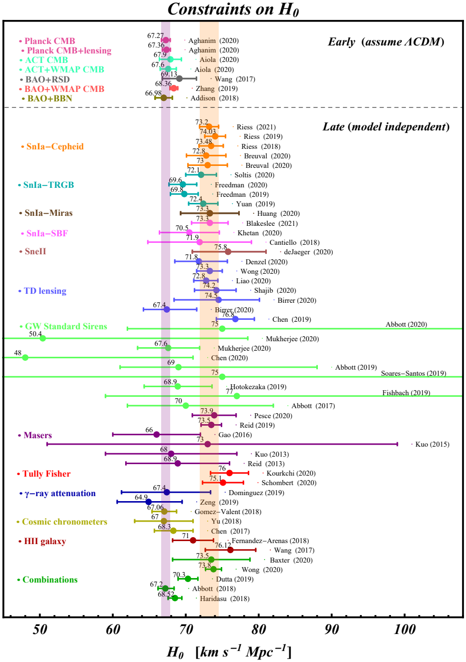


Type Ia Supernovae → redshift(z)
σ=0.135 mag
μ (z,H0=73.0,q0,j0)
SN Ia: m-M (mag)
Δ mag
Cepheids → Type Ia Supernovae
σ=0.130 mag, N=42
SN Ia: m-M (mag)
Cepheid: m-M (mag)
Geometry → Cepheids
N4258
M31
LMC
Milky Way
Cepheid: m-M (mag)
Geometry: 5 log D [Mpc] + 25

2211.04492


flat − ΛCDM
Early
67.4 +0.5 −0.5
Planck
67.4 +1.1 −1.2
DES+BAO+BBN
Late
74.0 +1.4 −1.4
SH0ES
69.8 +1.9 −1.9
CCHP
73.6 +3.9 −3.9
MIRAS
73.3 +1.7 −1.8
H0LiCOW
74.8 +3.1 −3.1
MCP
76.5 +4.0 −4.0
SBF
Early vs. Late
73.3 +0.8 −0.8 combining all
6.1σ
73.9 +1.0 −1.0 with Cepheids
5.8σ
72.5 +1.2 −1.2 with TRGB
4.0σ
73.8 +1.4 −1.3 with MIRAS
4.4σ
66
68
70
72
74
76
78
80
H_0 [km s^{-1} Mpc^{-1}]

1907.10625



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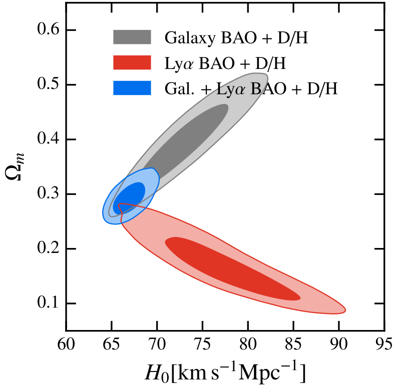
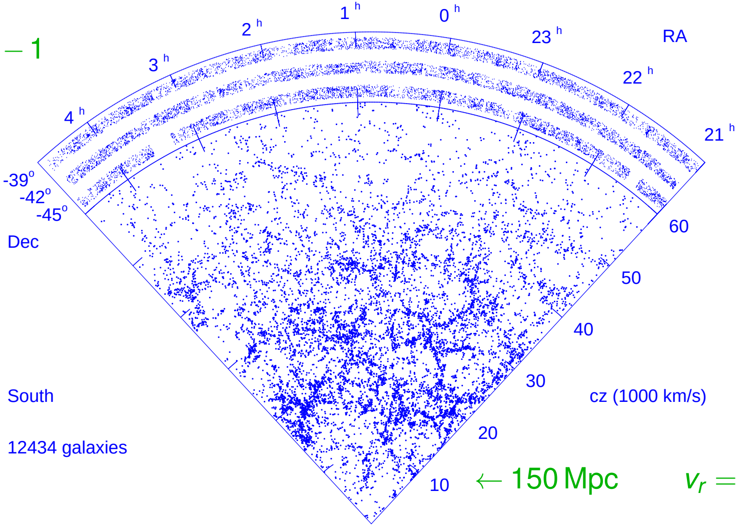


1707.06547

Universe is homogeneous and isotropic

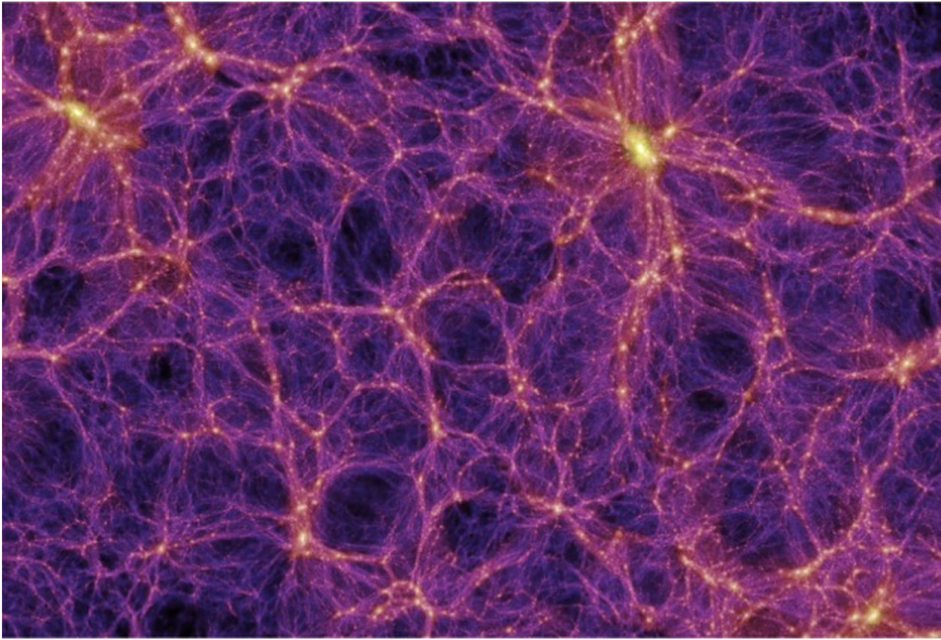
redshift

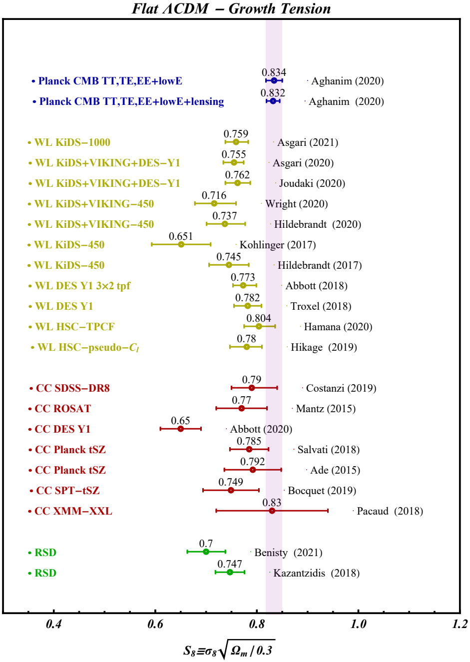
$$z \equiv \frac{\lambda_{\text{detector}}}{\lambda_{\text{source}}} - 1$$



$\leftarrow$ 150 Mpc

$v_r = c\,z$





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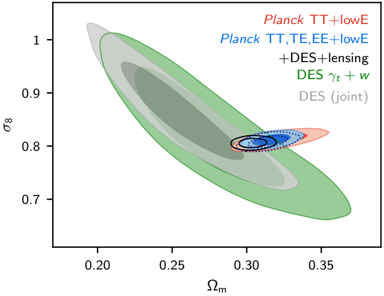


1807.06209

The Universe: age & geometry & energy density

$$[H_0] = L^{-1} = t^{-1}$$

time scale: $t_{H_0} = H_0^{-1} \approx 14 \times 10^9$ yr — age of our Universe

spatial scale: $l_{H_0} = H_0^{-1} \approx 4.3 \times 10^3$ Mpc — size of the visible Universe

t_{H_0} is in agreement with various observations

homogeneity and isotropy in 3d:

flat, spherical or hyperbolic

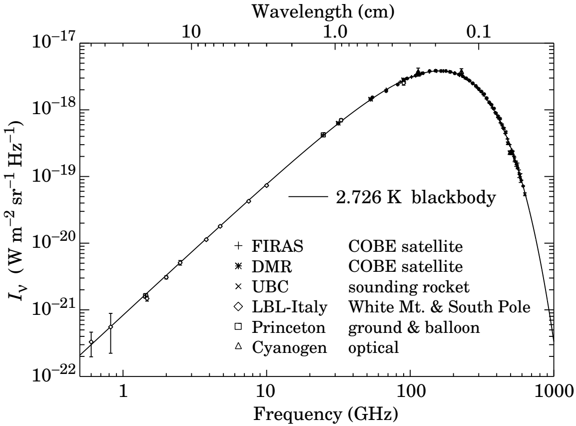
Observations: "very" flat $R_{curv} > 10 \times l_{H_0}$

order-of-magnitude estimate: $GM_U/l_U \sim G\rho_0 l_{H_0}^3/l_{H_0} \sim 1$

flat Universe

$$\rho_c = \frac{3}{8\pi} H_0^2 M_{Pl}^2 \approx 0.53 \times 10^{-5} \frac{\text{GeV}}{\text{cm}^3} \longrightarrow \text{5 protons in each 1 } m^3$$

Universe is occupied by “thermal” photons



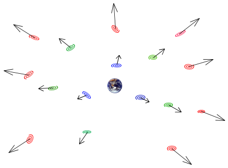
$T_0 = 2.726$ K

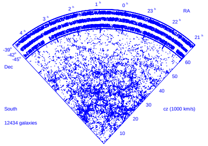
the spectrum (shape and normalization!) is thermal

$n_\gamma = 411\ \text{cm}^{-3}$

Conclusions from observations

The Universe is homogeneous, isotropic, hot and expanding...





Conclusions

- interval between events gets modified

$$\Delta s^2 = c^2\,\Delta t^2 - a^2(t)\,\Delta \mathbf{x}^2$$

in GR expansion is described by the Friedmann equation

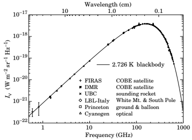
$$\left(\frac{\dot a}{a}\right)^2 = H^2(t) = \frac{8\pi}{3}\,G\rho^{\text{energy}}_{\text{density}}$$

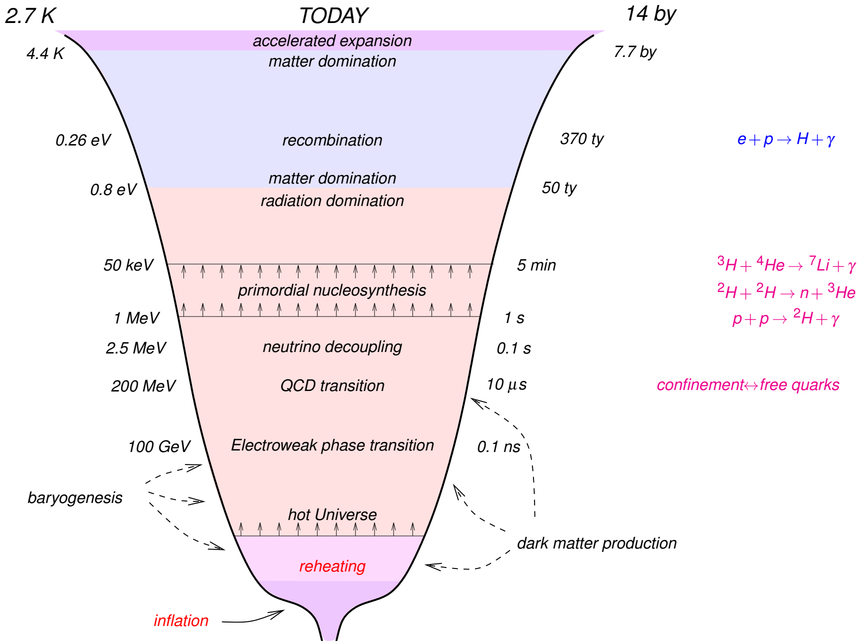
$$\rho^{\text{energy}}_{\text{density}} = \rho_{\text{radiation}} + \rho_{\text{matter}} + \dots$$

- in the past the matter density was higher, our Universe was "hotter" filled with electromagnetic plasma

$$\rho_{\text{matter}} \propto 1/a^3(t)\,,\;\; \rho_{\text{radiation}} \propto 1/a^4(t)\,,\;\; \rho_{\text{curvature}} \propto 1/a^2(t)$$

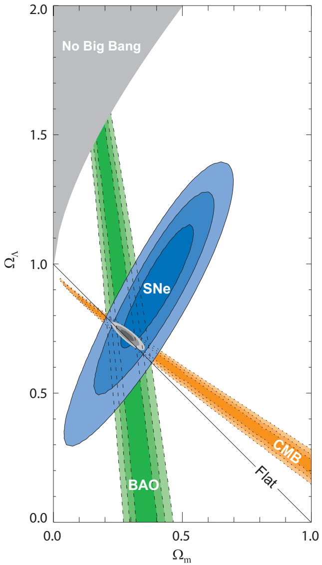
certainly known up to $T \sim 1\,\text{MeV} \sim 10^{10}\,\text{K}$




2.7 K
TODAY
14 by
accelerated expansion
4.4 K
matter domination
7.7 by
0.26 eV
recombination
370 ty
$e + p \to H + \gamma$
matter domination
0.8 eV
radiation domination
50 ty
50 keV
5 min
$^3H + {}^4He \to {}^7Li + \gamma$
primordial nucleosynthesis
$^2H + {}^2H \to n + {}^3He$
1 MeV
1 s
$p + p \to {}^2H + \gamma$
2.5 MeV
neutrino decoupling
0.1 s
200 MeV
QCD transition
10 μs
confinement↔free quarks
100 GeV
Electroweak phase transition
0.1 ns
baryogenesis
hot Universe
dark matter production
reheating
inflation

Outline

Astrophysical and cosmological data are in agreement



$$\left(\frac{\dot a}{a}\right)^2 = H^2(t) = \frac{8\pi}{3}\,G\rho^{\text{energy}}_{\text{density}}$$

$$\rho^{\text{energy}}_{\text{density}} = \rho_{\text{radiation}} + \rho^{\text{ordinary}}_{\text{matter}} + \rho^{\text{dark}}_{\text{matter}} + \rho_\Lambda$$

$$\rho_{\text{radiation}} \propto 1/a^4(t) \propto T^4(t)\,, \quad \rho_{\text{matter}} \propto 1/a^3(t)$$

$$\rho_\Lambda = \text{const}$$

$$\frac{3H_0^2}{8\pi G} = \rho^{\text{energy}}_{\text{density}}(t_0) \equiv \rho_c \approx 0.53\times 10^{-5}\,\frac{\text{GeV}}{\text{cm}^3}$$

radiation: $\Omega_\gamma \equiv \frac{\rho_\gamma}{\rho_c} = 0.5\times 10^{-4}$

Baryons (H, He): $\Omega_B \equiv \frac{\rho_B}{\rho_c} = 0.05$

Neutrino: $\Omega_\nu \equiv \frac{\sum \rho_{\nu_i}}{\rho_c} < 0.01$

Dark matter: $\Omega_{DM} \equiv \frac{\rho_{DM}}{\rho_c} = 0.27$

Dark energy: $\Omega_\Lambda \equiv \frac{\rho_\Lambda}{\rho_c} = 0.68$

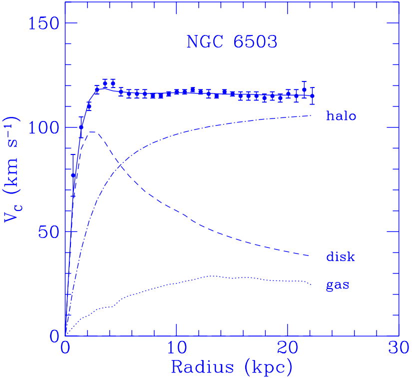
Why do we need dark components (within GR)?

- Astrophysical data favor Dark Matter
  - Observations in galaxies
  - Observations in galaxy clusters
- Cosmological data favor Dark Matter and Dark Energy
  - Observation of objects at cosmological distances (far=early)
  - Baryonic Aciustic (Sakharov) Oscillations (BAO) in two-point galaxy correlation function
  - Evolution of galaxy clusters in the Universe
  - Anisotropy of Cosmic Microwave Background (CMB)

Galactic dark halos: flat rotation curves

$$v(R) = \sqrt{G\frac{M(R)}{R}}$$

$$M(R) = 4\pi\int_0^R \rho(r) r^2 dr$$



observations: $v(R) \simeq$ const

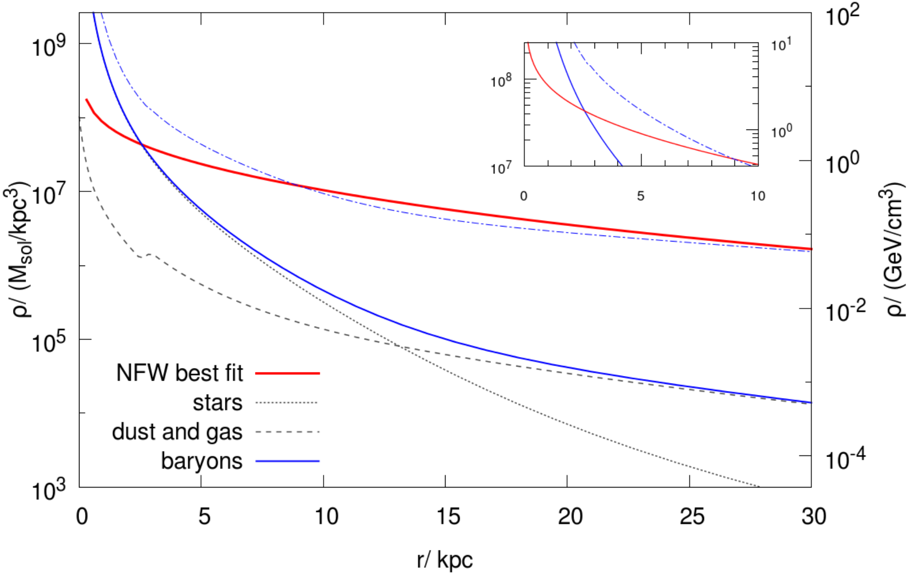
visible matter:

internal regions $v(R) \propto \sqrt{R}$

external ("empty") regions $v(R) \propto 1/\sqrt{R}$

Matter distribution in the Milky Way

1706.09850



Dark Matter in clusters

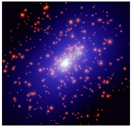
X-rays from hot gas in clusters

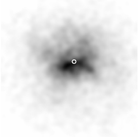
$$\frac{dP}{dR} = -\mu n_e(R) m_p \frac{GM(R)}{R^2}\,, \quad M(R) = 4\pi \int_0^R \rho(r) r^2 dr\,, \quad P(R) = n_e(R) T_e(R)$$

galaxies in clusters — virial theorem

$$U + 2E_k = 0$$

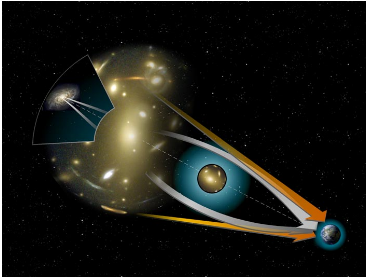
$$3M\langle v_r^2 \rangle = G\frac{M^2}{R}$$



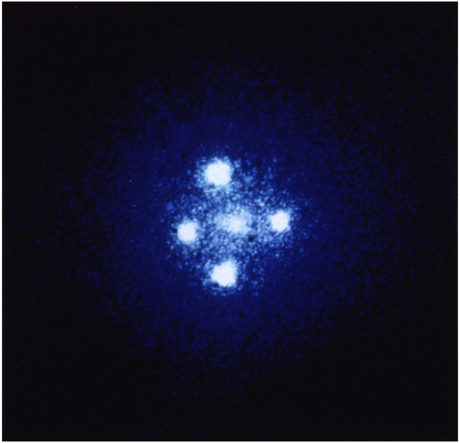


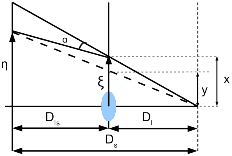
Milky Way: Virgo infall

Gravitational lensing in GR: $\alpha = 4GM/(c^2 b)$



Einstein Cross





$$\vec{\eta} = \frac{D_s}{D_l}\vec{\xi} - D_{ls}\vec{\alpha}\left(\vec{\xi}\right)$$

common lens with specific refraction coefficient

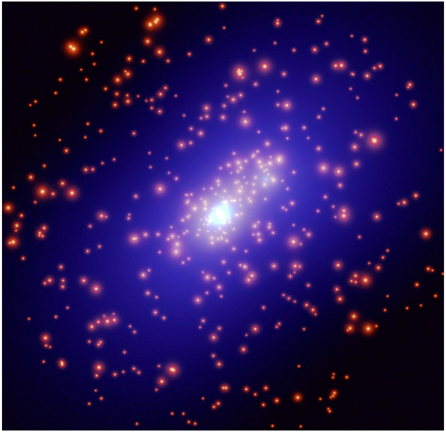
$$\vec{\alpha}\left(\vec{\xi}\right) = \frac{4G}{c}\int \frac{\vec{\xi}-\vec{\xi}'}{\left|\xi-\vec{\xi}'\right|^2} d^2\xi' \int \rho\left(\vec{\xi}',z\right) dz$$

source: quasar $D_s = 2.4$ Gpc

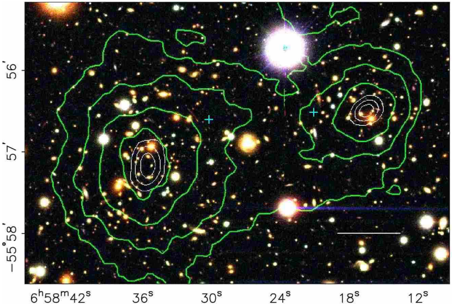
lens: galaxy $D_l = 120$ Mpc

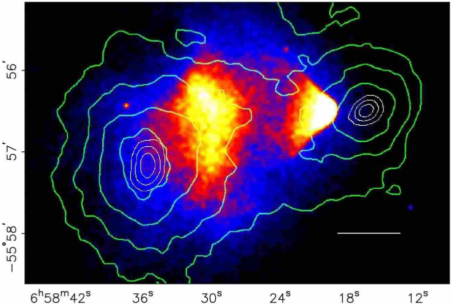
Dark Matter in clusters

gravitational lensing $\rho_B \approx 0.25\rho_{DM}$



Colliding clusters (Bullet clusters 1E0657-558)





gravitational lensing

Observations in X-rays

$M \simeq 10 \times m$

scale is 200 kpc

clusters are at 1.5 Gpc

Dark Matter Properties $p = 0$

(If) particles:

1. stable on cosmological time-scale
2. nonrelativistic long before RD/MD-transition (either Cold or Warm, $v_{RD/MD} \lesssim 10^{-3}$)
3. (almost) collisionless
4. (almost) electrically neutral

If were in thermal equilibrium: $M_X \gtrsim 1$ keV

If not: for bosons

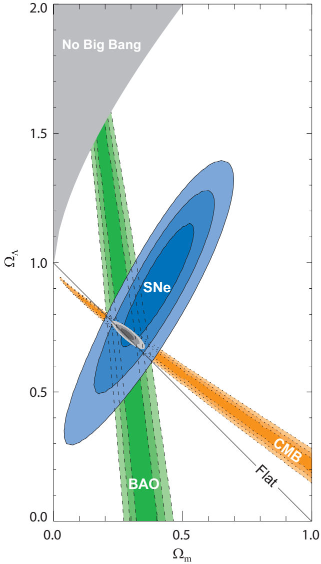
$\lambda = 2\pi/(M_X v_X)$, in a galaxy $v_X \sim 0.5 \cdot 10^{-3} \longrightarrow M_X \gtrsim 3 \cdot 10^{-22}$ eV

for fermions

Pauli blocking: $M_X \gtrsim 750$ eV

$$f(\mathbf{p},\mathbf{x}) = \frac{\rho_X(\mathbf{x})}{M_X} \cdot \frac{1}{\left(\sqrt{2\pi} M_X v_X\right)^3} \cdot e^{-\frac{\mathbf{p}^2}{2M_X^2 v_X^2}} \bigg|_{\mathbf{p}=0} \leq \frac{g_X}{(2\pi)^3}$$

Astrophysical and cosmological data are in agreement



$$\left(\frac{\dot a}{a}\right)^2 = H^2(t) = \frac{8\pi}{3} G\rho_{\text{density}}^{\text{energy}}$$

$$\rho_{\text{density}}^{\text{energy}} = \rho_{\text{radiation}} + \rho_{\text{matter}}^{\text{ordinary}} + \rho_{\text{matter}}^{\text{dark}} + \rho_\Lambda$$

$$\rho_{\text{radiation}} \propto 1/a^4(t) \propto T^4(t)\,, \quad \rho_{\text{matter}} \propto 1/a^3(t)$$

$$\rho_\Lambda = \text{const}$$

$$\frac{3H_0^2}{8\pi G} = \rho_{\text{density}}^{\text{energy}}(t_0) \equiv \rho_c \approx 0.53\times 10^{-5}\,\frac{\text{GeV}}{\text{cm}^3}$$

radiation: $\Omega_\gamma \equiv \frac{\rho_\gamma}{\rho_c} = 0.5\times 10^{-4}$

Baryons (H, He): $\Omega_B \equiv \frac{\rho_B}{\rho_c} = 0.05$

Neutrino: $\Omega_\nu \equiv \frac{\sum \rho_{\nu_i}}{\rho_c} < 0.01$

Dark matter: $\Omega_{DM} \equiv \frac{\rho_{DM}}{\rho_c} = 0.27$

Dark energy: $\Omega_\Lambda \equiv \frac{\rho_\Lambda}{\rho_c} = 0.68$

Determination of $a(t)$ reveals the composition of the present Universe

$\Delta s^2 = c^2\,\Delta t^2 - a^2(t)\,\Delta\vec{x}^2 \;\rightarrow\; ds^2 = g_{\mu\nu}dx^\mu dx^\nu$

Light propagation changes...

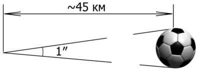
How do we check it?

by measuring distance L to an object!

- Measuring angular size θ of an object of known size d

single-type galaxies

$$\theta = \frac{d}{L}$$



- Measuring angular size $\theta(t)$ corresponding to physical size $d(t)$ with known evolution

– BAO in galaxy distribution
– lensing of CMB anisotropy

$$\theta(t) = \frac{d(t)}{L}$$

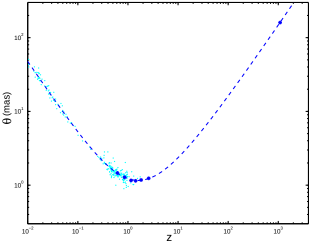
- Measuring brightness J of an object of known luminosity F

"standard candles"

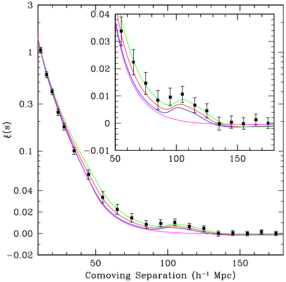
$$J = \frac{F}{4\pi L^2}$$

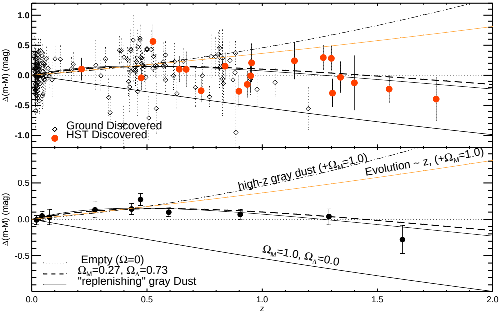
In the expanding Universe all these laws get modified

Results of distance measurements



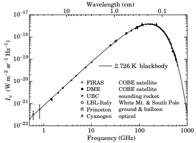
$$\Delta(m-M) = 5\log\frac{r_{ph}}{r_{ph}(\Omega_c = 0.8, \Omega_M = 0.2)}$$

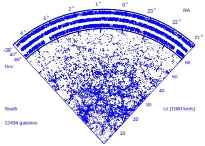


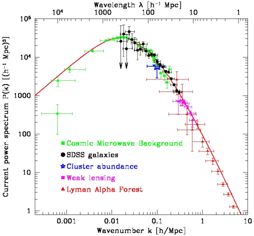


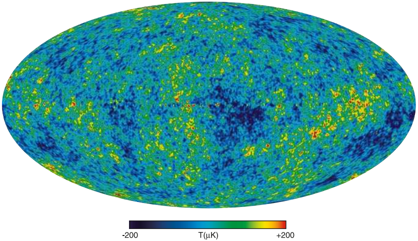
Key observable: matter perturbations

- CMB is isotropic, but “up to corrections, of course...”
  1. Earth movement with respect to CMB
     $$\frac{\Delta T_{\text{dipole}}}{T} \sim 10^{-3}$$
  2. More complex anisotropy!
     $$\frac{\Delta T}{T} \sim 10^{-4} - 10^{-5}$$
- There were matter inhomogenities $\Delta\rho/\rho \sim \Delta T/T$ at the stage of recombination ($e + p \to \gamma + H^*$)
- Jeans instability in the system of gravitating particles at rest $\Longrightarrow$ $\Delta\rho/\rho \nearrow$ $\Longrightarrow$ galaxies (CDM halos)









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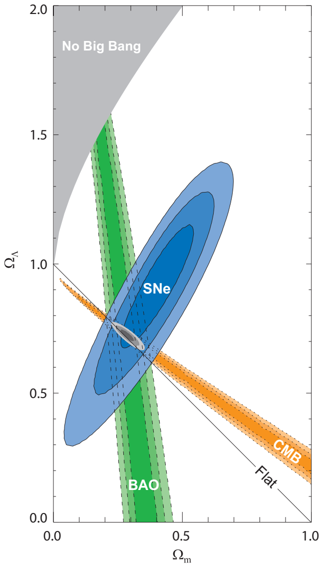
Outline

Dark Energy: nonclumping matter?



- estimates of Matter contribution confined in galaxies and clusters
  $\rho_c - \rho_M \neq 0$ but the Universe is flat, so $\rho_{curv} \simeq 0$
- corrections to the Hubble law : red shift – brightness curves for standard candles (SN Ia)
- The age of the Universe
- CMB anisotropy, large scale structures (galaxy clusters formation), etc

$$\rho_\Lambda = 0.68\rho_c$$

$$\rho_\Lambda \sim 10^{-5}\ \text{GeV/cm}^3 \sim \left(10^{-11.5}\ \text{GeV}\right)^4$$

Dark Energy: all evidences are from cosmology

Working hypothesis is cosmological constant $\Lambda \approx \left(2.5 \times 10^{-3}\,\text{eV}\right)^4$:
$p = w(t)\rho$, $w = \text{const} = -1$, $\rho = \Lambda$

$$S_\Lambda = -\Lambda \int d^4x \sqrt{-\det g_{\mu\nu}}$$

both parts contribute

$$S_{\text{grav}} = -\frac{1}{16\pi G}\int d^4x \sqrt{-\det g_{\mu\nu}}\, R\,,$$
$$S_{\text{matter}} = \int d^4x \sqrt{-\det g_{\mu\nu}} \left(\frac{1}{2} g^{\lambda\rho}\partial_\lambda\phi\partial_\rho\phi - V(\phi)\right)$$

natural values

$$\Lambda_{\text{grav}} \sim 1/G^2 \sim \left(10^{19}\,\text{GeV}\right)^4 , \quad \Lambda_{\text{matter}} \sim V(\phi_{vac}) \sim (100\,\text{GeV})^4, (100\,\text{MeV})^4, \ldots$$

Why Λ is small? Why $\Lambda \sim \rho_{\text{matter}}$? Why $\rho_B \sim \rho_{DM} \sim \rho_\Lambda$ today?

$$\left(\frac{\dot{a}}{a}\right)^2 = H^2(t) = \frac{8\pi}{3} G \rho_{\text{density}}^{\text{energy}}$$

$$\rho_{\text{density}}^{\text{energy}} = \rho_{\text{radiation}} + \rho_{\text{matter}}^{\text{ordinary}} + \rho_{\text{matter}}^{\text{dark}} + \rho_\Lambda$$

$$\rho_{\text{radiation}} \propto 1/a^4(t) \propto T^4(t)\,, \quad \rho_{\text{matter}} \propto 1/a^3(t)$$

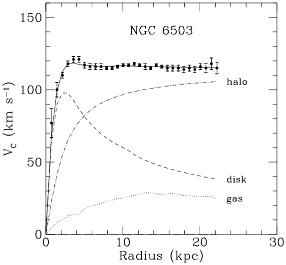
$$\rho_\Lambda = \text{const}$$

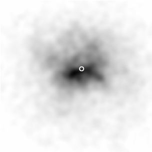
Why do we think it is most probably new particle physics
(new gravity if any is not enough) ?

DM at various spatial scales, BAU requires baryon number violation

Universe content from astrophysics

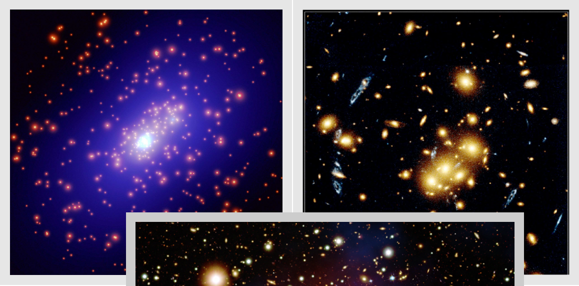
Rotational curves





X-rays from centers of galaxy clusters

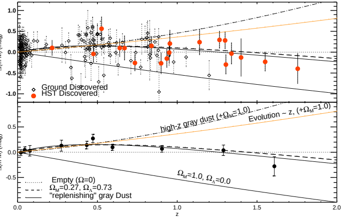
Gravitational lensing



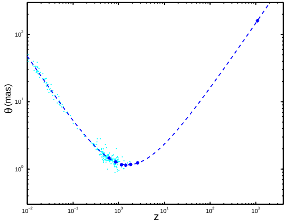
“Bullet” cluster

Universe content from cosmology

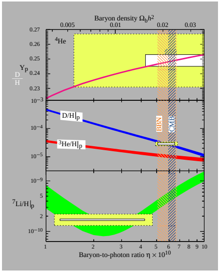
Standard candles

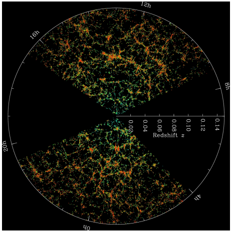


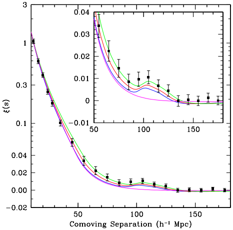
Angular distance

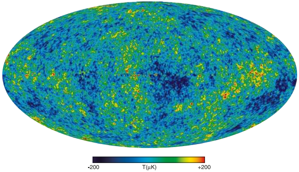


Nucleosynthesis





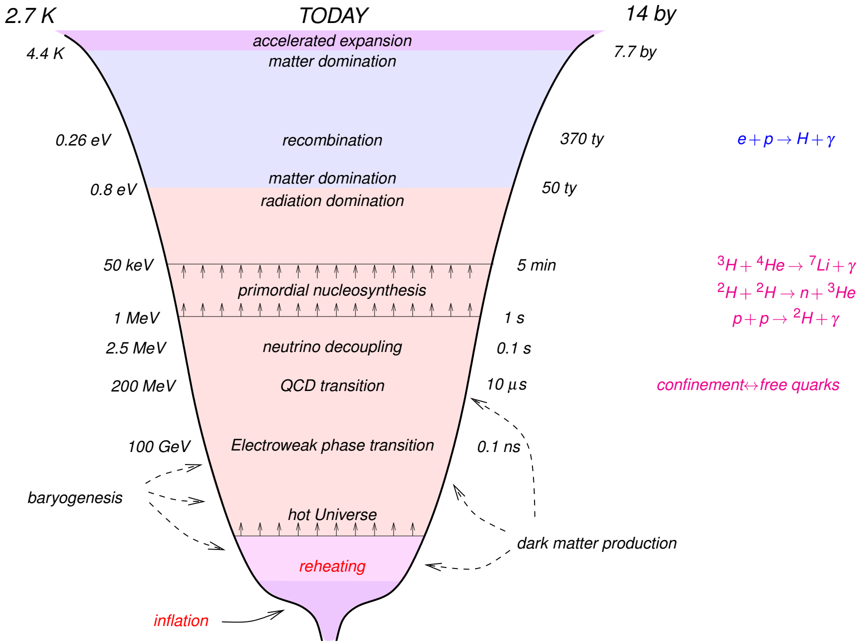




Large Scale Structures

Baryon acoustic oscillations

CMB anisotropy


2.7 K
TODAY
14 by
accelerated expansion
4.4 K
matter domination
7.7 by
0.26 eV
recombination
370 ty
$e + p \to H + \gamma$
matter domination
0.8 eV
radiation domination
50 ty
50 keV
5 min
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primordial nucleosynthesis
$^2H + {}^2H \to n + {}^3He$
1 MeV
1 s
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2.5 MeV
neutrino decoupling
0.1 s
200 MeV
QCD transition
10 µs
confinement↔free quarks
100 GeV
Electroweak phase transition
0.1 ns
baryogenesis
hot Universe
dark matter production
reheating
inflation

Friedmann equation for the present Universe

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3} G\left(\rho_{\mathrm{M}} + \rho_{rad} + \rho_\Lambda + \rho_{\mathrm{curv}}\right)$$

$$\frac{8\pi}{3} G\rho_{\mathrm{curv}} = -\frac{\varkappa}{a^2}\,, \quad \rho_c \equiv \frac{3}{8\pi G} H_0^2$$

$$\rho_c = \rho_{\mathrm{M},0} + \rho_{rad,0} + \rho_{\Lambda,0} = \rho_c = 0.53 \cdot 10^{-5}\,\frac{\mathrm{GeV}}{\mathrm{cm}^3}\,,$$

$$\Omega_X \equiv \frac{\rho_{X,0}}{\rho_c}$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3} G\rho_c \left[\Omega_{\mathrm{M}} \left(\frac{a_0}{a}\right)^3 + \Omega_{rad} \left(\frac{a_0}{a}\right)^4 + \Omega_\Lambda\right]$$

Outline

FLRW metric $g_{\mu\nu}$

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = dt^2 - a^2(t) dl^2 = dt^2 - a^2(t) \gamma_{ij} dx^i dx^j \,,$$

$$H(t) = \frac{\dot{a}(t)}{a(t)}$$

Special frame: different parts look similar
Also this is comoving frame: world lines of particles at rest are geodesics,

$$\frac{du^\mu}{ds} + \Gamma^\mu_{\nu\lambda} u^\nu u^\lambda = 0$$

$$\gamma_{ij} \approx \delta_{ij}$$

Photons in the expanding Universe

$$S = -\frac{1}{4}\int d^4x\sqrt{-g}g^{\mu\nu}g^{\lambda\rho}F_{\mu\lambda}F_{\nu\rho}$$

$dt = ad\eta$ conformally flat metric

$$ds^2 = dt^2 - a^2(t)\delta_{ij}dx^i dx^j \quad \longrightarrow \quad ds^2 = a^2(\eta)[d\eta^2 - \delta_{ij}dx^i dx^j]$$

$$S = -\frac{1}{4}\int d^4x\, \eta^{\mu\nu}\eta^{\lambda\rho}F_{\mu\lambda}F_{\nu\rho}\,, \qquad A_\mu^{(\alpha)} = e_\mu^{(\alpha)}\mathrm{e}^{ik\eta - i\mathbf{kx}}\,, \quad k = |\mathbf{k}|$$

$$\Delta x = 2\pi/k\,, \quad \Delta\eta = 2\pi/k$$

$$\lambda(t) = a(t)\Delta x = 2\pi\frac{a(t)}{k}\,, \quad T = a(t)\Delta\eta = 2\pi\frac{a(t)}{k}$$

Redshift and the Hubble law $\lambda_0 = \lambda_i \frac{a_0}{a(t_i)} \equiv \lambda_i (1 + z(t_i))$

$$\mathbf{p}(t) = \frac{\mathbf{k}}{a(t)}\ , \quad \omega(t) = \frac{k}{a(t)}$$

for not very distant objects — $1\ \text{pc} \approx 3\ \text{ly}$

$$a(t_i) = a_0 - \dot{a}(t_0)(t_0 - t_i) \quad \longrightarrow \quad a(t_i) = a_0 [1 - H_0(t_0 - t_i)]$$

$$z(t_i) = H_0(t_0 - t_i) = H_0 r\ , \qquad z \ll 1$$

$$H_0 = h \cdot 100\ \frac{\text{km}}{\text{s} \cdot \text{Mpc}}\ , \qquad h \approx 0.68$$

similar reddening for other relativistic particles (small H, $\dot{H}$, etc.)

$\mathbf{p} = \frac{\mathbf{k}}{a(t)}$ is true for massive particles as well

Gas of free particles in the expanding Universe

homogeneous gas $dN = f(\mathbf{p}, t) d^3\mathbf{X} d^3\mathbf{p}$

in comoving coordinates:

$$d^3\mathbf{x} = \text{const}\,, \quad d^3\mathbf{k} = \text{const}\,, \quad f(k) = \text{const}$$

$$f(k) d^3\mathbf{x} d^3\mathbf{k} = \text{const}$$

comoving volume equals physical volume

$$d^3\mathbf{x} d^3\mathbf{k} = d^3(a\mathbf{x}) d^3\left(\frac{\mathbf{k}}{a}\right) = d^3\mathbf{X} d^3\mathbf{p}$$

$$f(\mathbf{p}, t) = f(\mathbf{k}) = f[a(t) \cdot \mathbf{p}]\,.$$

$$t = t_i\ :\quad f_i(\mathbf{p}) \ \longrightarrow\ f(\mathbf{p}, t) = f_i\left(\frac{a(t)}{a(t_i)}\mathbf{p}\right)$$

Massless bosons (photons) fermions

$$f_i(\mathbf{p}) = f_{\rm Pl}\left(\frac{|\mathbf{p}|}{T_i}\right) = \frac{1}{(2\pi)^3}\frac{1}{e^{|\mathbf{p}|/T_i} \mp 1}$$

$$f(\mathbf{p},t) = f_{\rm Pl}\left(\frac{a(t)|\mathbf{p}|}{a_i T_i}\right) = f_{\rm Pl}\left(\frac{|\mathbf{p}|}{T_{eff}(t)}\right)$$

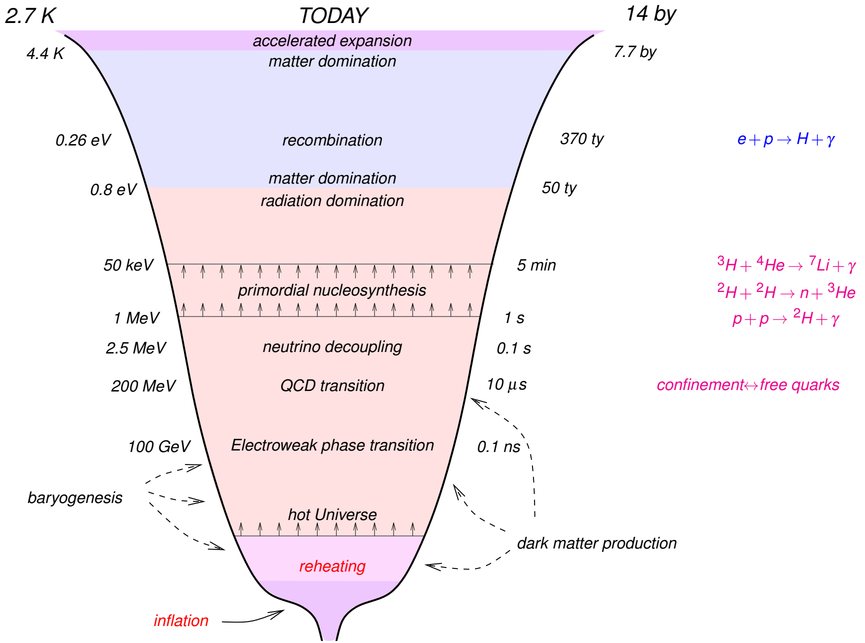
$$T_{eff}(t) = \frac{a_i}{a(t)} T_i$$

decoupling at $T \gg m$: neutrinos, hot(warm) dark matter

decoupling at $T \ll m$: $f(\mathbf{p}) = \frac{1}{(2\pi)^3}\exp\left(-\frac{m-\mu_i}{T_i}\right)\exp\left(-\frac{a^2(t)\mathbf{p}^2}{2ma_i^2 T_i}\right)$

$$f(\mathbf{p},t) = \frac{1}{(2\pi)^3}\exp\left(-\frac{m-\mu_{eff}}{T_{eff}}\right)\exp\left(-\frac{\mathbf{p}^2}{2mT_{eff}}\right)$$

$$T_{eff}(t) = \left(\frac{a_i}{a(t)}\right)^2 T_i\,, \qquad \frac{m-\mu_{eff}(t)}{T_{eff}} = \frac{m-\mu_i}{T_i}$$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Electroweak phase transition
0.1 ns
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Outline

Einstein equations

$T_{\mu\nu}$: macroscopic description $\qquad \frac{1}{2}\int d^4x\sqrt{-g}\,T_{\mu\nu}\delta g^{\mu\nu}$

$T_{\mu\nu} = (p+\rho)u_\mu u_\nu - g_{\mu\nu}p$ $\qquad$ ideal fluid with $\rho(t)$ and $p(t)$

in the comoving frame $u^0 = 1$, $\mathbf{u} = 0$ $\qquad$ (almost) always works

$$T_\mu^\nu = diag(\rho, \, -p)$$

$$ds^2 = dt^2 - a^2(t)\gamma_{ij}dx^i dx^j \; ,$$

$$S_{EH} = -\frac{1}{16\pi G}\int d^4x\sqrt{-g}R \; : \quad R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G T_{\mu\nu}$$

$$(00): \quad \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3}G\rho - \frac{\varkappa}{a^2}$$

Friedmann equation (00) : $\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3}G\rho - \frac{\varkappa}{a^2}$

$$\nabla_\mu T^{\mu 0} = 0 \quad \longrightarrow \quad \dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = 0$$

the equation of state

$$p = p(\rho)$$

many-component liquid,
in case of thermal equilibrium

other equations

$$-3d(\ln a) = \frac{d\rho}{p+\rho} = d(\ln s)$$

entropy of cosmic primordial plasma is conserved in a comoving frame

$$sa^3 = \text{const}$$

Examples of cosmological solutions

$$\varkappa = 0 \qquad \left(\frac{\dot a}{a}\right)^2 = \frac{8\pi}{3} G\rho$$

dust: $p = 0$ — singular at $t = t_s$

$$\rho = \frac{\text{const}}{a^3}\,, \quad a(t) = const\cdot(t - t_s)^{2/3}\,, \quad \rho(t) = \frac{\text{const}}{(t-t_s)^2}$$

$$t_s = 0\,, \quad H(t) = \frac{\dot a}{a}(t) = \frac{2}{3t}\,, \quad \rho = \frac{3}{8\pi G} H^2 = \frac{1}{6\pi G}\frac{1}{t^2}$$

the Universe is too young

$$t_0 = \frac{2}{3H_0} = 0.9\times 10^{10}\ \text{yr} \quad (h = 0.7)$$

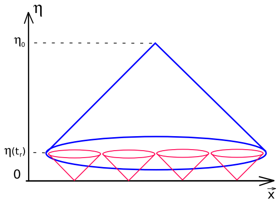
Cosmological (particle) horizon $l_H(t)$

distance covered by photons emitted at $t = 0$

the size of causally-connected region — the size of the visible part of the Universe

in conformal coordinates:
$ds^2 = 0 \longrightarrow |d\mathbf{x}| = d\eta$
coordinate size of the horizon equals
$\eta(t) = \int d\eta$

$$l_H(t) = a(t)\eta(t) = a(t)\int_0^t \frac{dt'}{a(t')}$$



dust

$$l_H(t) = 3t = \frac{2}{H(t)}\,, \quad l_{H,0} = 2.6\times 10^{28}\text{cm} \quad (h = 0.7)$$

Examples of cosmological solutions

radiation: $p = \frac{1}{3}\rho$ singular at $t = t_s$

$$\rho = \frac{\text{const}}{a^4}\,, \quad a(t) = const \cdot (t - t_s)^{1/2}\,, \quad \rho(t) = \frac{\text{const}}{(t - t_s)^2}$$

$$t_s = 0\,, \quad H(t) = \frac{\dot{a}}{a}(t) = \frac{1}{2t}\,, \quad \rho = \frac{3}{8\pi G}H^2 = \frac{3}{32\pi G}\frac{1}{t^2}$$

$$l_H(t) = a(t)\int_0^t \frac{dt'}{a(t')} = 2t = \frac{1}{H(t)}\,.$$

In case of thermal equilibrium $T = \text{const}/a$

$$\rho_b = \frac{\pi^2}{30}g_b T^4\,, \quad \rho_f = \frac{7}{8}\frac{\pi^2}{30}g_f T^4$$

$$\rho = \frac{\pi^2}{30}g_* T^4\,, \quad g_* = \sum_b g_b + \frac{7}{8}\sum_f g_f = g_*(T)$$

Evolution of energy and entropy densities

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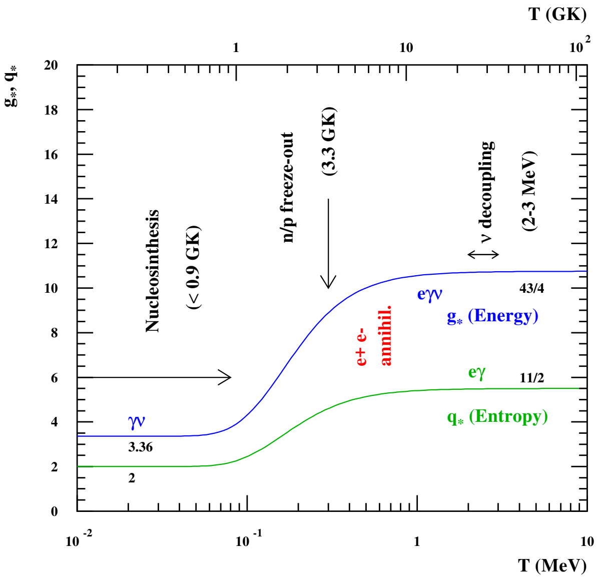


Examples of cosmological solutions

vacuum: $T_{\mu\nu} = \rho_{vac}\eta_{\mu\nu}$ $\quad p = -\rho$

$$S_G = -\frac{1}{16\pi G}\int R\sqrt{-g}d^4x\,, \quad S_\Lambda = -\Lambda\int\sqrt{-g}d^4x\,.$$

$$a = \text{const}\cdot \mathrm{e}^{H_{\text{dS}}t}\,, \quad H_{\text{dS}} = \sqrt{\frac{8\pi}{3}G\rho_{vac}}$$

de Sitter space: space-time of constant curvature

$$ds^2 = dt^2 - \mathrm{e}^{2H_{dS}t}d\mathbf{x}^2$$

$\ddot{a} > 0$, no initial singularity

$ds^2 = dt^2 - e^{2H_{dS}t} d\mathbf{x}^2$

no cosmological horizon: $l_{\rm H}(t) = e^{H_{dS}t} \int_{-\infty}^{t} dt' e^{-H_{dS}t'} = \infty$

de Sitter (events) horizon ($\mathbf{x} = 0$, t):

from which distance $l(t)$ one can detect light emitted at t?

in conformal coordinates: $ds^2 = 0 \longrightarrow |d\mathbf{x}| = d\eta$

coordinate size: $\eta(t \to \infty) - \eta(t) = \int_t^\infty \frac{dt'}{a(t')}$

physical size: $l_{dS} = a(t) \int_t^\infty \frac{dt'}{a(t')} = \frac{1}{H_{dS}}$

observer will never be informed what happens at distances larger than $l_{dS} = H_{dS}^{-1}$

Our future? with $H_{dS} = 0.8 \times H_0$

Outline

Friedmann equation for the present Universe

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3}G\left(\rho_{\mathrm{M}} + \rho_{rad} + \rho_{\Lambda} + \rho_{\mathrm{curv}}\right)$$

$$\frac{8\pi}{3}G\rho_{\mathrm{curv}} = -\frac{\varkappa}{a^2}\,, \quad \rho_c \equiv \frac{3}{8\pi G}H_0^2$$

$$\rho_c = \rho_{\mathrm{M},0} + \rho_{rad,0} + \rho_{\Lambda,0} = \rho_c = 0.52\cdot 10^{-5}\,\frac{\mathrm{GeV}}{\mathrm{cm}^3}\,, \qquad \text{for } \; h = 0.7$$

$$\Omega_X \equiv \frac{\rho_{X,0}}{\rho_c}$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3}G\rho_c\left[\Omega_{\mathrm{M}}\left(\frac{a_0}{a}\right)^3 + \Omega_{rad}\left(\frac{a_0}{a}\right)^4 + \Omega_{\Lambda} + \Omega_{curv}\left(\frac{a_0}{a}\right)^2\right]$$

Homogeneous and isotropic 3d manifolds

$$dl^2 = d\rho^2 + r^2(\rho)(d\theta^2 + \sin^2\theta d\phi^2)$$

$$r(\rho) = \begin{cases} R\sin(\rho/R)\,, & \text{3-sphere} \\ \rho\,, & \text{3-plane} \\ R\sinh(\rho/R)\,, & \text{3-hyperboloid} \end{cases}$$

ρ is a geodesic distance; $S = 4\pi r^2(\rho)$; $\Delta\theta = \frac{l}{r(\rho)}$

$$d\rho^2 = \frac{dr^2}{\cosh^2\frac{\rho}{R}} = \frac{dr^2}{1+\frac{r^2}{R^2}} \qquad d\rho^2 = \frac{dr^2}{\cos^2\frac{\rho}{R}} = \frac{dr^2}{1-\frac{r^2}{R^2}}$$

$$dl^2 = \frac{dr^2}{1-\varkappa\frac{r^2}{R^2}} + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

Brightness–redshift dependence in the Universe

$$ds^2 = dt^2 - a^2(t)\left[d\chi^2 + \sinh^2\chi\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)\right]$$

coordinate distance $\rho/R \to \chi = \int_{t_i}^{t_0} \frac{dt}{a(t)}$ $\qquad z(t) = \frac{a_0}{a(t)} - 1$

$$\chi(z) = \int_0^z \frac{dz'}{a_0 H_0} \frac{1}{\sqrt{\Omega_M (z'+1)^3 + \Omega_\Lambda + \Omega_{curv}(z'+1)^2}}$$

$$a_0^2 H_0^2 \Omega_{curv} = 1\,, \quad \Omega_M + \Omega_\Lambda + \Omega_{curv} = 1$$

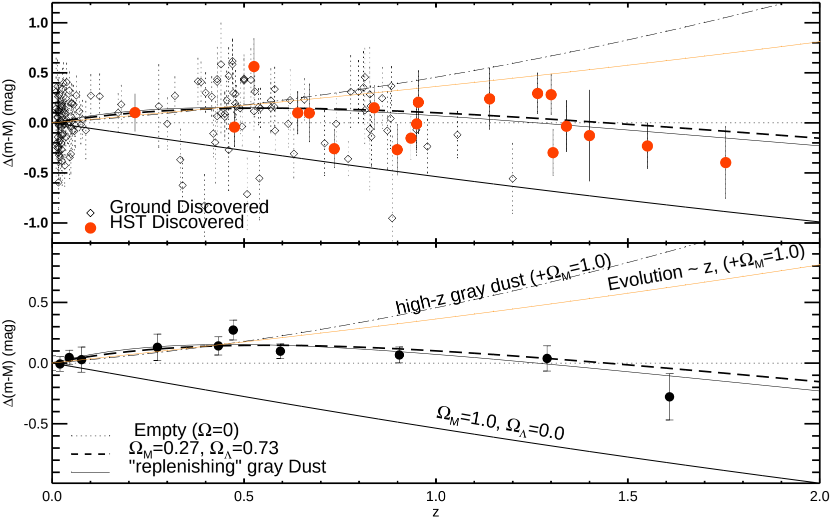
$$S(z) = 4\pi r^2(z)\,, \qquad r(z) = a_0 \sinh\chi(z)$$

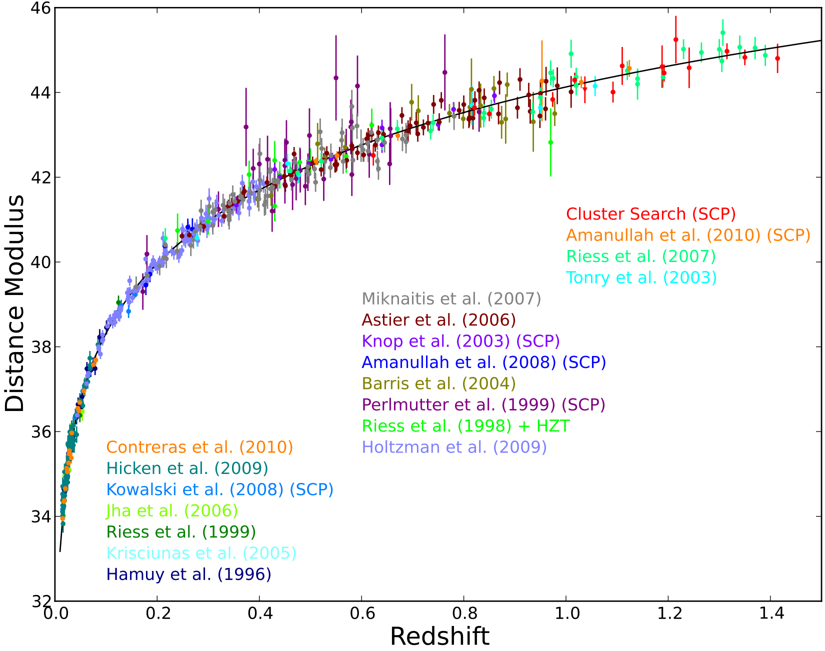
detector: $N_\gamma \propto S^{-1}$, $\omega = \omega_i/(1+z)$, $dt_0 = (1+z)dt_i$
hence the brightness (energy flux measured by a detector) is

$$J = \frac{L}{(1+z)^2 S(z)} \equiv \frac{L}{4\pi r_{ph}^2}\,, \quad r_{ph} = (1+z)\cdot r(z)$$

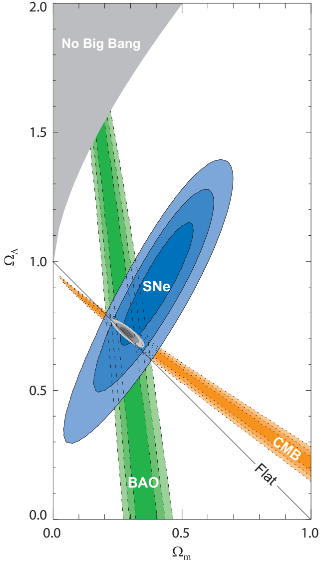
Brightness–redshift dependence: SNe Ia

$$\Delta(m-M) = 5\log\frac{r_{ph}}{r_{ph}(\Omega_c = 0.8, \Omega_M = 0.2)}$$





Distance Modulus
Redshift
46
44
42
40
38
36
34
32
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
Cluster Search (SCP)
Amanullah et al. (2010) (SCP)
Riess et al. (2007)
Tonry et al. (2003)
Miknaitis et al. (2007)
Astier et al. (2006)
Knop et al. (2003) (SCP)
Amanullah et al. (2008) (SCP)
Barris et al. (2004)
Perlmutter et al. (1999) (SCP)
Riess et al. (1998) + HZT
Holtzman et al. (2009)
Contreras et al. (2010)
Hicken et al. (2009)
Kowalski et al. (2008) (SCP)
Jha et al. (2006)
Riess et al. (1999)
Krisciunas et al. (2005)
Hamuy et al. (1996)


No Big Bang
SNe
CMB
Flat
BAO
Ω_Λ
Ω_m
2.0
1.5
1.0
0.5
0.0
0.0
0.5
1.0

Last scattering: $\gamma e \to \gamma e$

$$\sigma_{\mathrm{T}} = \frac{8\pi}{3}\frac{\alpha^2}{m_e^2} \approx 0.67\cdot 10^{-24}\ \mathrm{cm}^2\,, \qquad \tau_\gamma = \frac{1}{\sigma_{\mathrm{T}}\cdot n_e(T)}$$

last scattering: $\tau_\gamma(T_f) \simeq H^{-1}(T_f) \simeq t_f$

$$T_f = 0.26\ \mathrm{eV}\,, \quad z = 1100\,, \quad t_f = 370\,000\ \mathrm{yr}$$

for general processes one should solve kinetic equations

$$\frac{dn_{X_i}}{dt} + 3Hn_{X_i} = \int (production - destruction)$$

Boltzmann equation in a comoving volume: $\frac{d}{dt}\left(n a^3\right) = a^3 \int \ldots$

Recombination: horizon

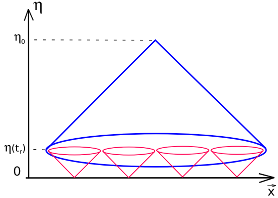
matter domination: $l_{\mathrm{H},r} = 2H_r^{-1}$

$$H_r^2 = \frac{8\pi}{3} G\rho_{\mathrm{M}}(t_r) = \frac{8\pi}{3} G\rho_{\mathrm{M},0}\left(\frac{a_0}{a_r}\right)^3 = \frac{8\pi}{3} G\rho_c \Omega_{\mathrm{M},0}(1+z_r)^3 \, .$$

at recombination: $l_{\mathrm{H},r} = \frac{2}{H_0\sqrt{\Omega_{\mathrm{M}}}} \frac{1}{(1+z_r)^{3/2}}$

today: $l_{\mathrm{H},r}(t_0) = l_{\mathrm{H},r} \times \frac{a_0}{a_r} = \frac{2}{H_0\sqrt{\Omega_{\mathrm{M}}}} \frac{1}{\sqrt{1+z_r}}$

$$\frac{l_{H_0}}{l_{\mathrm{H},r}(t_0)} \sim \sqrt{1+z_r} \simeq 30$$



Recombination: angle

angular distance: $$d_{ph} = r_a(z)\,\Delta\theta$$

$$\chi_r = \int_{t_r}^{t_0} \frac{dt}{a(t)}\,, \quad \Delta\theta_r = \frac{l_{H,r}}{r_a(z_r)}$$

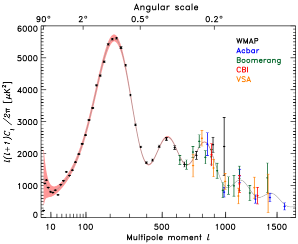
$$d_{conf} = \sinh\chi_r\,\Delta\theta$$

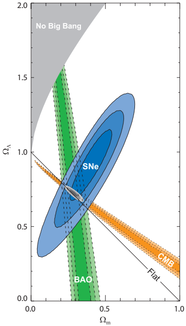
$$r_a(z_r) = (1+z_r)^{-1}\cdot a_0 \cdot \sinh\chi_r$$

$$\Delta\theta_r = \frac{1}{\sqrt{z_r+1}}\,, \quad \Omega_{curv} = \Omega_\Lambda = 0\,.$$

$$\Delta\theta_r = \frac{1}{\sqrt{z_r+1}}\,\frac{2\sqrt{\Omega_{curv}/\Omega_M}}{\sinh\left(2\sqrt{\Omega_{curv}/\Omega_M}\,I\right)}\,.$$

$$I = \int_0^1 \frac{dy}{\sqrt{1+\frac{\Omega_\Lambda}{\Omega_M}y^6}}$$

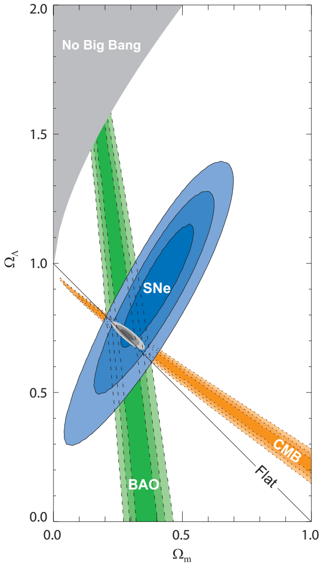


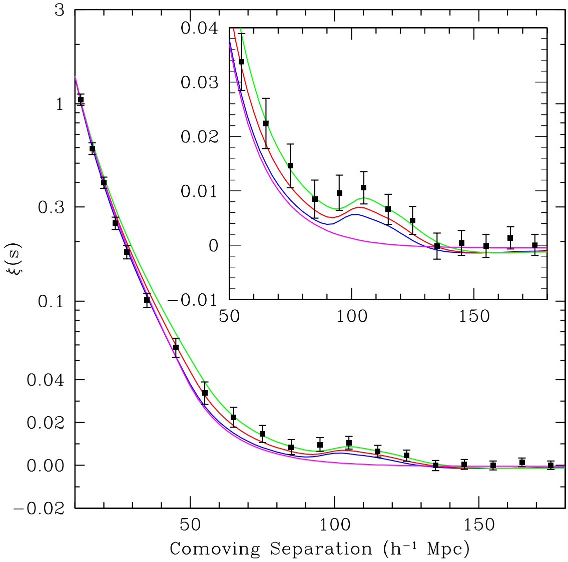


Acoustic oscillations in relativistic plasma:
What matters is the sound horizon:

$$l_{s,r} = l_{H,r}\cdot v_s \approx l_{H,r}/\sqrt{3}$$

Then $$\Delta\theta_{r,s} = \frac{1}{\sqrt{3}}\,\frac{1}{\sqrt{1+z}} \times \frac{180^\circ}{\pi} \simeq 1^\circ$$





$$110/0.7\,\text{Mpc} \simeq l_{H,r}(t_0) \times \sqrt{v_s^2} \simeq l_{H_0}/\sqrt{3}/\sqrt{1+z_r}$$