Inflation and reheating in the early Universe
Lecture #2
Introduction: observables in Hot Big Bang Theory

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“Quantum fields:
from gravity and cosmology
to physics of condensed matter”
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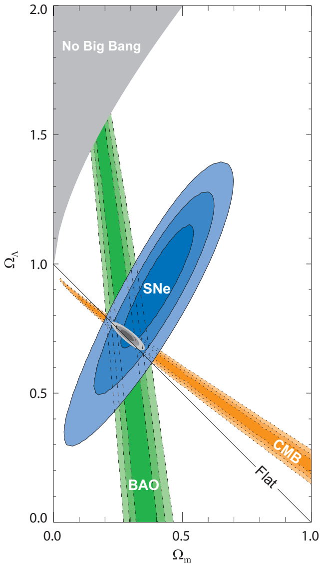
Astrophysical and cosmological data are in agreement



$$\left(\frac{\dot{a}}{a}\right)^2 = H^2(t) = \frac{8\pi}{3} G \rho^{\text{energy}}_{\text{density}}$$

$$\rho^{\text{energy}}_{\text{density}} = \rho_{\text{radiation}} + \rho^{\text{ordinary}}_{\text{matter}} + \rho^{\text{dark}}_{\text{matter}} + \rho_\Lambda$$

$$\rho_{\text{radiation}} \propto 1/a^4(t) \propto T^4(t)\,, \quad \rho_{\text{matter}} \propto 1/a^3(t)$$

$$\rho_\Lambda = \text{const}$$

$$\frac{3H_0^2}{8\pi G} = \rho^{\text{energy}}_{\text{density}}(t_0) \equiv \rho_c \approx 0.53 \times 10^{-5}\,\frac{\text{GeV}}{\text{cm}^3}$$

radiation: $\Omega_\gamma \equiv \frac{\rho_\gamma}{\rho_c} = 0.5 \times 10^{-4}$

Baryons (H, He): $\Omega_B \equiv \frac{\rho_B}{\rho_c} = 0.05$

Neutrino: $\Omega_\nu \equiv \frac{\sum \rho_{\nu_i}}{\rho_c} < 0.01$

Dark matter: $\Omega_{DM} \equiv \frac{\rho_{DM}}{\rho_c} = 0.27$

Dark energy: $\Omega_\Lambda \equiv \frac{\rho_\Lambda}{\rho_c} = 0.68$

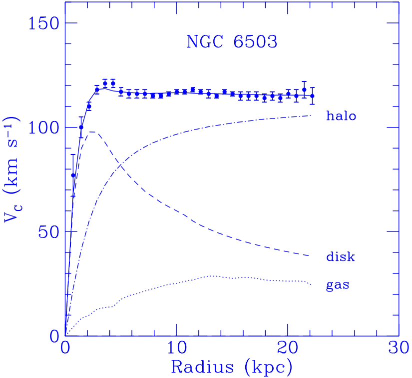
Why do we need dark components (within GR)?

- Astrophysical data favor Dark Matter
  - Observations in galaxies
  - Observations in galaxy clusters
- Cosmological data favor Dark Matter and Dark Energy
  - Observation of objects at cosmological distances (far=early)
  - Baryonic Aciustic (Sakharov) Oscillations (BAO) in two-point galaxy correlation function
  - Evolution of galaxy clusters in the Universe
  - Anisotropy of Cosmic Microwave Background (CMB)

Galactic dark halos: flat rotation curves

$$v(R) = \sqrt{G\frac{M(R)}{R}}$$

$$M(R) = 4\pi \int_0^R \rho(r) r^2 dr$$



observations: $v(R) \simeq \text{const}$

visible matter:

internal regions $v(R) \propto \sqrt{R}$

external ("empty") regions $v(R) \propto 1/\sqrt{R}$

Matter distribution in the Milky Way

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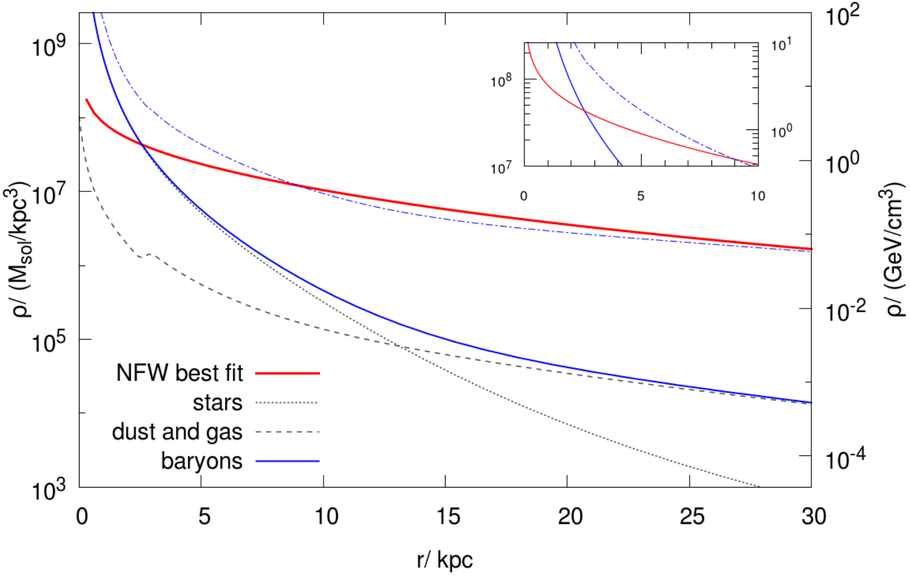


Dark Matter in clusters

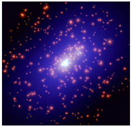
X-rays from hot gas in clusters

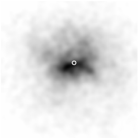
$$\frac{dP}{dR} = -\mu n_e(R) m_p \frac{GM(R)}{R^2}\,, \quad M(R) = 4\pi \int_0^R \rho(r) r^2 dr\,, \quad P(R) = n_e(R) T_e(R)$$

galaxies in clusters — virial theorem

$$U + 2E_k = 0$$

$$3M\langle v_r^2 \rangle = G\frac{M^2}{R}$$

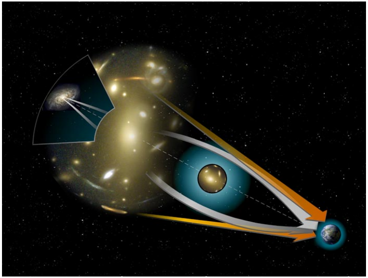




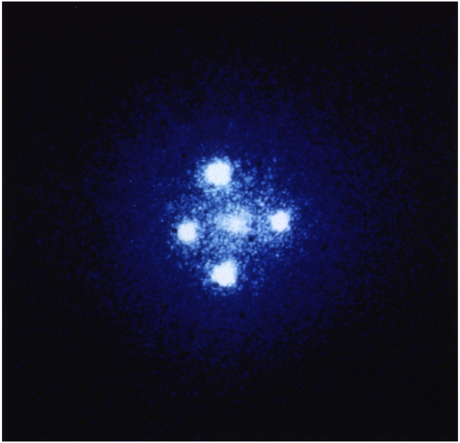
Milky Way: Virgo infall

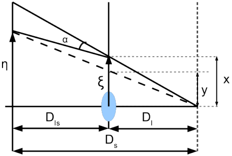
Gravitational lensing in GR:

$$\alpha = 4GM/(c^2 b)$$



Einstein Cross





$$\vec{\eta} = \frac{D_s}{D_l}\vec{\xi} - D_{ls}\vec{\alpha}\left(\vec{\xi}\right)$$

common lens with specific refraction coefficient

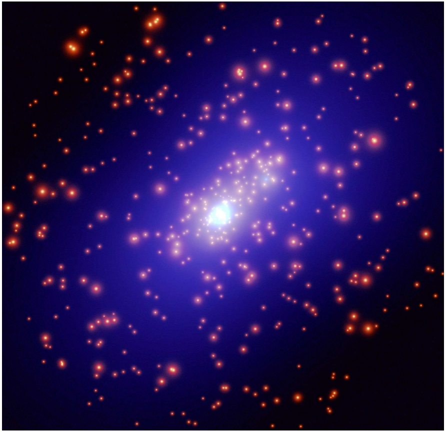
$$\vec{\alpha}\left(\vec{\xi}\right) = \frac{4G}{c}\int \frac{\vec{\xi}-\vec{\xi}'}{\left|\xi-\vec{\xi}'\right|^2} d^2\xi' \int \rho\left(\vec{\xi}',z\right) dz$$

source: quasar $D_s = 2.4$ Gpc

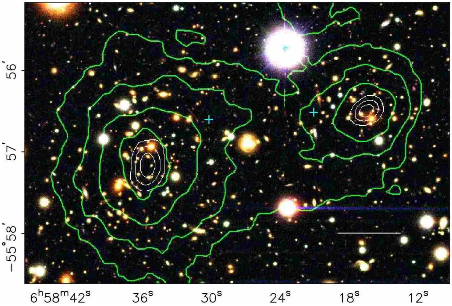
lens: galaxy $D_l = 120$ Mpc

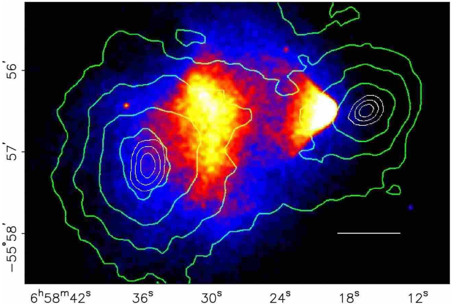
Dark Matter in clusters

gravitational lensing $\rho_B \approx 0.25 \rho_{DM}$



Colliding clusters (Bullet clusters 1E0657-558)





gravitational lensing

Observations in X-rays

$M \simeq 10 \times m$

scale is 200 kpc

clusters are at 1.5 Gpc

Dark Matter Properties $p = 0$

(If) particles:

1. stable on cosmological time-scale
2. nonrelativistic long before RD/MD-transition (either Cold or Warm, $v_{RD/MD} \lesssim 10^{-3}$)
3. (almost) collisionless
4. (almost) electrically neutral

If were in thermal equilibrium: $M_X \gtrsim 1$ keV

If not: for bosons

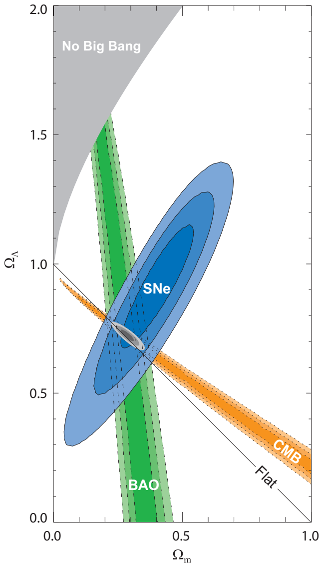
$\lambda = 2\pi/(M_X v_X)$, in a galaxy $v_X \sim 0.5 \cdot 10^{-3} \longrightarrow M_X \gtrsim 3 \cdot 10^{-22}$ eV

for fermions

Pauli blocking: $M_X \gtrsim 750$ eV

$$f(\mathbf{p},\mathbf{x}) = \frac{\rho_X(\mathbf{x})}{M_X} \cdot \frac{1}{\left(\sqrt{2\pi} M_X v_X\right)^3} \cdot e^{-\frac{\mathbf{p}^2}{2M_X^2 v_X^2}} \Bigg|_{\mathbf{p}=0} \leq \frac{g_X}{(2\pi)^3}$$

Astrophysical and cosmological data are in agreement



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Determination of $a(t)$ reveals the composition of the present Universe

$\Delta s^2 = c^2 \Delta t^2 - a^2(t) \Delta \vec{x}^2 \;\; \rightarrow \;\; ds^2 = g_{\mu\nu} dx^\mu dx^\nu$

Light propagation changes...

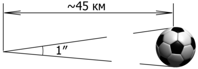
How do we check it?

by measuring distance L to an object!

- Measuring angular size θ of an object of known size d

single-type galaxies

$$\theta = \frac{d}{L}$$



- Measuring angular size $\theta(t)$ corresponding to physical size $d(t)$ with known evolution

– BAO in galaxy distribution

– lensing of CMB anisotropy

$$\theta(t) = \frac{d(t)}{L}$$

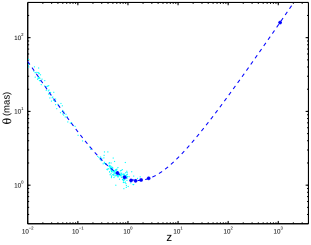
- Measuring brightness J of an object of known luminosity F

"standard candles"

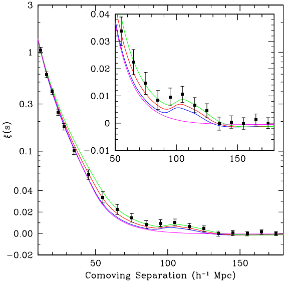
$$J = \frac{F}{4\pi L^2}$$

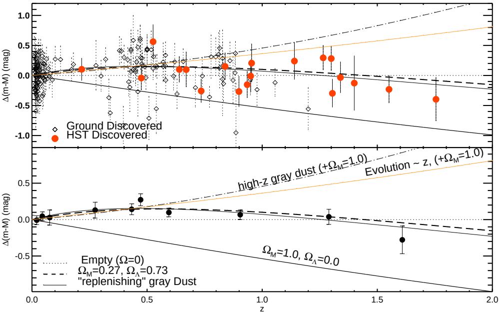
In the expanding Universe all these laws get modified

Results of distance measurements



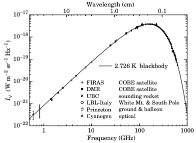
$$\Delta(m-M) = 5\log\frac{r_{ph}}{r_{ph}(\Omega_c=0.8,\Omega_M=0.2)}$$

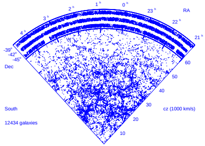


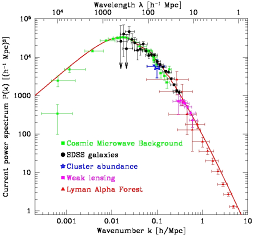


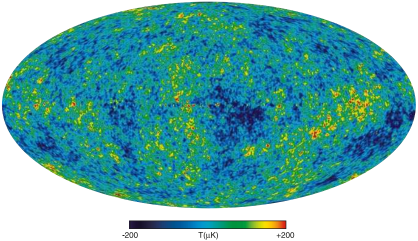
Key observable: matter perturbations

- CMB is isotropic, but “up to corrections, of course...”
  1. Earth movement with respect to CMB
     $$\frac{\Delta T_{\text{dipole}}}{T} \sim 10^{-3}$$
  2. More complex anisotropy!
     $$\frac{\Delta T}{T} \sim 10^{-4} - 10^{-5}$$
- There were matter inhomogenities $\Delta\rho/\rho \sim \Delta T/T$ at the stage of recombination ($e + p \to \gamma + H^*$)
- Jeans instability in the system of gravitating particles at rest $\Longrightarrow$ $\Delta\rho/\rho \nearrow$ $\Longrightarrow$ galaxies (CDM halos)









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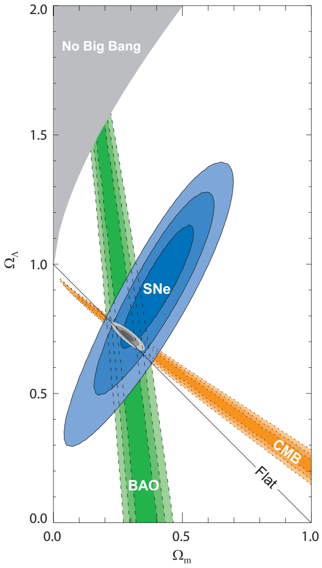
Dark Energy: nonclumping matter?



- estimates of Matter contribution confined in galaxies and clusters
  $\rho_c - \rho_M \neq 0$ but the Universe is flat, so $\rho_{curv} \simeq 0$
- corrections to the Hubble law : red shift – brightness curves for standard candles (SN Ia)
- The age of the Universe
- CMB anisotropy, large scale structures (galaxy clusters formation), etc

$$\rho_\Lambda = 0.68\rho_c$$

$$\rho_\Lambda \sim 10^{-5}\ \text{GeV/cm}^3 \sim \left(10^{-11.5}\ \text{GeV}\right)^4$$

Dark Energy: all evidences are from cosmology

Working hypothesis is cosmological constant $\Lambda \approx \left(2.5\times 10^{-3}\,\text{eV}\right)^4$:
$p = w(t)\,\rho$, $w = \text{const} = -1$, $\rho = \Lambda$

$$S_\Lambda = -\Lambda \int d^4x\,\sqrt{-\det g_{\mu\nu}}$$

both parts contribute

$$S_{\text{grav}} = -\frac{1}{16\pi G}\int d^4x\,\sqrt{-\det g_{\mu\nu}}\,R\,,$$
$$S_{\text{matter}} = \int d^4x\,\sqrt{-\det g_{\mu\nu}}\left(\frac{1}{2}\,g^{\lambda\rho}\,\partial_\lambda\phi\,\partial_\rho\phi - V(\phi)\right)$$

natural values

$$\Lambda_{\text{grav}} \sim 1/G^2 \sim \left(10^{19}\,\text{GeV}\right)^4\,,\quad \Lambda_{\text{matter}} \sim V(\phi_{vac}) \sim (100\,\text{GeV})^4\,,(100\,\text{MeV})^4\,,\ldots$$

Why Λ is small? Why $\Lambda \sim \rho_{\text{matter}}$? Why $\rho_B \sim \rho_{DM} \sim \rho_\Lambda$ today?

$$\left(\frac{\dot{a}}{a}\right)^2 = H^2(t) = \frac{8\pi}{3} G \rho^{\text{energy}}_{\text{density}}$$

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$$\rho_{\text{radiation}} \propto 1/a^4(t) \propto T^4(t)\,, \quad \rho_{\text{matter}} \propto 1/a^3(t)$$

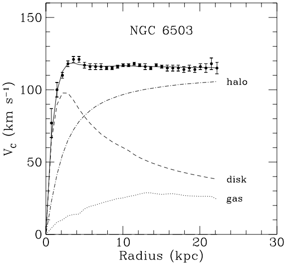
$$\rho_\Lambda = \text{const}$$

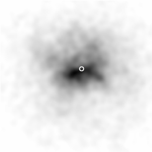
Why do we think it is most probably new particle physics (new gravity if any is not enough) ?

DM at various spatial scales, BAU requires baryon number violation

Universe content from astrophysics

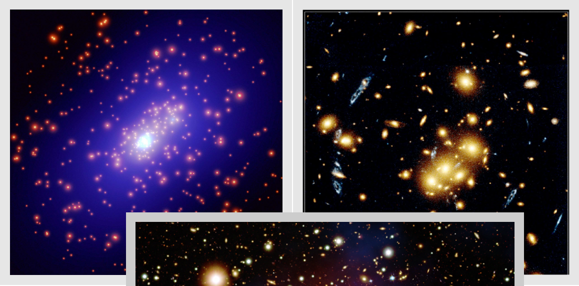
Rotational curves





X-rays from centers of galaxy clusters

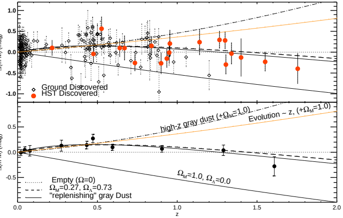
Gravitational lensing



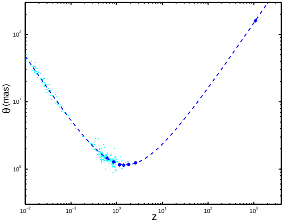
“Bullet” cluster

Universe content from cosmology

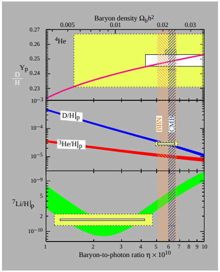
Standard candles

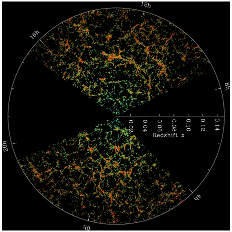


Angular distance

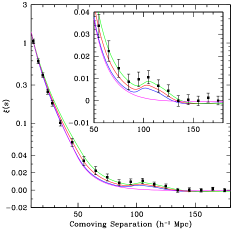


Nucleosynthesis

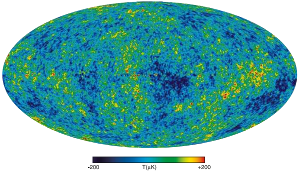




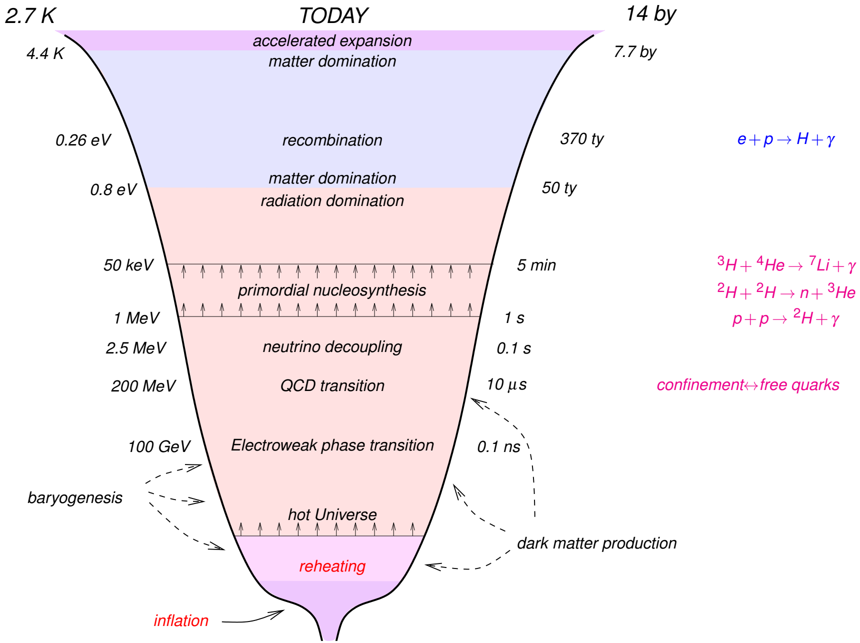
Large Scale Structures



Baryon acoustic oscillations



CMB anisotropy


2.7 K
TODAY
14 by
accelerated expansion
4.4 K
matter domination
7.7 by
0.26 eV
recombination
370 ty
$e + p \to H + \gamma$
matter domination
0.8 eV
50 ty
radiation domination
50 keV
5 min
$^3H + {}^4He \to {}^7Li + \gamma$
primordial nucleosynthesis
$^2H + {}^2H \to n + {}^3He$
1 MeV
1 s
$p + p \to {}^2H + \gamma$
2.5 MeV
neutrino decoupling
0.1 s
200 MeV
QCD transition
10 μs
confinement↔free quarks
100 GeV
Electroweak phase transition
0.1 ns
baryogenesis
hot Universe
dark matter production
reheating
inflation

Friedmann equation for the present Universe

$$H^2 \equiv \left(\frac{\dot a}{a}\right)^2 = \frac{8\pi}{3} G\left(\rho_{\rm M} + \rho_{rad} + \rho_\Lambda + \rho_{\rm curv}\right)$$

$$\frac{8\pi}{3} G\rho_{\rm curv} = -\frac{\varkappa}{a^2}\,, \quad \rho_c \equiv \frac{3}{8\pi G} H_0^2$$

$$\rho_c = \rho_{{\rm M},0} + \rho_{rad,0} + \rho_{\Lambda,0} = \rho_c = 0.53\cdot 10^{-5}\,\frac{\rm GeV}{\rm cm^3}\,,$$

$$\Omega_X \equiv \frac{\rho_{X,0}}{\rho_c}$$

$$\left(\frac{\dot a}{a}\right)^2 = \frac{8\pi}{3} G\rho_c\left[\Omega_{\rm M}\left(\frac{a_0}{a}\right)^3 + \Omega_{rad}\left(\frac{a_0}{a}\right)^4 + \Omega_\Lambda\right]$$

FLRW metric $g_{\mu\nu}$

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = dt^2 - a^2(t)dl^2 = dt^2 - a^2(t)\gamma_{ij}dx^i dx^j \ ,$$

$$H(t) = \frac{\dot{a}(t)}{a(t)}$$

Special frame: different parts look similar
Also this is comoving frame: world lines of particles at rest are geodesics,

$$\frac{du^\mu}{ds} + \Gamma^\mu_{\nu\lambda}u^\nu u^\lambda = 0$$

$$\gamma_{ij} \approx \delta_{ij}$$

Photons in the expanding Universe

$$S = -\frac{1}{4}\int d^4x\sqrt{-g}g^{\mu\nu}g^{\lambda\rho}F_{\mu\lambda}F_{\nu\rho}$$

$dt = ad\eta$ conformally flat metric

$$ds^2 = dt^2 - a^2(t)\delta_{ij}dx^i dx^j \longrightarrow ds^2 = a^2(\eta)[d\eta^2 - \delta_{ij}dx^i dx^j]$$

$$S = -\frac{1}{4}\int d^4x\, \eta^{\mu\nu}\eta^{\lambda\rho}F_{\mu\lambda}F_{\nu\rho}\,, \qquad A_\mu^{(\alpha)} = e_\mu^{(\alpha)}\mathrm{e}^{ik\eta - i\mathbf{kx}}\,, \quad k = |\mathbf{k}|$$

$$\Delta x = 2\pi/k\,, \quad \Delta\eta = 2\pi/k$$

$$\lambda(t) = a(t)\Delta x = 2\pi\frac{a(t)}{k}\,, \quad T = a(t)\Delta\eta = 2\pi\frac{a(t)}{k}$$

Redshift and the Hubble law $\lambda_0 = \lambda_i \frac{a_0}{a(t_i)} \equiv \lambda_i (1 + z(t_i))$

$$\mathbf{p}(t) = \frac{\mathbf{k}}{a(t)}\,, \quad \omega(t) = \frac{k}{a(t)}$$

for not very distant objects $\quad$ 1 pc $\approx$ 3 ly

$$a(t_i) = a_0 - \dot{a}(t_0)(t_0 - t_i) \quad \longrightarrow \quad a(t_i) = a_0[1 - H_0(t_0 - t_i)]$$

$$z(t_i) = H_0(t_0 - t_i) = H_0 r\,, \qquad z \ll 1$$

$$H_0 = h \cdot 100\,\frac{\text{km}}{\text{s} \cdot \text{Mpc}}\,, \qquad h \approx 0.68$$

similar reddening for other relativistic particles (small H, $\dot{H}$, etc.)

$\mathbf{p} = \frac{\mathbf{k}}{a(t)}$ is true for massive particles as well

Gas of free particles in the expanding Universe

homogeneous gas $dN = f(\mathbf{p},t)d^3\mathbf{X}d^3\mathbf{p}$

in comoving coordinates:

$$d^3\mathbf{x} = \text{const}\,, \quad d^3\mathbf{k} = \text{const}\,, \quad f(k) = \text{const}$$

$$f(k)d^3\mathbf{x}d^3\mathbf{k} = \text{const}$$

comoving volume equals physical volume

$$d^3\mathbf{x}d^3\mathbf{k} = d^3(a\mathbf{x})d^3\left(\frac{\mathbf{k}}{a}\right) = d^3\mathbf{X}d^3\mathbf{p}$$

$$f(\mathbf{p},t) = f(\mathbf{k}) = f[a(t)\cdot\mathbf{p}]\,.$$

$$t = t_i\ :\quad f_i(\mathbf{p}) \ \longrightarrow\ f(\mathbf{p},t) = f_i\left(\frac{a(t)}{a(t_i)}\mathbf{p}\right)$$

Massless bosons (photons) fermions

$$f_i(\mathbf{p}) = f_{Pl}\left(\frac{|\mathbf{p}|}{T_i}\right) = \frac{1}{(2\pi)^3}\frac{1}{e^{|\mathbf{p}|/T_i} - 1}$$

$$f(\mathbf{p}, t) = f_{Pl}\left(\frac{a(t)|\mathbf{p}|}{a_i T_i}\right) = f_{Pl}\left(\frac{|\mathbf{p}|}{T_{eff}(t)}\right)$$

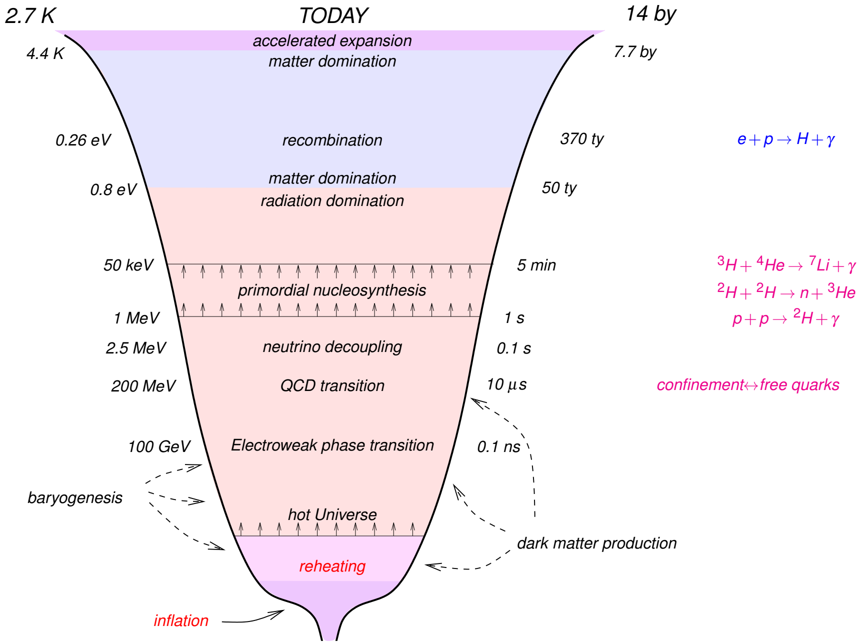
$$T_{eff}(t) = \frac{a_i}{a(t)} T_i$$

decoupling at $T \gg m$: neutrinos, hot(warm) dark matter

decoupling at $T \ll m$: $f(\mathbf{p}) = \frac{1}{(2\pi)^3} \exp\left(-\frac{m-\mu_i}{T_i}\right) \exp\left(-\frac{a^2(t)\mathbf{p}^2}{2ma_i^2 T_i}\right)$

$$f(\mathbf{p}, t) = \frac{1}{(2\pi)^3} \exp\left(-\frac{m - \mu_{eff}}{T_{eff}}\right) \exp\left(-\frac{\mathbf{p}^2}{2mT_{eff}}\right)$$

$$T_{eff}(t) = \left(\frac{a_i}{a(t)}\right)^2 T_i\,, \qquad \frac{m - \mu_{eff}(t)}{T_{eff}} = \frac{m - \mu_i}{T_i}$$


2.7 K
TODAY
14 by
accelerated expansion
4.4 K
matter domination
7.7 by
0.26 eV
recombination
370 ty
$e + p \to H + \gamma$
0.8 eV
matter domination
radiation domination
50 ty
50 keV
5 min
$^3H + ^4He \to ^7Li + \gamma$
primordial nucleosynthesis
$^2H + ^2H \to n + ^3He$
1 MeV
1 s
$p + p \to ^2H + \gamma$
2.5 MeV
neutrino decoupling
0.1 s
200 MeV
QCD transition
10 μs
confinement↔free quarks
100 GeV
Electroweak phase transition
0.1 ns
baryogenesis
hot Universe
dark matter production
reheating
inflation

Einstein equations

$T_{\mu\nu}$: macroscopic description

$\frac{1}{2}\int d^4x\sqrt{-g}T_{\mu\nu}\delta g^{\mu\nu}$

$T_{\mu\nu} = (p+\rho)u_\mu u_\nu - g_{\mu\nu}p$

ideal fluid with $\rho(t)$ and $p(t)$

in the comoving frame $u^0 = 1$, $\mathbf{u} = 0$

(almost) always works

$$T_\mu^\nu = diag(\rho, \, -p)$$

$$ds^2 = dt^2 - a^2(t)\gamma_{ij}dx^i dx^j \ ,$$

$$S_{EH} = -\frac{1}{16\pi G}\int d^4x\sqrt{-g}R \ : \quad R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G T_{\mu\nu}$$

$$(00): \quad \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3}G\rho - \frac{\varkappa}{a^2}$$

Friedmann equation (00) : $\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3}G\rho - \frac{\varkappa}{a^2}$

$$\nabla_\mu T^{\mu 0} = 0 \quad \longrightarrow \quad \dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = 0$$

the equation of state

$$p = p(\rho)$$

many-component liquid, in case of thermal equilibrium

other equations

$$-3d(\ln a) = \frac{d\rho}{p+\rho} = d(\ln s)$$

entropy of cosmic primordial plasma is conserved in a comoving frame

$$sa^3 = \text{const}$$

Examples of cosmological solutions

$\varkappa = 0$ $$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3} G\rho$$

dust: $p = 0$ singular at $t = t_s$

$$\rho = \frac{\text{const}}{a^3}\,, \quad a(t) = const \cdot (t - t_s)^{2/3}\,, \quad \rho(t) = \frac{\text{const}}{(t - t_s)^2}$$

$$t_s = 0\,, \quad H(t) = \frac{\dot{a}}{a}(t) = \frac{2}{3t}\,, \quad \rho = \frac{3}{8\pi G} H^2 = \frac{1}{6\pi G}\frac{1}{t^2}$$

the Universe is too young

$$t_0 = \frac{2}{3H_0} = 0.9 \times 10^{10}\ \text{yr} \quad (h = 0.7)$$

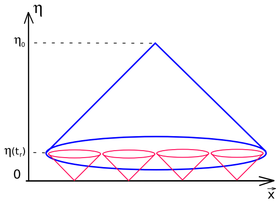
Cosmological (particle) horizon $l_H(t)$

distance covered by photons emitted at $t = 0$

the size of causally-connected region — the size of the visible part of the Universe

in conformal coordinates:
$ds^2 = 0 \longrightarrow |d\mathbf{x}| = d\eta$
coordinate size of the horizon equals
$\eta(t) = \int d\eta$

$$l_H(t) = a(t)\eta(t) = a(t)\int_0^t \frac{dt'}{a(t')}$$



dust

$$l_H(t) = 3t = \frac{2}{H(t)}\,, \qquad l_{H,0} = 2.6\times 10^{28}\text{cm} \quad (h = 0.7)$$

Examples of cosmological solutions

radiation: $p = \frac{1}{3}\rho$ singular at $t = t_s$

$$\rho = \frac{\text{const}}{a^4}\,, \quad a(t) = const \cdot (t - t_s)^{1/2}\,, \quad \rho(t) = \frac{\text{const}}{(t - t_s)^2}$$

$$t_s = 0\,, \quad H(t) = \frac{\dot{a}}{a}(t) = \frac{1}{2t}\,, \quad \rho = \frac{3}{8\pi G}H^2 = \frac{3}{32\pi G}\frac{1}{t^2}$$

$$l_H(t) = a(t)\int_0^t \frac{dt'}{a(t')} = 2t = \frac{1}{H(t)}\,.$$

In case of thermal equilibrium $T = \text{const}/a$

$$\rho_b = \frac{\pi^2}{30} g_b T^4\,, \quad \rho_f = \frac{7}{8}\frac{\pi^2}{30} g_f T^4$$

$$\rho = \frac{\pi^2}{30} g_* T^4\,, \quad g_* = \sum_b g_b + \frac{7}{8}\sum_f g_f = g_*(T)$$

Evolution of energy and entropy densities

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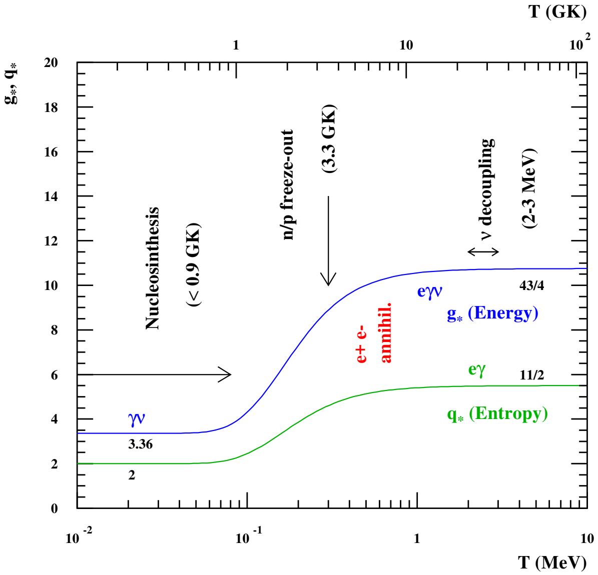


Examples of cosmological solutions

vacuum: $T_{\mu\nu} = \rho_{vac}\eta_{\mu\nu}$ $p = -\rho$

$$S_G = -\frac{1}{16\pi G}\int R\sqrt{-g}d^4x\,, \quad S_\Lambda = -\Lambda\int\sqrt{-g}d^4x\,.$$

$$a = \text{const}\cdot \mathrm{e}^{H_{\text{dS}}t}\,, \quad H_{\text{dS}} = \sqrt{\frac{8\pi}{3}G\rho_{vac}}$$

de Sitter space: space-time of constant curvature

$$ds^2 = dt^2 - \mathrm{e}^{2H_{dS}t}d\mathbf{x}^2$$

$\ddot{a} > 0$, no initial singularity

$ds^2 = dt^2 - e^{2H_{dS}t} d\mathbf{x}^2$

no cosmological horizon: $l_{\mathrm{H}}(t) = e^{H_{dS}t} \int_{-\infty}^{t} dt' e^{-H_{dS}t'} = \infty$

de Sitter (events) horizon ($\mathbf{x} = 0$, t):

from which distance $l(t)$ one can detect light emitted at t?

in conformal coordinates: $ds^2 = 0 \longrightarrow |d\mathbf{x}| = d\eta$

coordinate size: $\eta(t \to \infty) - \eta(t) = \int_t^\infty \frac{dt'}{a(t')}$

physical size: $l_{dS} = a(t) \int_t^\infty \frac{dt'}{a(t')} = \frac{1}{H_{dS}}$

observer will never be informed what happens at distances larger than $l_{dS} = H_{dS}^{-1}$

Our future? with $H_{dS} = 0.8 \times H_0$

Friedmann equation for the present Universe

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3} G \left(\rho_{\rm M} + \rho_{rad} + \rho_\Lambda + \rho_{\rm curv}\right)$$

$$\frac{8\pi}{3} G\rho_{\rm curv} = -\frac{\varkappa}{a^2}\,, \quad \rho_c \equiv \frac{3}{8\pi G} H_0^2$$

$$\rho_c = \rho_{{\rm M},0} + \rho_{rad,0} + \rho_{\Lambda,0} = \rho_c = 0.52 \cdot 10^{-5} \frac{\rm GeV}{\rm cm^3}\,, \qquad \text{for } \; h = 0.7$$

$$\Omega_X \equiv \frac{\rho_{X,0}}{\rho_c}$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3} G\rho_c \left[\Omega_{\rm M} \left(\frac{a_0}{a}\right)^3 + \Omega_{rad} \left(\frac{a_0}{a}\right)^4 + \Omega_\Lambda + \Omega_{curv} \left(\frac{a_0}{a}\right)^2\right]$$

Homogeneous and isotropic 3d manifolds

$$dl^2 = d\rho^2 + r^2(\rho)(d\theta^2 + \sin^2\theta d\phi^2)$$

$$r(\rho) = \begin{cases} R\sin(\rho/R)\,, & \text{3-sphere} \\ \rho\,, & \text{3-plane} \\ R\sinh(\rho/R)\,, & \text{3-hyperboloid} \end{cases}$$

ρ is a geodesic distance; $S = 4\pi r^2(\rho)$; $\Delta\theta = \frac{l}{r(\rho)}$

$$d\rho^2 = \frac{dr^2}{\cosh^2\frac{\rho}{R}} = \frac{dr^2}{1+\frac{r^2}{R^2}} \qquad d\rho^2 = \frac{dr^2}{\cos^2\frac{\rho}{R}} = \frac{dr^2}{1-\frac{r^2}{R^2}}$$

$$dl^2 = \frac{dr^2}{1-\varkappa\frac{r^2}{R^2}} + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

Brightness–redshift dependence in the Universe

$$ds^2 = dt^2 - a^2(t)\left[d\chi^2 + \sinh^2\chi\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)\right]$$

coordinate distance $\rho/R \to \chi = \int_{t_i}^{t_0} \frac{dt}{a(t)}$ $\qquad z(t) = \frac{a_0}{a(t)} - 1$

$$\chi(z) = \int_0^z \frac{dz'}{a_0 H_0} \frac{1}{\sqrt{\Omega_M (z'+1)^3 + \Omega_\Lambda + \Omega_{curv}(z'+1)^2}}$$

$$a_0^2 H_0^2 \Omega_{curv} = 1\,, \quad \Omega_M + \Omega_\Lambda + \Omega_{curv} = 1$$

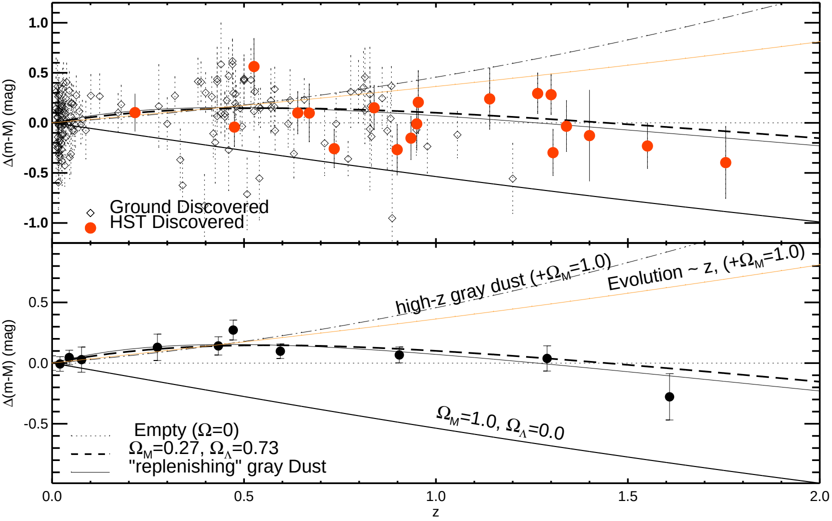
$$S(z) = 4\pi r^2(z)\,, \qquad r(z) = a_0 \sinh\chi(z)$$

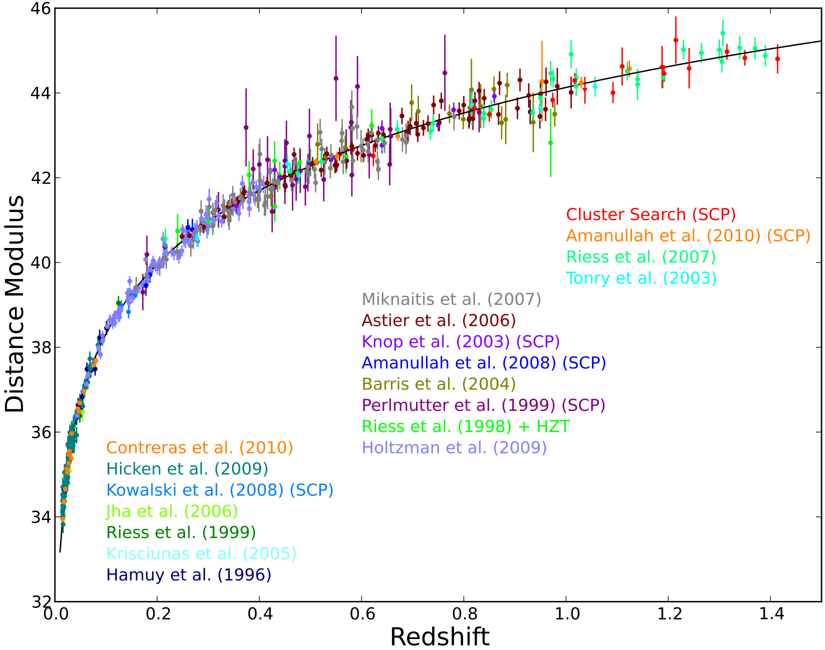
detector: $N_\gamma \propto S^{-1}$, $\omega = \omega_i/(1+z)$, $dt_0 = (1+z)dt_i$
hence the brightness (energy flux measured by a detector) is

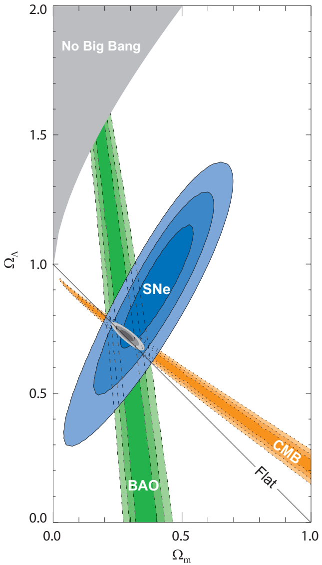
$$J = \frac{L}{(1+z)^2 S(z)} \equiv \frac{L}{4\pi r_{ph}^2}\,, \quad r_{ph} = (1+z)\cdot r(z)$$

Brightness–redshift dependence: SNe Ia

$$\Delta(m-M) = 5\log\frac{r_{ph}}{r_{ph}(\Omega_c = 0.8, \Omega_M = 0.2)}$$




Distance Modulus
Redshift
46
44
42
40
38
36
34
32
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
Cluster Search (SCP)
Amanullah et al. (2010) (SCP)
Riess et al. (2007)
Tonry et al. (2003)
Miknaitis et al. (2007)
Astier et al. (2006)
Knop et al. (2003) (SCP)
Amanullah et al. (2008) (SCP)
Barris et al. (2004)
Perlmutter et al. (1999) (SCP)
Riess et al. (1998) + HZT
Holtzman et al. (2009)
Contreras et al. (2010)
Hicken et al. (2009)
Kowalski et al. (2008) (SCP)
Jha et al. (2006)
Riess et al. (1999)
Krisciunas et al. (2005)
Hamuy et al. (1996)


2.0
1.5
1.0
0.5
0.0
0.0
0.5
1.0
Ω_Λ
Ω_m
No Big Bang
SNe
BAO
CMB
Flat

Microscopic processes in the expanding Universe

A competition between scattering, decays, etc and expansion

for general processes one should solve kinetic equations

$$\frac{dn_{X_i}}{dt} + 3Hn_{X_i} = \sum(production - destruction)$$

Boltzmann equation in a comoving volume: $\frac{d}{dt}\left(n\,a^3\right) = a^3 \int \dots$

production:

$$\sigma(A+B \to X+C)n_A n_B, \quad \Gamma(D \to E+X)n_D \cdot M_D/E_D, \quad \text{etc}$$

desrtuction:

$$\sigma(A+X \to C+B)n_A n_X, \quad \Gamma(X \to F+G)n_X \cdot M_X/E_X, \quad \text{etc}$$

Fast direct and inverse processes, $\Gamma \gtrsim H$, are in equilibrium,

$\sum(\;) = 0$ and thermalize particles

Last scattering: $\gamma e \to \gamma e$

$$\sigma_{\mathrm{T}} = \frac{8\pi}{3}\frac{\alpha^2}{m_e^2} \approx 0.67 \cdot 10^{-24}\ \mathrm{cm}^2\,, \qquad \tau_\gamma = \frac{1}{\sigma_{\mathrm{T}} \cdot n_e(T)}$$

last scattering: $\tau_\gamma(T_f) \simeq H^{-1}(T_f) \simeq t_f$

$$T_f = 0.26\ \mathrm{eV}\,, \quad z = 1100\,, \quad t_f = 370\,000\ \mathrm{yr}$$

for general processes one should solve kinetic equations

$$\frac{dn_{X_i}}{dt} + 3Hn_{X_i} = \int (production - destruction)$$

Boltzmann equation in a comoving volume: $\frac{d}{dt}\left(n\,a^3\right) = a^3 \int \dots$

Recombination: horizon

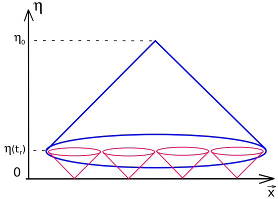
matter domination: $l_{\mathrm{H},r} = 2H_r^{-1}$

$$H_r^2 = \frac{8\pi}{3} G\rho_{\mathrm{M}}(t_r) = \frac{8\pi}{3} G\rho_{\mathrm{M},0}\left(\frac{a_0}{a_r}\right)^3 = \frac{8\pi}{3} G\rho_c \Omega_{\mathrm{M},0}(1+z_r)^3 \ .$$

at recombination: $l_{\mathrm{H},r} = \frac{2}{H_0\sqrt{\Omega_{\mathrm{M}}}}\frac{1}{(1+z_r)^{3/2}}$

today: $l_{\mathrm{H},r}(t_0) = l_{\mathrm{H},r} \times \frac{a_0}{a_r} = \frac{2}{H_0\sqrt{\Omega_{\mathrm{M}}}}\frac{1}{\sqrt{1+z_r}}$

$$\frac{l_{H_0}}{l_{\mathrm{H},r}(t_0)} \sim \sqrt{1+z_r} \simeq 30$$



Recombination: angle

angular distance: $$d_{ph} = r_a(z)\,\Delta\theta$$

$$\chi_r = \int_{t_r}^{t_0} \frac{dt}{a(t)}\,, \quad \Delta\theta_r = \frac{l_{H,r}}{r_a(z_r)}$$

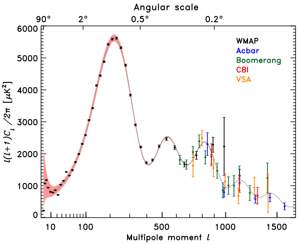
$$d_{conf} = \sinh\chi_r\,\Delta\theta$$

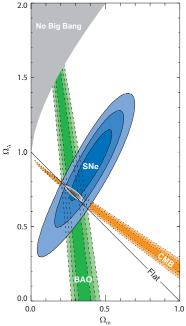
$$r_a(z_r) = (1+z_r)^{-1}\cdot a_0 \cdot \sinh\chi_r$$

$$\Delta\theta_r = \frac{1}{\sqrt{z_r+1}}\,, \quad \Omega_{curv} = \Omega_\Lambda = 0\,.$$

$$\Delta\theta_r = \frac{1}{\sqrt{z_r+1}}\frac{2\sqrt{\Omega_{curv}/\Omega_M}}{\sinh\left(2\sqrt{\Omega_{curv}/\Omega_M}\,I\right)}\,.$$

$$I = \int_0^1 \frac{dy}{\sqrt{1+\frac{\Omega_\Lambda}{\Omega_M}y^6}}$$

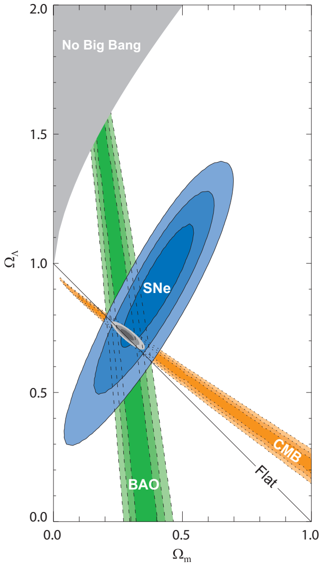


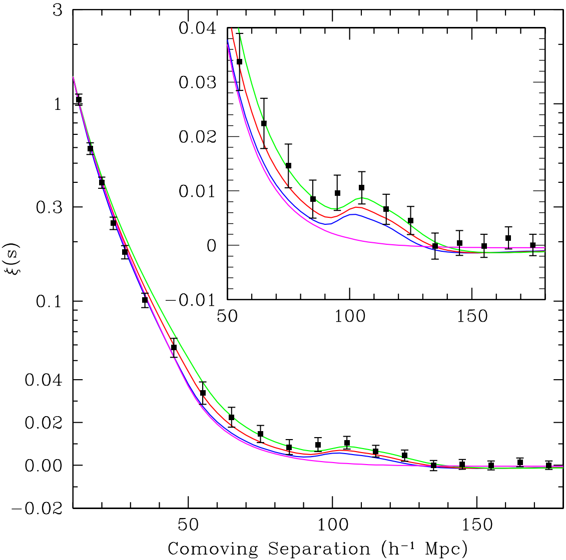


Acoustic oscillations in relativistic plasma:
What matters is the sound horizon:

$$l_{s,r} = l_{H,r}\cdot v_s \approx l_{H,r}/\sqrt{3}$$

Then $$\Delta\theta_{r,s} = \frac{1}{\sqrt{3}}\frac{1}{\sqrt{1+z}}\times\frac{180^\circ}{\pi} \simeq 1^\circ$$





$$110/0.7\,\text{Mpc} \simeq l_{H,r}(t_0) \times \sqrt{v_s^2} \simeq l_{H_0}/\sqrt{3}/\sqrt{1+z_r}$$

Neutrino freeze-out

$T > m_e$ $\qquad$ $e^+e^- \leftrightarrow \nu\bar{\nu},\ e\nu \leftrightarrow e\nu$

$$\sigma_\nu \sim G_F^2 E^2$$

neutrino interaction rate

$$\tau_\nu = \frac{1}{\langle \sigma_\nu n v \rangle} \sim \frac{1}{G_F^2 T^5} \qquad H^2 = \frac{8\pi}{3 M_{Pl}^2} \frac{\pi^2}{30} g_* T^4 \equiv \frac{T^4}{M_{Pl}^{*2}}$$

$$\tau_\nu(T) \sim H^{-1}(T) = \frac{M_{Pl}^*}{T^2}$$

$$T_{\nu,f} \sim \left(\frac{1}{G_F^2 M_{Pl}^*} \right)^{1/3} \sim 2 \div 3\ \text{MeV}$$

Neutron decoupling

$$p + e \longleftrightarrow n + \nu_e$$

typical energy scales

$$T \gtrsim \Delta m = 1.3 \text{ MeV}, \qquad T \gtrsim m_e = 0.5 \text{ MeV}$$

neutron interaction rate

$$\tau_{n\leftrightarrow p} = \frac{1}{\Gamma_{n\leftrightarrow p}} = \frac{1}{C_n G_F^2 T^5}$$

neutron decoupling

$$\Gamma_{n\leftrightarrow p}(T) \sim H(T) = T^2/M_{Pl}^*$$

$$T_n = \frac{1}{\left(C_n M_{Pl}^* G_F^2\right)^{1/3}} \approx 0.8 \text{ MeV}$$

$$g_* = 2 + \tfrac{7}{8}\cdot 4 + \tfrac{7}{8}\cdot 2 \cdot N_\nu \qquad t = \frac{1}{2H(T_n)} = \frac{M_{Pl}^*}{2T_n^2} \approx 1 \text{ s}$$

Neutron density at decoupling

$$n_n = g_n \left(\frac{m_n T}{2\pi} \right)^{3/2} e^{\frac{\mu_n - m_n}{T}} \qquad \mu_n + \mu_\nu = \mu_p + \mu_e$$

$$\frac{n_n}{n_p} = e^{-\frac{m_n - m_p}{T}} e^{\frac{\mu_n - \mu_p}{T}}$$

for relativistic e^+ and e^-

$$n_{e^-} - n_{e^+} \sim \mu_e T^2 \longrightarrow \frac{\mu_e}{T} \sim \frac{n_{e^-} - n_{e^+}}{T^3} = \frac{n_p}{T^3} \sim \eta_B \equiv \frac{n_p}{n_\gamma} = 6 \times 10^{-10}$$

$$\frac{n_n}{n_p} = e^{-\frac{m_n - m_p}{T_n}} \equiv e^{-\frac{\Delta m}{T_n}} \approx \frac{1}{5} e^{-\frac{\mu_\nu}{T_n}}$$

$p+n \longrightarrow \mathrm{D}+\gamma$ Saha equation

$$n_n = 2\left(\frac{m_p T}{2\pi}\right)^{3/2} \mathrm{e}^{\frac{\mu_n - m_n}{T}}\,, \qquad n_p = 2\left(\frac{m_p T}{2\pi}\right)^{3/2} \mathrm{e}^{\frac{\mu_p - m_p}{T}}\,,$$

Chemical equilibrium for nuclei: $\mu_A = \mu_p \cdot Z + \mu_n \cdot (A - Z)$

$$n_D \sim \left(\frac{m_D T}{2\pi}\right)^{3/2} \mathrm{e}^{\frac{\mu_D - m_D}{T}}\,,$$

Temperature of BBN T_{NS}: $n_{\mathrm{D}} \sim n_n$

$\Delta_{\mathrm{D}} = 2.23$ MeV $t_{NS} \approx 3\,\mathrm{min}$

$$n_{\mathrm{D}}/n_p(T_{NS}) \sim \eta_{\mathrm{B}} \left(\frac{2.5\,T_{NS}}{m_p}\right)^{3/2} \mathrm{e}^{\frac{\Delta_{\mathrm{D}}}{T_{NS}}} \sim 1 \longrightarrow T_{NS} \approx 50\ \mathrm{keV}$$

Helium abundance (NO chemical equilibrium)

Neutrons remain mostly in helium

$$n_{^4\text{He}}(T_{NS}) = \frac{1}{2} n_n(T_{NS}) \,,$$

neutron-to-proton ratio $\tau_n \approx 880$ s

$$\frac{n_n(T_{NS})}{n_p(T_{NS})} \approx \frac{1}{5} \cdot e^{-\frac{t_{NS}}{\tau_n}} \cdot e^{-\frac{\mu_\nu}{T_n}} \approx \frac{1}{7} \,,$$

$$Y_p \equiv X_{^4\text{He}} = \frac{m_{^4\text{He}} \cdot n_{^4\text{He}}(T_{NS})}{m_p\left(n_p(T_{NS}) + n_n(T_{NS})\right)} = \frac{2}{\frac{n_p(T_{NS})}{n_n(T_{NS})} + 1} \approx 25\%$$

from observations of relic helium abundance:

$$\Delta N_{\nu,eff} \leq 1 \,, \qquad \left|\frac{\mu_\nu}{T_n}\right| \lesssim 0.01$$

Main nuclear reactions

1. $p(n,\gamma)$D — deuterium production, BBN starts.
2. D$(p,\gamma)^3$He, D(D,n)^{3}He, D(D,p)T, ^{3}He(n,p)T — intermediate stage.
3. T(D,n)^{4}He, ^{3}He(D,p)^{4}He — production of ^{4}He.
4. T$(\alpha,\gamma)^7$Li, ^{3}He$(\alpha,\gamma)^7$Be, ^{7}Be$(n,p)^7$Li — production of the heaviest baryonic relics.
5. ^{7}Li$(p,\alpha)^4$He — ^{7}Li burning.

One has to compare reaction rates to the expansion rate

$$H(T_{NS} = 50\text{ keV}) = 10^{-2}\text{ s}^{-1}$$

to obtain nonequilibrium concentrations

Neutron burning $p + n \longrightarrow \mathrm{D} + \gamma$

@ $T = T_{NS} = 65$ keV

$$(\sigma v)_{p(n,\gamma)D} \approx 6 \cdot 10^{-20}\ \frac{\mathrm{cm}^3}{\mathrm{s}}\ .$$

for the rate (neutron disappearence when meets proton)

$$\Gamma_{p(n,\gamma)D} = n_p \cdot (\sigma v)_{p(n,\gamma)D} = \eta_B \cdot 2\frac{\zeta(3)}{\pi^2} T^3 \cdot (\sigma v)_{p(n,\gamma)D} = 0.31\ \mathrm{s}^{-1}$$

for $\eta_B = 6.15 \cdot 10^{-10}$ and $T = T_{NS}$

So, neutrons disappear very rapidly

$$\Gamma_{p(n,\gamma)D} \gg H(T_{NS}) = 4 \cdot 10^{-3}\ \mathrm{s}^{-1}$$

Deuterium burning $D(D,n)^3He$, $D(D,p)T$

Coloumb barier: tunneling $T_9 \equiv T/(10^9\ \mathrm{K}) = T/(86\ \mathrm{keV})$

$$\langle\sigma v\rangle_{DD} = 3\cdot 10^{-15}\frac{\mathrm{cm}^3}{\mathrm{s}}\cdot T_9^{-2/3}\cdot e^{-4.26\cdot T_9^{-1/3}}\ .$$

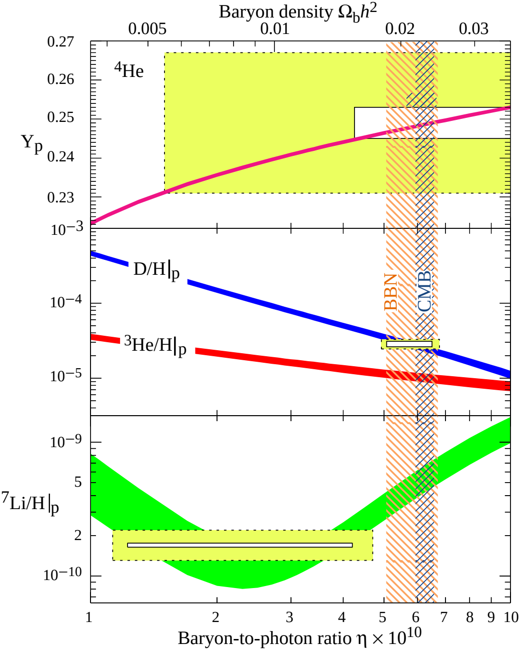
deuterium stops burning when $T = T_{NS}(T_9 = 0.75)$

$$\Gamma_{DD} = n_D(T)\cdot\langle\sigma v\rangle_{DD}(T) \sim H(T)\ .$$

Then relic deuterium abundance is estimated as

$$\frac{n_D}{n_p} = \frac{1}{0.75\eta_B}\cdot\frac{n_D}{n_\gamma(T_{NS})} = 0.3\cdot 10^{-4}$$

for $\eta_B = 6.15\cdot 10^{-10}$



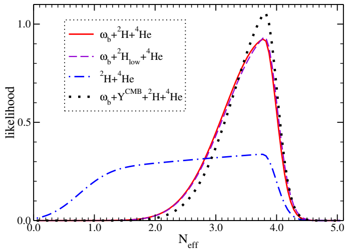
Measurement of $\eta_B = n_B/n_\gamma$ at $T \sim 1$ MeV

Lack of Lithium. . . Exotics needed?

$$Y_p = 0.2581 \pm 0.025\,,$$

$$D/H\big|_p = (2.87 \pm 0.21) \times 10^{-5}$$

1103.1261

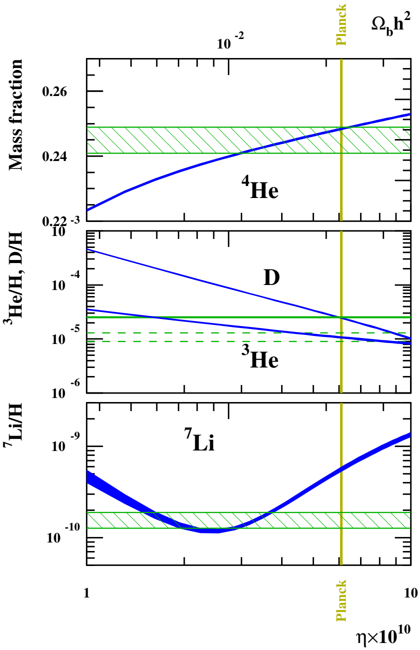


similar results from other recent studies including structure formation

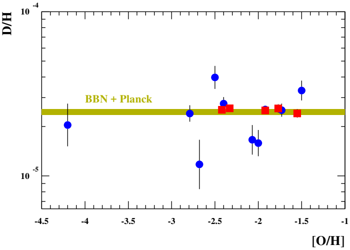
1001.4440, 1001.5218, 1202.2889

$N_{\nu,eff} < 4.2$ @ 95%CL

$N_{\nu,eff} < 4.0$ from D/H 1205.3785

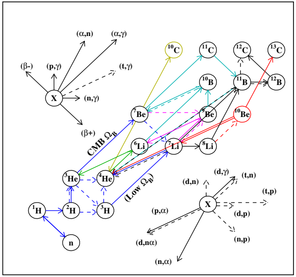


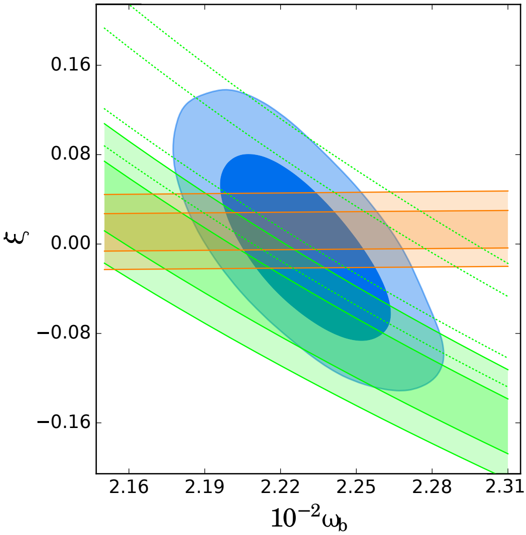
1707.01004 Measurement of $\eta_B = n_B/n_\gamma$ at $T \sim 1$ MeV



Lack of Lithium. . .

Exotics needed?





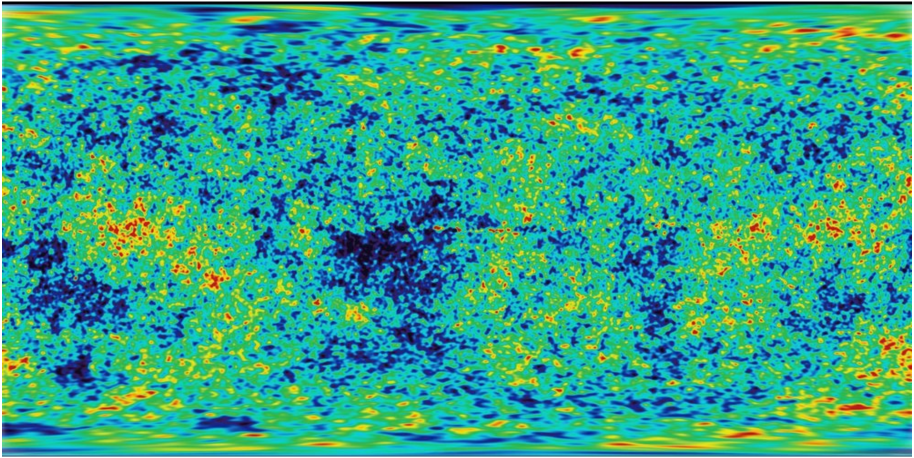
1706.01705

Primordial Element Abundance Observations:

- Lack of Lithium. . . Exotics needed?
- Measurement of $\eta_B = n_B/n_\gamma$ at $T \sim 1$ MeV consistent with present and recombination values
  – no “decaying relics”
- measurement of the Universe expansion rate $H^2 \sim \rho_{relativistic}$
  in particular:
  – neutrino number $N_\nu \approx 3$
  – no “dark radiation”

Baryon asymmetry must be produced before BBN !!

CMB map



Mode evolution

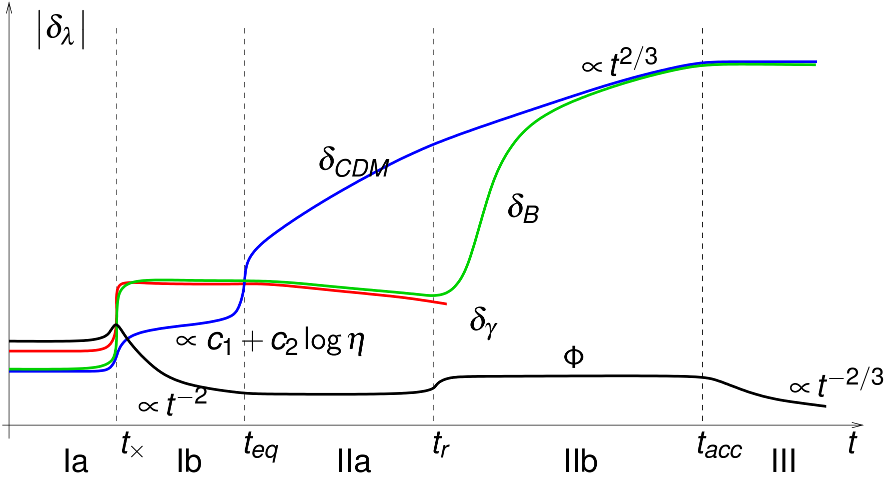
- Amplitude remains constant, while superhorizon, e.g. $k/a < H$
- Subhorizon Inhomogeneities of DM start to grow at MD-stage, $\delta\rho_{CDM}/\rho_{CDM} \propto a$ from $T \approx 0.8\,\text{eV}$
  Smaller objects (first stars, dwarf galaxies) are first to form
- Subhorizon Inhomogeneities of baryons join those of DM only after recombination, $\delta\rho_{CDM}/\rho_{CDM} \propto a$ from $T_{rec} \approx 0.25\,\text{eV}$
- at recombination $\delta\rho_B/\rho_B \sim \delta T/T \sim 10^{-4}$ and would grow only by a factor $T_{rec}/T_0 \sim 10^3$ without DM
- Subhorizon Inhomogeneities of photons $\delta\rho_\gamma/\rho_\gamma$ oscillate with constant amplitude at RD and with decreasing amplitude at MD, thus we can measure $T_{RD/MD}/T_{rec}$
- Phase of oscillations decoupled after recombination depends on the wave-length, recombination time and sound speed

$$\delta\rho_\gamma/\rho_\gamma \propto \cos\left(k\int_0^{t_r} \frac{v_s\,dt}{a(t)}\right) = cos(kl_{sound})$$

-

$$\delta T(\theta,\varphi) = \sum a_{lm} Y_{lm}(\theta,\varphi)\,, \qquad \langle a_{lm}^* a_{lm}\rangle = C_l \equiv 2\pi\mathscr{D}_l/(l(l+1))$$

Mode evolution at various stages



On formulas...

- short waves, $k\eta_{eq} \gg 1$ $\qquad R_B \equiv 3\rho_B/4\rho_\gamma$

$$\delta_\gamma = \Phi_{(i)} \cdot \left[-324 \cdot (1+R_B)\, l^2 \frac{\Omega_{CDM}}{\Omega_M} (1+z_{eq}) \frac{\log(0.2k\eta_{eq})}{(k\eta_0)^2} + \frac{6}{(1+R_B)^{1/4}} \cos\left(k \int_0^\eta d\tilde{\eta}\, u_s \right) \right] ,$$

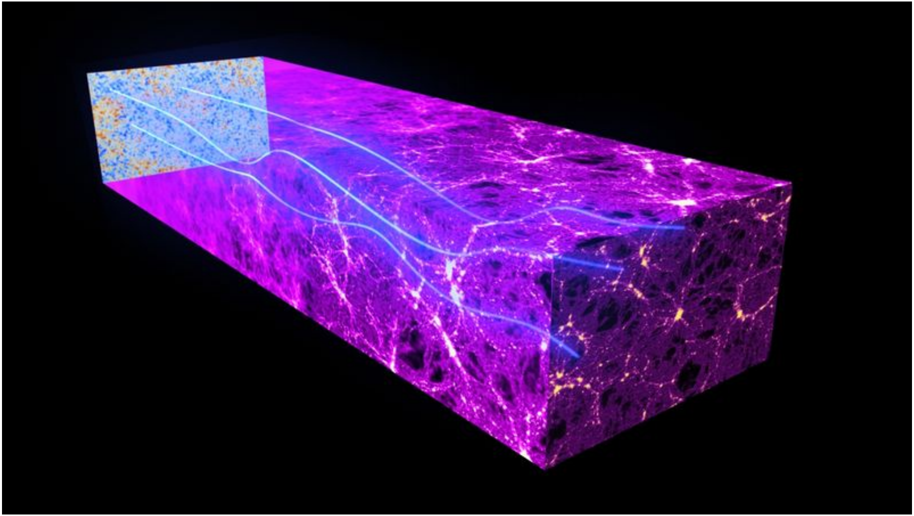
- long waves, $k\eta_{rec} \ll 1$

$$\delta_\gamma = -\frac{12}{5}\Phi_{(i)} = \text{const}$$

- intermediate waves ...

$$\delta_\gamma(\mathbf{k},\eta) = -4\left[1+R_B(\eta)\right]\Phi(\mathbf{k},\eta) + 4\Phi_{(i)}(\mathbf{k}) \cdot A(k,\eta) \cos\left(k \int_0^\eta u_s d\tilde{\eta} \right) ,$$

On top of that: propagation in expanding Universe



On formulas...

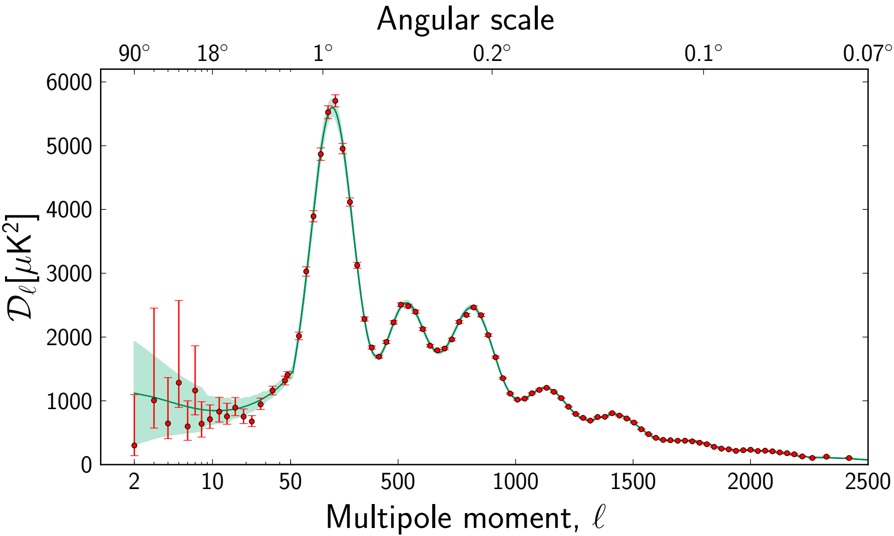
From linear approximation to the geodesic equation...
for scalar perturbations

$$\frac{\delta T}{T}(\mathbf{n},\eta_0) = \frac{1}{4}\delta_\gamma(\eta_r) + (\Phi(\eta_r) - \Phi(\eta_0)) + \int_{\eta_r}^{\eta_0} \left(\Phi' - \Psi'\right) d\eta + \mathbf{nv}(\eta_r) - \mathbf{nv}(\eta_0)\,.$$

for tensor perturbations

$$\frac{\delta T}{T}(\mathbf{n},\eta_0) = \frac{1}{2}\int_{\eta_r}^{\eta_0} d\eta\, n_i h_{ij}^{TT\,\prime} n_j\,,$$

CMB measurements (Planck) $\theta, \Omega_{DM}, \Omega_B, \tau, \Delta_{\mathscr{R}}, n_s$



Standard cosmological model $ds^2 = dt^2 - a^2(t)dx^2$

$$\left(\frac{\dot{a}}{a}\right)^2 \equiv H^2 = H_0^2 \left[\Omega_\Lambda + (\Omega_{DM} + \Omega_B + \Omega_{\nu,m\neq 0})\left(\frac{a_0}{a}\right)^3 + (\Omega_\gamma + \Omega_{\nu,m=0})\left(\frac{a_0}{a}\right)^4\right]$$

- $T_\gamma = 2.735\,\text{K}, \quad \Longrightarrow \quad \Omega_\gamma \sim 10^{-5}$
- $N_\nu \approx 3, \quad \sum m_\nu < 0.2\,\text{eV} \quad \Longrightarrow \quad \Omega_{\nu,\neq 0}, \Omega_{\nu,0} \sim 10^{-5}$?
- $\Omega_B = 4.5\% \quad \Longrightarrow \quad \eta_B \equiv n_B/n_\gamma = 6 \times 10^{-10}$
- $\Omega_{DM} = 27.5\%$
- $H_0 = 67\,\text{km/s/Mpc} \quad \Longrightarrow \quad \rho_0 = 5\,\text{GeV/m}^3$
- $\Omega_\Lambda = 68\% \quad \Longrightarrow$ flat space
- adiabatic, gaussian matter perturbations

$$\langle\left(\frac{\delta\rho}{\rho}\right)^2\rangle \sim A_S \int \frac{dk}{k}\left(\frac{k}{k_*}\right)^{n_S - 1}$$

  with $A_S = 3 \times 10^{-9}$ and $n_S = 0.97$
- no tensor perturbations, $r \equiv A_T/A_S < 0.05$
- reionization at $z \equiv a_0/a = 10$