Inflation and reheating in the early Universe
Lecture #3
Introduction: more on observables in Hot Big Bang Theory

Dmitry Gorbunov

Institute for Nuclear Research of RAS, Moscow

BASIS School
"Quantum fields:
from gravity and cosmology
to physics of condensed matter"
Velich country club, Moscow region, Russia

Friedmann equation (00) : $\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3} G\rho - \frac{\varkappa}{a^2}$

$$\nabla_\mu T^{\mu 0} = 0 \quad \longrightarrow \quad \dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = 0$$

the equation of state

$$p = p(\rho)$$

many-component liquid, in case of thermal equilibrium

other equations

$$-3d(\ln a) = \frac{d\rho}{p+\rho} = d(\ln s)$$

entropy of cosmic primordial plasma is conserved in a comoving frame

$$sa^3 = \text{const}$$

Examples of cosmological solutions

$$\varkappa = 0 \qquad \left(\frac{\dot a}{a}\right)^2 = \frac{8\pi}{3} G\rho$$

dust: $p = 0$ singular at $t = t_s$

$$\rho = \frac{\text{const}}{a^3}\,, \quad a(t) = const \cdot (t - t_s)^{2/3}\,, \quad \rho(t) = \frac{\text{const}}{(t - t_s)^2}$$

$$t_s = 0\,, \quad H(t) = \frac{\dot a}{a}(t) = \frac{2}{3t}\,, \quad \rho = \frac{3}{8\pi G} H^2 = \frac{1}{6\pi G}\frac{1}{t^2}$$

the Universe is too young

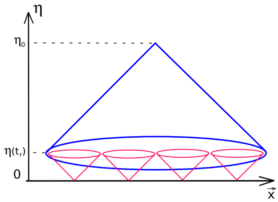
$$t_0 = \frac{2}{3H_0} = 0.9 \times 10^{10}\ \text{yr} \quad (h = 0.7)$$

Cosmological (particle) horizon $l_H(t)$

distance covered by photons emitted at $t = 0$

the size of causally-connected region — the size of the visible part of the Universe

in conformal coordinates:
$ds^2 = 0 \longrightarrow |d\mathbf{x}| = d\eta$
coordinate size of the horizon equals
$\eta(t) = \int d\eta$



$$l_H(t) = a(t)\eta(t) = a(t)\int_0^t \frac{dt'}{a(t')}$$

dust

$$l_H(t) = 3t = \frac{2}{H(t)}\,, \quad l_{H,0} = 2.6 \times 10^{28}\text{cm} \quad (h = 0.7)$$

Examples of cosmological solutions

radiation: $p = \frac{1}{3}\rho$ singular at $t = t_s$

$$\rho = \frac{\text{const}}{a^4}\,, \quad a(t) = const\cdot(t-t_s)^{1/2}\,, \quad \rho(t) = \frac{\text{const}}{(t-t_s)^2}$$

$$t_s = 0\,, \quad H(t) = \frac{\dot{a}}{a}(t) = \frac{1}{2t}\,, \quad \rho = \frac{3}{8\pi G}H^2 = \frac{3}{32\pi G}\frac{1}{t^2}$$

$$l_H(t) = a(t)\int_0^t \frac{dt'}{a(t')} = 2t = \frac{1}{H(t)}\,.$$

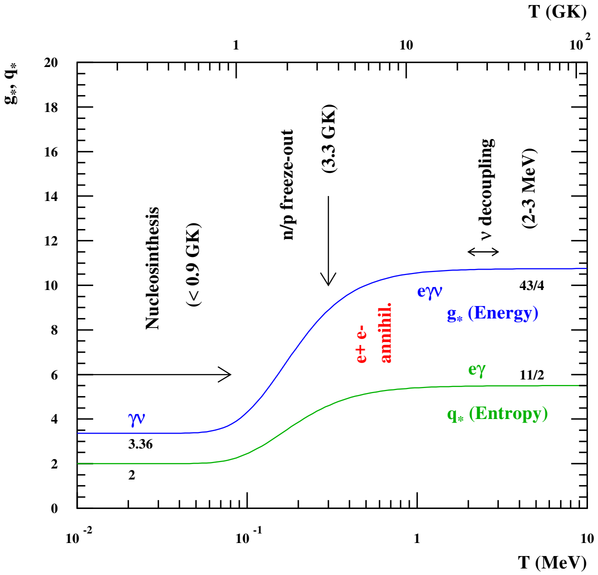
In case of thermal equilibrium $T = \text{const}/a$

$$\rho_b = \frac{\pi^2}{30}g_b T^4\,, \quad \rho_f = \frac{7}{8}\frac{\pi^2}{30}g_f T^4$$

$$\rho = \frac{\pi^2}{30}g_* T^4\,, \qquad g_* = \sum_b g_b + \frac{7}{8}\sum_f g_f = g_*(T)$$

Evolution of energy and entropy densities

1707.01004



Examples of cosmological solutions

vacuum: $T_{\mu\nu} = \rho_{vac}\eta_{\mu\nu}$ $\quad p = -\rho$

$$S_G = -\frac{1}{16\pi G}\int R\sqrt{-g}d^4x\,, \quad S_\Lambda = -\Lambda\int\sqrt{-g}d^4x\,.$$

$$a = \text{const}\cdot e^{H_{\text{dS}}t}\,, \quad H_{\text{dS}} = \sqrt{\frac{8\pi}{3}G\rho_{vac}}$$

de Sitter space: space-time of constant curvature

$$ds^2 = dt^2 - e^{2H_{dS}t}d\mathbf{x}^2$$

$\ddot{a} > 0$, no initial singularity

$ds^2 = dt^2 - e^{2H_{dS}t} d\mathbf{x}^2$

no cosmological horizon: $l_{\mathrm{H}}(t) = e^{H_{dS}t} \int_{-\infty}^{t} dt' e^{-H_{dS}t'} = \infty$

de Sitter (events) horizon ($\mathbf{x} = 0$, t):

from which distance $l(t)$ one can detect light emitted at t?

in conformal coordinates: $ds^2 = 0 \longrightarrow |d\mathbf{x}| = d\eta$

coordinate size: $\eta(t \to \infty) - \eta(t) = \int_t^\infty \frac{dt'}{a(t')}$

physical size: $l_{dS} = a(t) \int_t^\infty \frac{dt'}{a(t')} = \frac{1}{H_{dS}}$

observer will never be informed what happens at distances larger than $l_{dS} = H_{dS}^{-1}$

Our future? with $H_{dS} = 0.8 \times H_0$

Friedmann equation for the present Universe

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3} G \left(\rho_{\rm M} + \rho_{rad} + \rho_\Lambda + \rho_{\rm curv}\right)$$

$$\frac{8\pi}{3} G\rho_{\rm curv} = -\frac{\varkappa}{a^2}\,, \quad \rho_c \equiv \frac{3}{8\pi G} H_0^2$$

$$\rho_c = \rho_{\rm M,0} + \rho_{rad,0} + \rho_{\Lambda,0} = \rho_c = 0.52 \cdot 10^{-5} \frac{\rm GeV}{\rm cm^3}\,, \qquad \text{for } \; h = 0.7$$

$$\Omega_X \equiv \frac{\rho_{X,0}}{\rho_c}$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3} G\rho_c \left[\Omega_{\rm M} \left(\frac{a_0}{a}\right)^3 + \Omega_{rad} \left(\frac{a_0}{a}\right)^4 + \Omega_\Lambda + \Omega_{curv} \left(\frac{a_0}{a}\right)^2\right]$$

Homogeneous and isotropic 3d manifolds

$$dl^2 = d\rho^2 + r^2(\rho)(d\theta^2 + \sin^2\theta d\phi^2)$$

$$r(\rho) = \begin{cases} R\sin(\rho/R)\,, & \text{3-sphere} \\ \rho\,, & \text{3-plane} \\ R\sinh(\rho/R)\,, & \text{3-hyperboloid} \end{cases}$$

ρ is a geodesic distance; $S = 4\pi r^2(\rho)$; $\Delta\theta = \frac{l}{r(\rho)}$

$$d\rho^2 = \frac{dr^2}{\cosh^2\frac{\rho}{R}} = \frac{dr^2}{1+\frac{r^2}{R^2}} \qquad d\rho^2 = \frac{dr^2}{\cos^2\frac{\rho}{R}} = \frac{dr^2}{1-\frac{r^2}{R^2}}$$

$$dl^2 = \frac{dr^2}{1-\varkappa\frac{r^2}{R^2}} + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

Brightness–redshift dependence in the Universe

$$ds^2 = dt^2 - a^2(t)\left[d\chi^2 + \sinh^2\chi\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)\right]$$

coordinate distance $\rho/R \to \chi = \int_{t_i}^{t_0} \frac{dt}{a(t)}$ $\qquad z(t) = \frac{a_0}{a(t)} - 1$

$$\chi(z) = \int_0^z \frac{dz'}{a_0 H_0} \frac{1}{\sqrt{\Omega_M (z'+1)^3 + \Omega_\Lambda + \Omega_{curv}(z'+1)^2}}$$

$$a_0^2 H_0^2 \Omega_{curv} = 1\,, \quad \Omega_M + \Omega_\Lambda + \Omega_{curv} = 1$$

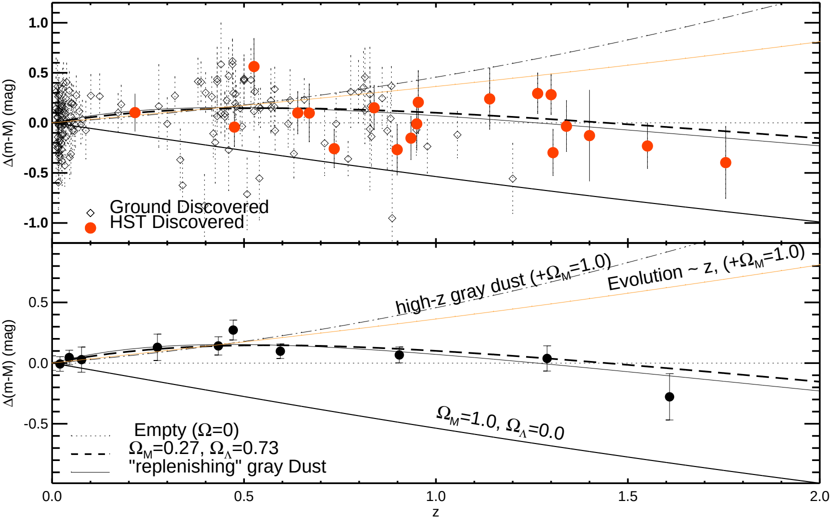
$$S(z) = 4\pi r^2(z)\,, \qquad r(z) = a_0 \sinh\chi(z)$$

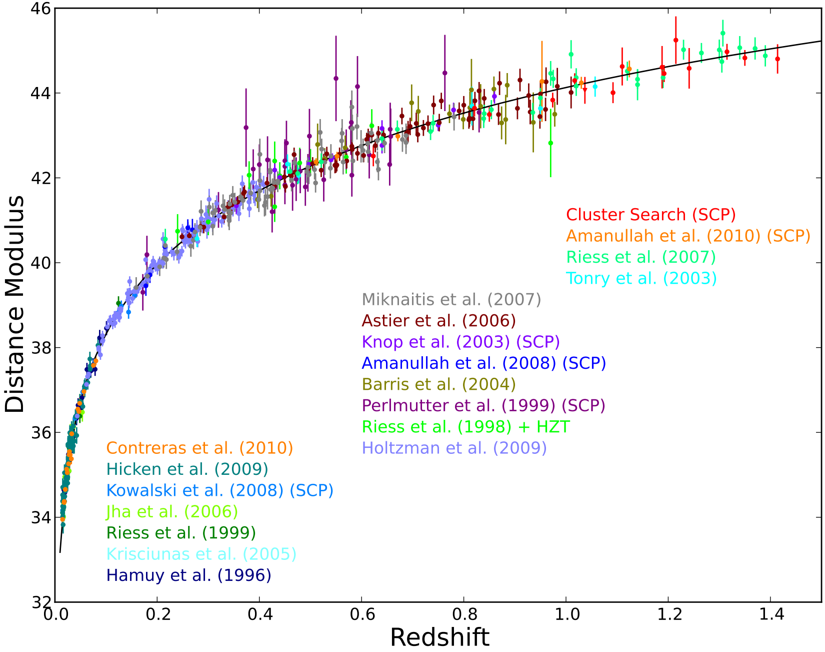
detector: $N_\gamma \propto S^{-1}$, $\omega = \omega_i/(1+z)$, $dt_0 = (1+z)dt_i$
hence the brightness (energy flux measured by a detector) is

$$J = \frac{L}{(1+z)^2 S(z)} \equiv \frac{L}{4\pi r_{ph}^2}\,, \quad r_{ph} = (1+z)\cdot r(z)$$

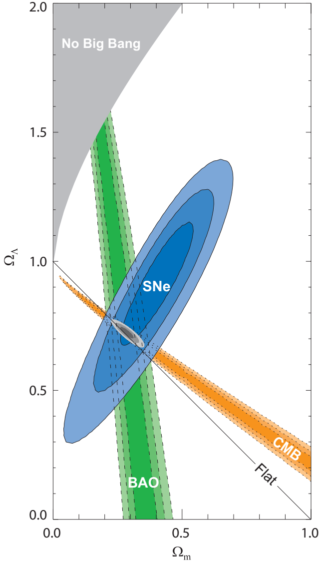
Brightness–redshift dependence: SNe Ia

$$\Delta(m-M) = 5\log\frac{r_{ph}}{r_{ph}(\Omega_c = 0.8, \Omega_M = 0.2)}$$





46
44
42
40
38
36
34
32
Distance Modulus
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
Redshift
Cluster Search (SCP)
Amanullah et al. (2010) (SCP)
Riess et al. (2007)
Tonry et al. (2003)
Miknaitis et al. (2007)
Astier et al. (2006)
Knop et al. (2003) (SCP)
Amanullah et al. (2008) (SCP)
Barris et al. (2004)
Perlmutter et al. (1999) (SCP)
Riess et al. (1998) + HZT
Holtzman et al. (2009)
Contreras et al. (2010)
Hicken et al. (2009)
Kowalski et al. (2008) (SCP)
Jha et al. (2006)
Riess et al. (1999)
Krisciunas et al. (2005)
Hamuy et al. (1996)


2.0
1.5
1.0
0.5
0.0
0.0
0.5
1.0
Ω_Λ
Ω_m
No Big Bang
SNe
CMB
BAO
Flat

Microscopic processes in the expanding Universe

A competition between scattering, decays, etc and expansion

for general processes one should solve kinetic equations

$$\frac{dn_{X_i}}{dt} + 3Hn_{X_i} = \sum(production - destruction)$$

Boltzmann equation in a comoving volume: $\frac{d}{dt}\left(n\,a^3\right) = a^3 \int \dots$

production:

$$\sigma(A+B \to X+C)n_A n_B, \quad \Gamma(D \to E+X)n_D \cdot M_D/E_D, \quad \text{etc}$$

desrtuction:

$$\sigma(A+X \to C+B)n_A n_X, \quad \Gamma(X \to F+G)n_X \cdot M_X/E_X, \quad \text{etc}$$

Fast direct and inverse processes, $\Gamma \gtrsim H$, are in equilibrium,

$\sum(\ ) = 0$ and thermalize particles

Last scattering: $\gamma e \to \gamma e$

$$\sigma_{\rm T} = \frac{8\pi}{3}\frac{\alpha^2}{m_e^2} \approx 0.67\cdot 10^{-24}\ \text{cm}^2\,, \qquad \tau_\gamma = \frac{1}{\sigma_{\rm T}\cdot n_e(T)}$$

last scattering: $\tau_\gamma(T_f) \simeq H^{-1}(T_f) \simeq t_f$

$$T_f = 0.26\ \text{eV}\,, \quad z = 1100\,, \quad t_f = 370\,000\ \text{yr}$$

for general processes one should solve kinetic equations

$$\frac{dn_{X_i}}{dt} + 3Hn_{X_i} = \int (production - destruction)$$

Boltzmann equation in a comoving volume: $\frac{d}{dt}\left(n\,a^3\right) = a^3 \int \dots$

Recombination: horizon

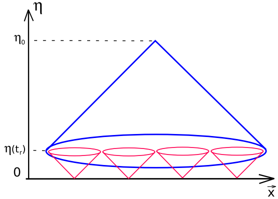
matter domination: $l_{\mathrm{H},r} = 2H_r^{-1}$

$$H_r^2 = \frac{8\pi}{3} G\rho_{\mathrm{M}}(t_r) = \frac{8\pi}{3} G\rho_{\mathrm{M},0}\left(\frac{a_0}{a_r}\right)^3 = \frac{8\pi}{3} G\rho_c \Omega_{\mathrm{M},0}(1+z_r)^3 \,.$$

at recombination: $l_{\mathrm{H},r} = \frac{2}{H_0\sqrt{\Omega_{\mathrm{M}}}}\frac{1}{(1+z_r)^{3/2}}$

today: $l_{\mathrm{H},r}(t_0) = l_{\mathrm{H},r} \times \frac{a_0}{a_r} = \frac{2}{H_0\sqrt{\Omega_{\mathrm{M}}}}\frac{1}{\sqrt{1+z_r}}$

$$\frac{l_{H_0}}{l_{\mathrm{H},r}(t_0)} \sim \sqrt{1+z_r} \simeq 30$$



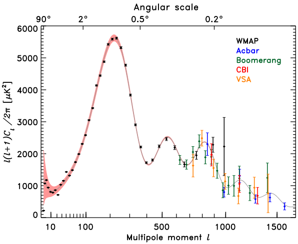
Recombination: angle

angular distance: $$d_{ph} = r_a(z)\,\Delta\theta$$

$$\chi_r = \int_{t_r}^{t_0} \frac{dt}{a(t)}\,, \quad \Delta\theta_r = \frac{l_{H,r}}{r_a(z_r)}$$

$$d_{conf} = \sinh\chi_r\,\Delta\theta$$

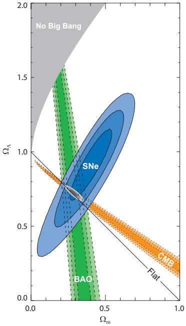
$$r_a(z_r) = (1+z_r)^{-1}\cdot a_0 \cdot \sinh\chi_r$$



$$\Delta\theta_r = \frac{1}{\sqrt{z_r+1}}\,, \quad \Omega_{curv} = \Omega_\Lambda = 0\,.$$

$$\Delta\theta_r = \frac{1}{\sqrt{z_r+1}}\,\frac{2\sqrt{\Omega_{curv}/\Omega_M}}{\sinh\left(2\sqrt{\Omega_{curv}/\Omega_M}\,I\right)}\,.$$

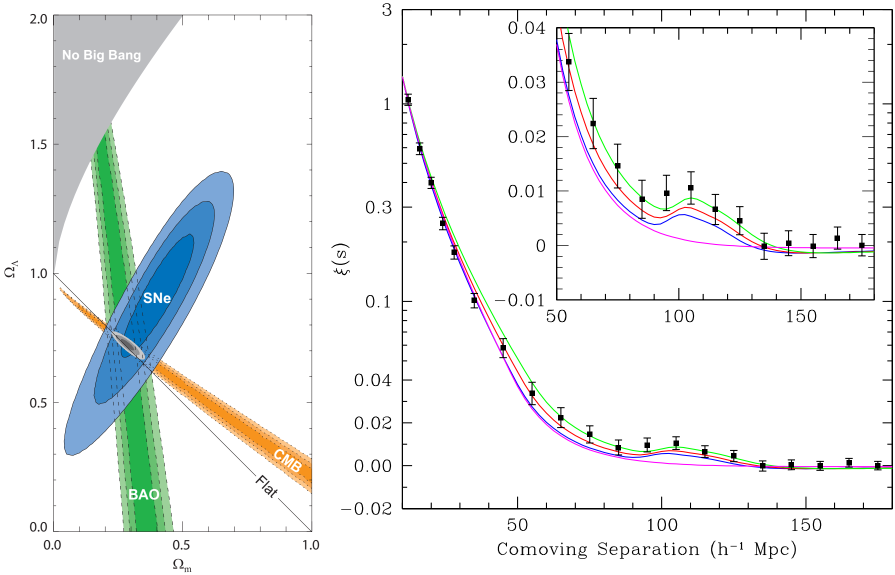
$$I = \int_0^1 \frac{dy}{\sqrt{1+\frac{\Omega_\Lambda}{\Omega_M}y^6}}$$



Acoustic oscillations in relativistic plasma:
What matters is the sound horizon:

$$l_{s,r} = l_{H,r}\cdot v_s \approx l_{H,r}/\sqrt{3}$$

Then $$\Delta\theta_{r,s} = \frac{1}{\sqrt{3}}\,\frac{1}{\sqrt{1+z}} \times \frac{180^\circ}{\pi} \simeq 1^\circ$$



$$110/0.7\,\mathrm{Mpc} \simeq l_{H,r}(t_0) \times \sqrt{v_s^2} \simeq l_{H_0}/\sqrt{3}/\sqrt{1+z_r}$$

Neutrino freeze-out

$T > m_e$ $\quad e^+e^- \leftrightarrow \nu\bar{\nu},\ e\nu \leftrightarrow e\nu$

$$\sigma_\nu \sim G_F^2 E^2$$

neutrino interaction rate

$$\tau_\nu = \frac{1}{\langle \sigma_\nu n v \rangle} \sim \frac{1}{G_F^2 T^5} \qquad H^2 = \frac{8\pi}{3 M_{Pl}^2} \frac{\pi^2}{30} g_* T^4 \equiv \frac{T^4}{M_{Pl}^{*2}}$$

$$\tau_\nu(T) \sim H^{-1}(T) = \frac{M_{Pl}^*}{T^2}$$

$$T_{\nu,f} \sim \left(\frac{1}{G_F^2 M_{Pl}^*} \right)^{1/3} \sim 2 \div 3\ \text{MeV}$$

Neutron decoupling $p + e \longleftrightarrow n + \nu_e$

typical energy scales

$$T \gtrsim \Delta m = 1.3\ \text{MeV}, \qquad T \gtrsim m_e = 0.5\ \text{MeV}$$

neutron interaction rate

$$\tau_{n\leftrightarrow p} = \frac{1}{\Gamma_{n\leftrightarrow p}} = \frac{1}{C_n G_F^2 T^5}$$

neutron decoupling $\Gamma_{n\leftrightarrow p}(T) \sim H(T) = T^2/M_{Pl}^*$

$$T_n = \frac{1}{\left(C_n M_{Pl}^* G_F^2\right)^{1/3}} \approx 0.8\ \text{MeV}$$

$$g_* = 2 + \frac{7}{8}\cdot 4 + \frac{7}{8}\cdot 2\cdot N_\nu \qquad t = \frac{1}{2H(T_n)} = \frac{M_{Pl}^*}{2T_n^2} \approx 1\ \text{s}$$

Neutron density at decoupling

$$n_n = g_n \left(\frac{m_n T}{2\pi}\right)^{3/2} e^{\frac{\mu_n - m_n}{T}} \qquad \mu_n + \mu_\nu = \mu_p + \mu_e$$

$$\frac{n_n}{n_p} = e^{-\frac{m_n - m_p}{T}} e^{\frac{\mu_n - \mu_p}{T}}$$

for relativistic e^+ and e^-

$$n_{e^-} - n_{e^+} \sim \mu_e T^2 \longrightarrow \frac{\mu_e}{T} \sim \frac{n_{e^-} - n_{e^+}}{T^3} = \frac{n_p}{T^3} \sim \eta_B \equiv \frac{n_p}{n_\gamma} = 6 \times 10^{-10}$$

$$\frac{n_n}{n_p} = e^{-\frac{m_n - m_p}{T_n}} \equiv e^{-\frac{\Delta m}{T_n}} \approx \frac{1}{5} e^{-\frac{\mu_\nu}{T_n}}$$

$p + n \longrightarrow \mathrm{D} + \gamma$ Saha equation

$$n_n = 2\left(\frac{m_p T}{2\pi}\right)^{3/2} \mathrm{e}^{\frac{\mu_n - m_n}{T}}\,, \qquad n_p = 2\left(\frac{m_p T}{2\pi}\right)^{3/2} \mathrm{e}^{\frac{\mu_p - m_p}{T}}\,,$$

Chemical equilibrium for nuclei: $\mu_A = \mu_p \cdot Z + \mu_n \cdot (A - Z)$

$$n_D \sim \left(\frac{m_D T}{2\pi}\right)^{3/2} \mathrm{e}^{\frac{\mu_D - m_D}{T}}\,,$$

Temperature of BBN T_{NS}: $n_\mathrm{D} \sim n_n$

$\Delta_\mathrm{D} = 2.23$ MeV $t_{NS} \approx 3\,\mathrm{min}$

$$n_\mathrm{D}/n_p(T_{NS}) \sim \eta_\mathrm{B} \left(\frac{2.5\,T_{NS}}{m_p}\right)^{3/2} \mathrm{e}^{\frac{\Delta_\mathrm{D}}{T_{NS}}} \sim 1 \longrightarrow T_{NS} \approx 50\ \mathrm{keV}$$

Helium abundance (NO chemical equilibrium)

Neutrons remain mostly in helium

$$n_{^4\mathrm{He}}(T_{NS}) = \frac{1}{2} n_n(T_{NS}) \,,$$

neutron-to-proton ratio $\tau_n \approx 880$ s

$$\frac{n_n(T_{NS})}{n_p(T_{NS})} \approx \frac{1}{5} \cdot e^{-\frac{t_{NS}}{\tau_n}} \cdot e^{-\frac{\mu_\nu}{T_n}} \approx \frac{1}{7} \,,$$

$$Y_p \equiv X_{^4\mathrm{He}} = \frac{m_{^4\mathrm{He}} \cdot n_{^4\mathrm{He}}(T_{NS})}{m_p\,(n_p(T_{NS}) + n_n(T_{NS}))} = \frac{2}{\frac{n_p(T_{NS})}{n_n(T_{NS})} + 1} \approx 25\%$$

from observations of relic helium abundance:

$$\Delta N_{\nu,eff} \leq 1 \,, \qquad \left| \frac{\mu_\nu}{T_n} \right| \lesssim 0.01$$

Main nuclear reactions

1. $p(n,\gamma)$D — deuterium production, BBN starts.
2. D$(p,\gamma)^3$He, D(D,n)^{3}He, D(D,p)T, ^{3}He(n,p)T — intermediate stage.
3. T(D,n)^{4}He, ^{3}He(D,p)^{4}He — production of ^{4}He.
4. T$(\alpha,\gamma)^7$Li, ^{3}He$(\alpha,\gamma)^7$Be, ^{7}Be$(n,p)^7$Li — production of the heaviest baryonic relics.
5. ^{7}Li$(p,\alpha)^4$He — ^{7}Li burning.

One has to compare reaction rates to the expansion rate

$$H(T_{NS} = 50\text{ keV}) = 10^{-2}\text{ s}^{-1}$$

to obtain nonequilibrium concentrations

Neutron burning $p + n \longrightarrow \mathrm{D} + \gamma$

@ $T = T_{NS} = 65$ keV

$$(\sigma v)_{p(n,\gamma)D} \approx 6 \cdot 10^{-20} \; \frac{\mathrm{cm}^3}{\mathrm{s}} \; .$$

for the rate (neutron disappearence when meets proton)

$$\Gamma_{p(n,\gamma)D} = n_p \cdot (\sigma v)_{p(n,\gamma)D} = \eta_B \cdot 2 \frac{\zeta(3)}{\pi^2} T^3 \cdot (\sigma v)_{p(n,\gamma)D} = 0.31 \; \mathrm{s}^{-1}$$

for $\eta_B = 6.15 \cdot 10^{-10}$ and $T = T_{NS}$

So, neutrons disappear very rapidly

$$\Gamma_{p(n,\gamma)D} \gg H(T_{NS}) = 4 \cdot 10^{-3} \; \mathrm{s}^{-1}$$

Deuterium burning $D(D,n)^3He$, $D(D,p)T$

Coloumb barier: tunneling $T_9 \equiv T/(10^9\ \text{K}) = T/(86\ \text{keV})$

$$\langle \sigma v \rangle_{DD} = 3 \cdot 10^{-15} \frac{\text{cm}^3}{\text{s}} \cdot T_9^{-2/3} \cdot e^{-4.26 \cdot T_9^{-1/3}} .$$

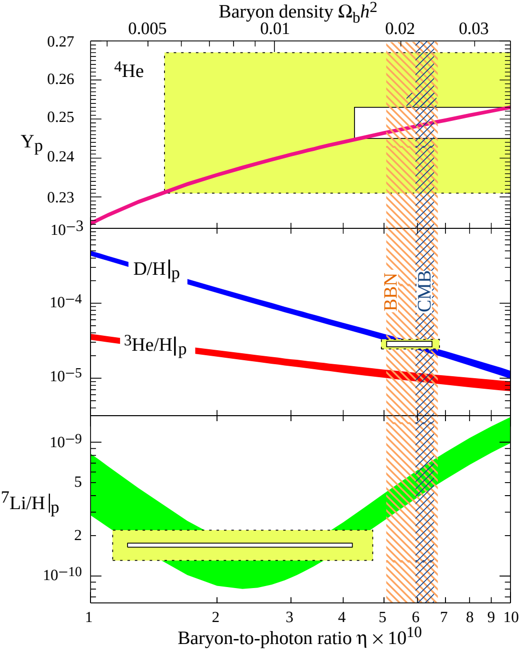
deuterium stops burning when $T = T_{NS} (T_9 = 0.75)$

$$\Gamma_{DD} = n_D(T) \cdot \langle \sigma v \rangle_{DD}(T) \sim H(T) .$$

Then relic deuterium abundance is estimated as

$$\frac{n_D}{n_p} = \frac{1}{0.75 \eta_B} \cdot \frac{n_D}{n_\gamma(T_{NS})} = 0.3 \cdot 10^{-4}$$

for $\eta_B = 6.15 \cdot 10^{-10}$



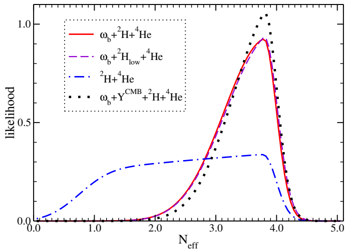
Measurement of $\eta_B = n_B/n_\gamma$ at $T \sim 1$ MeV

Lack of Lithium. . . Exotics needed?

$$Y_p = 0.2581 \pm 0.025\,,$$

$$D/H\big|_p = (2.87 \pm 0.21) \times 10^{-5}$$

1103.1261

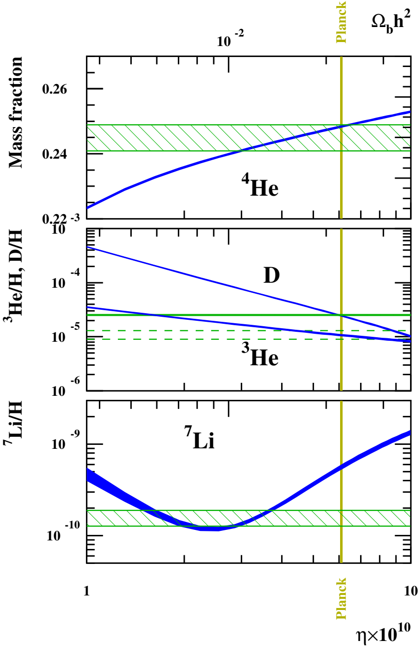


similar results from other recent studies including structure formation

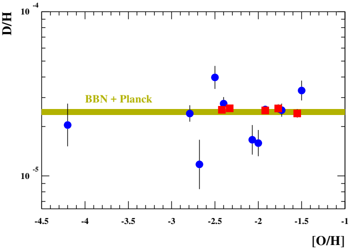
1001.4440, 1001.5218, 1202.2889

$N_{\nu,eff} < 4.2$ @ 95%CL

$N_{\nu,eff} < 4.0$ from D/H 1205.3785

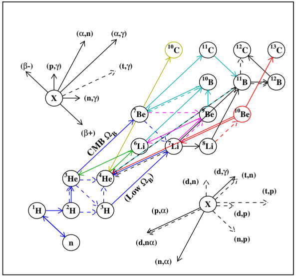


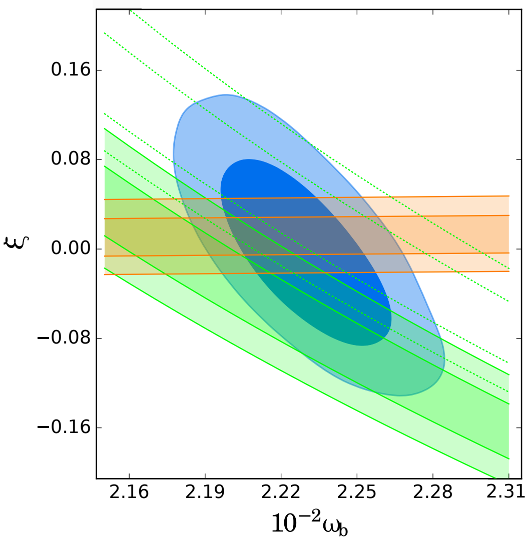
1707.01004 Measurement of $\eta_B = n_B/n_\gamma$ at $T \sim 1$ MeV



Lack of Lithium. . .

Exotics needed?





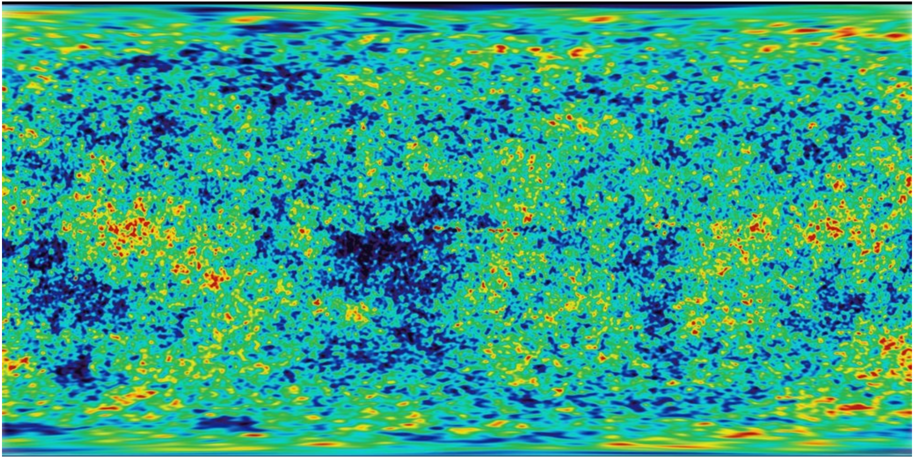
1706.01705

Primordial Element Abundance Observations:

- Lack of Lithium. . . Exotics needed?
- Measurement of $\eta_B = n_B/n_\gamma$ at $T \sim 1$ MeV consistent with present and recombination values
  – no "decaying relics"
- measurement of the Universe expansion rate $H^2 \sim \rho_{relativistic}$
  in particular:
  – neutrino number $N_\nu \approx 3$
  – no "dark radiation"

Baryon asymmetry must be produced before BBN !!

CMB map



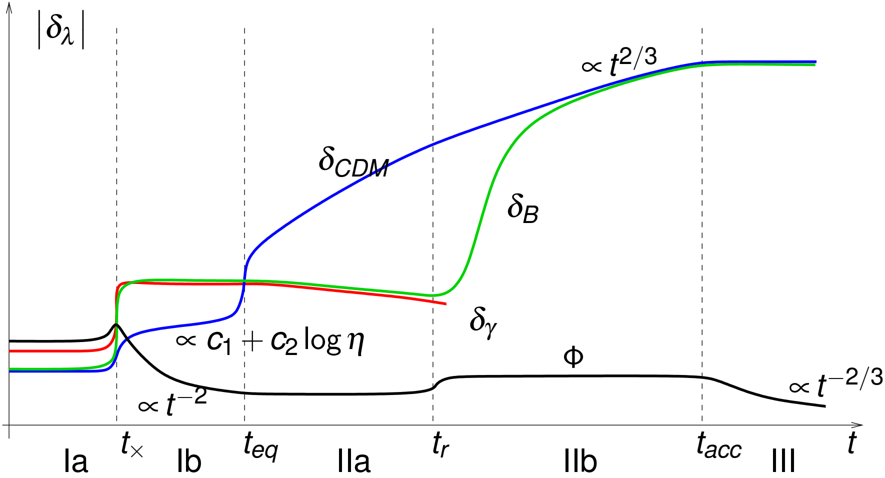
Mode evolution

- Amplitude remains constant, while superhorizon, e.g. $k/a < H$
- Subhorizon Inhomogeneities of DM start to grow at MD-stage, $\delta\rho_{CDM}/\rho_{CDM} \propto a$ from $T \approx 0.8\,\text{eV}$
  Smaller objects (first stars, dwarf galaxies) are first to form
- Subhorizon Inhomogeneities of baryons join those of DM only after recombination, $\delta\rho_{CDM}/\rho_{CDM} \propto a$ from $T_{rec} \approx 0.25\,\text{eV}$
- at recombination $\delta\rho_B/\rho_B \sim \delta T/T \sim 10^{-4}$ and would grow only by a factor $T_{rec}/T_0 \sim 10^3$ without DM
- Subhorizon Inhomogeneities of photons $\delta\rho_\gamma/\rho_\gamma$ oscillate with constant amplitude at RD and with decreasing amplitude at MD, thus we can measure $T_{RD/MD}/T_{rec}$
- Phase of oscillations decoupled after recombination depends on the wave-length, recombination time and sound speed

$$\delta\rho_\gamma/\rho_\gamma \propto \cos\left(k\int_0^{t_r}\frac{v_s\,dt}{a(t)}\right) = cos(kl_{sound})$$

- $$\delta T(\theta,\varphi) = \sum a_{lm} Y_{lm}(\theta,\varphi)\,, \qquad \langle a_{lm}^* a_{lm}\rangle = C_l \equiv 2\pi\mathscr{D}_l/(l(l+1))$$

Mode evolution at various stages



On formulas...

- short waves, $k\eta_{eq} \gg 1$ $R_B \equiv 3\rho_B/4\rho_\gamma$

$$\delta_\gamma = \Phi_{(i)} \cdot \left[-324 \cdot (1+R_B)\, l^2 \frac{\Omega_{CDM}}{\Omega_M} (1+z_{eq}) \frac{\log(0.2k\eta_{eq})}{(k\eta_0)^2} + \frac{6}{(1+R_B)^{1/4}} \cos\left(k \int_0^\eta d\tilde{\eta}\, u_s \right) \right] ,$$

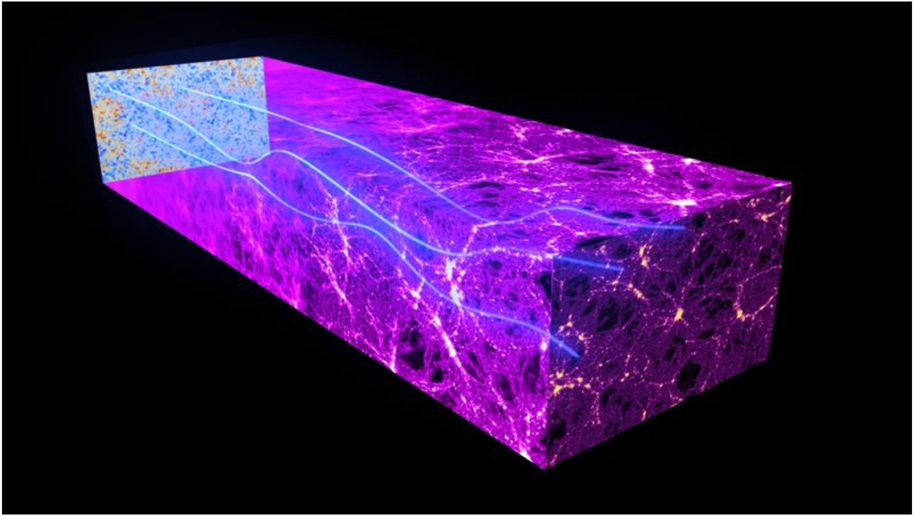
- long waves, $k\eta_{rec} \ll 1$

$$\delta_\gamma = -\frac{12}{5}\Phi_{(i)} = \text{const}$$

- intermediate waves ...

$$\delta_\gamma(\mathbf{k},\eta) = -4\left[1+R_B(\eta)\right]\Phi(\mathbf{k},\eta) + 4\Phi_{(i)}(\mathbf{k}) \cdot A(k,\eta) \cos\left(k \int_0^\eta u_s d\tilde{\eta} \right) ,$$

On top of that: propagation in expanding Universe



On formulas. . .

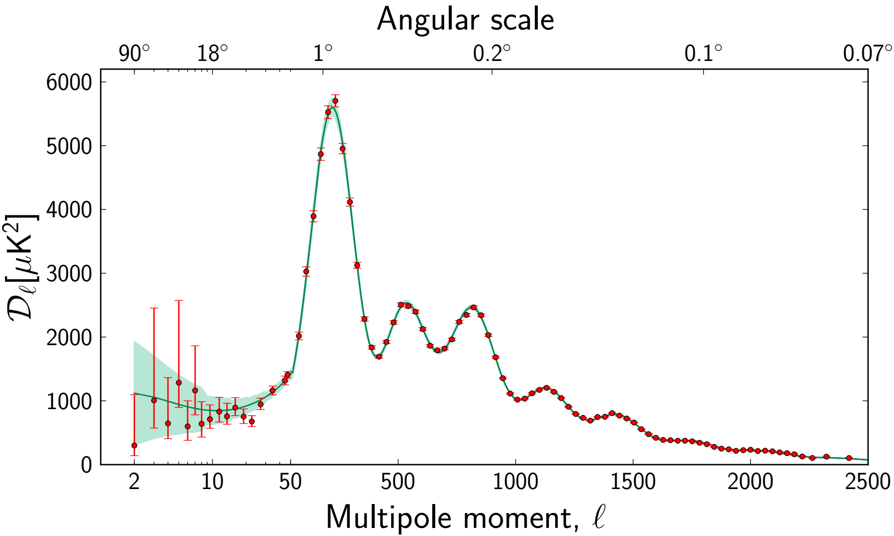
From linear approximation to the geodesic equation. . .
for scalar perturbations

$$\frac{\delta T}{T}(\mathbf{n},\eta_0) = \frac{1}{4}\delta_\gamma(\eta_r) + (\Phi(\eta_r) - \Phi(\eta_0)) + \int_{\eta_r}^{\eta_0} \left(\Phi' - \Psi'\right) d\eta + \mathbf{nv}(\eta_r) - \mathbf{nv}(\eta_0)\,.$$

for tensor perturbations

$$\frac{\delta T}{T}(\mathbf{n},\eta_0) = \frac{1}{2}\int_{\eta_r}^{\eta_0} d\eta\, n_i h_{ij}^{TT\,\prime} n_j\,,$$

CMB measurements (Planck) $\theta, \Omega_{DM}, \Omega_B, \tau, \Delta_{\mathscr{R}}, n_s$



Standard cosmological model $ds^2 = dt^2 - a^2(t)dx^2$

$$\left(\frac{\dot{a}}{a}\right)^2 \equiv H^2 = H_0^2 \left[\Omega_\Lambda + (\Omega_{DM} + \Omega_B + \Omega_{\nu,m\neq 0})\left(\frac{a_0}{a}\right)^3 + (\Omega_\gamma + \Omega_{\nu,m=0})\left(\frac{a_0}{a}\right)^4\right]$$

- $T_\gamma = 2.735\,\mathrm{K}, \quad \Longrightarrow \quad \Omega_\gamma \sim 10^{-5}$
- $N_\nu \approx 3, \quad \sum m_\nu < 0.2\,\mathrm{eV} \quad \Longrightarrow \quad \Omega_{\nu,\neq 0}, \Omega_{\nu,0} \sim 10^{-5}$?
- $\Omega_B = 4.5\% \quad \Longrightarrow \quad \eta_B \equiv n_B/n_\gamma = 6 \times 10^{-10}$
- $\Omega_{DM} = 27.5\%$
- $H_0 = 67\,\mathrm{km/s/Mpc} \quad \Longrightarrow \quad \rho_0 = 5\,\mathrm{GeV/m^3}$
- $\Omega_\Lambda = 68\% \quad \Longrightarrow$ flat space
- adiabatic, gaussian matter perturbations

$$\langle \left(\frac{\delta\rho}{\rho}\right)^2 \rangle \sim A_S \int \frac{dk}{k}\left(\frac{k}{k_*}\right)^{n_S - 1}$$

  with $A_S = 3 \times 10^{-9}$ and $n_S = 0.97$
- no tensor perturbations, $r \equiv A_T/A_S < 0.05$
- reionization at $z \equiv a_0/a = 10$