Inflation and reheating in the early Universe
Lecture #5
Inflation nad reheating

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“Quantum fields:
from gravity and cosmology
to physics of condensed matter”
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Outline

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Inflation: general remarks

- How long? Solution to the horizon problem:

$$1 \lesssim \frac{l_{H,e}(t_0)}{l_{H,0}} \Rightarrow N_e^{tot} \gtrsim \log \frac{T_0}{H_0} + \ln \frac{a(t_e)}{a_{reh}} + \ln \frac{H(t_{Pl})}{T_{reh}} \simeq 50 - 60$$

Inflation lasts not less than (accepting $H^2 \sim \rho / M_{Pl}^2$)

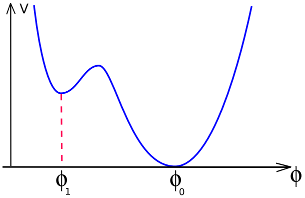
$$\Delta t_{infl} \sim N_e^{tot} / H_e \sim 10^{-11}\,\text{c} \cdot \left(\frac{1\,\text{TeV}}{T_{reh}} \right)^2$$

we must reheat the Universe then!

- In realistic models $N_e^{tot} \ggg 100$!!!
  Inflatinary stage may be short, but expansion is enormous!

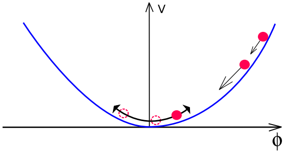
Inflatinary stage: simplest models

“Old inflation” by Guth



does not work in fact!
starts from a hot stage
and ends up in a false vacuum
reheating due to percollations
However: for sufficiently long inflationary stage requires
$\Gamma < H^4_{infl}$
hence the bubbles never collide!

“Chaotic inflation”



needs superplanckian field values!

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$

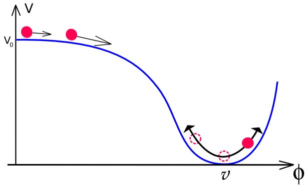
$$p = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

$$\varepsilon = \frac{M_{Pl}^2}{16\pi}\left(\frac{V'}{V}\right)^2 \, , \; \eta = \frac{M_{Pl}^2}{8\pi}\frac{V''}{V} \, ,$$

$V(\phi) \propto \phi^n \;\Rightarrow \varepsilon, \eta \sim M_{Pl}^2/\phi^2 \ll 1$ $\leftarrow$ slow-roll conditions

“New inflation”



Initial condition is very specific!

$$H^2 = \frac{8\pi}{3M_P^2}V(\phi) \, , \quad a(t) \propto e^{Ht}$$

and we require

$$V(\phi) < M_{Pl}^4$$

Unexpected bonus: generation of perturbations

- Quantum fluctuations of wavelength λ of a free massless field φ have an amplitude of $\delta\varphi_\lambda \simeq 1/\lambda$
- In the expanding Universe: $\lambda \propto a$

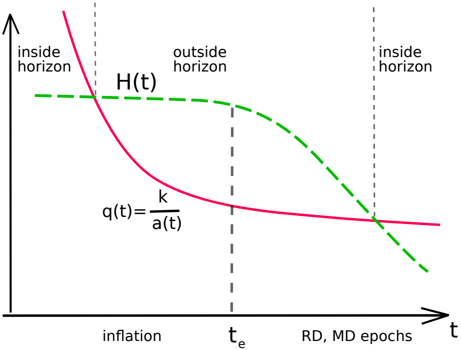
  inflation: $l_H \sim 1/H = \text{const}$, so modes "exit horizon"
  Ordinary stage: $l_H \sim 1/H \propto t$, $l_H/\lambda \nearrow$, modes "enter horizon"

Evolution at inflation

- inside horizon: $\lambda < l_H$
  $\lambda \propto a \Rightarrow$ $\delta\varphi_\lambda \propto 1/\lambda \propto 1/a$

- outside horison: $\lambda > l_H$
  $\lambda \propto a \Rightarrow$ $\delta\varphi_\lambda = \text{const} = H_{infl}$!!!

- got "classical" fluctuations:
  $\delta\varphi_\lambda = \delta\varphi_\lambda^{quantum} \times e^{N_e}$



Power spectrum of perturbations

In the Minkowski space-time:

- fluctuations of a free quantum field φ are gaussian its power spectrum is defined as

$$\int_0^\infty \frac{dq}{q}\,\mathscr{P}_\varphi(q) \equiv \langle \varphi^2(x) \rangle = \int_0^\infty \frac{dq}{q}\,\frac{q^2}{(2\pi)^2}$$

We define amplitude as $\delta\varphi(q) \equiv \sqrt{\mathscr{P}_\varphi} = q/(2\pi)$

- In the expanding Universe momenta $q = k/a$ gets redshifted
- Cast the solution in terms $\phi(\mathbf{x},t) = \phi_c(t) + \varphi(\mathbf{x},t)$, $\varphi(\mathbf{x},t) \propto e^{\pm i\mathbf{kx}}\,\varphi(\mathbf{k},t)$
  φ solves the equation

$$\ddot{\varphi} + 3H\dot{\varphi} + \frac{k^2}{a^2}\,\varphi + V''(\phi_c)\varphi = 0$$

- $q = k/a \gg H$ $\Rightarrow$ as in Minkowski space-time
- $q = k/a \ll H$ $\Rightarrow$ for inflaton $\varphi = \text{const}$
- Matching at t_k: $q(t_k) = k/a(t_k) = H(t_k) \equiv H_k$ gives

$$\delta\varphi(q) = \frac{H_k}{2\pi} \;\Rightarrow\; \mathscr{P}_\varphi(q) = \frac{H_k^2}{(2\pi)^2}$$

amplification $H_k/q = e^{N_e(k)}$!!!

$H_k \approx \text{const} = H_{infl}$ hence (almost) flat spectrum

Transfer to matter perturbations: simple models

Illustration: Local delay(advance)δt in evolution due to impact of $\delta\phi$ of all modes with $\lambda > H$:

$$\delta\phi = \dot{\phi}_c\,\delta t\,, \quad \delta\rho \sim \dot{\rho}\,\delta t$$

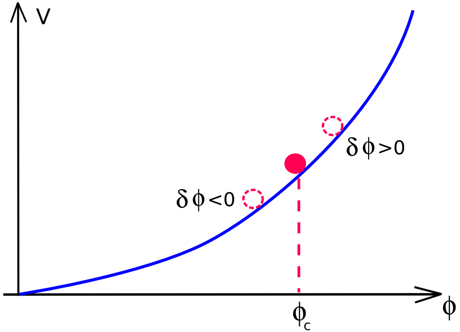
at the end of inflation $\dot{\rho} \sim -H\rho$, then

$$\frac{\delta\rho}{\rho} \sim \frac{H}{\dot{\phi}_c}\,\delta\phi$$

Hence, $\delta\rho/\rho$ is also gaussian.
Power spectrum of scalar perturbations

$$\mathscr{P}_{\mathscr{R}}(k) = \left(\frac{H^2}{2\pi\,\dot{\phi}_c}\right)^2\,,$$

everything is calculated at $t = t_k$: $H = k/a$



Analogously for the tensor perturbations: each of the two polarizations of the gravity waves solves the free scalar field equation!

$$\mathscr{P}_T(k) = \frac{16}{\pi}\,\frac{H_k^2}{M_{Pl}^2}$$

To the leading order no k-dependence: both spectra are “flat” (scale-invariant)!

Inflaton parameters and spectral parameters

- Observation of CMB anisotropy gives $\delta T/T$

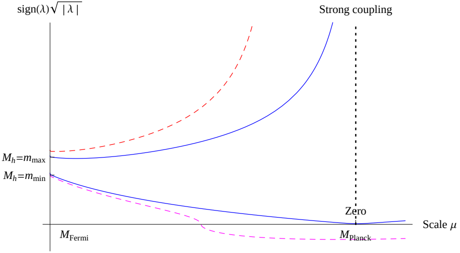
$$\frac{\delta T}{T} \sim \frac{\delta\rho}{\rho} \Rightarrow \Delta_{\mathscr{R}} \equiv \sqrt{\mathscr{P}_{\mathscr{R}}} = 5\times 10^{-5}$$

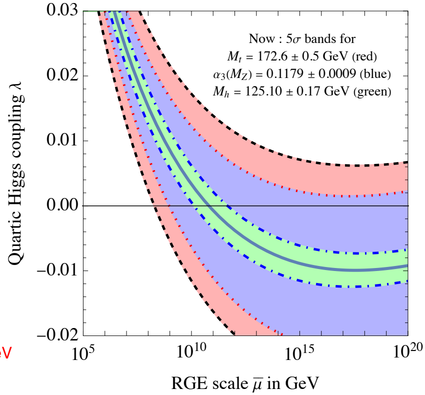
- These are so-called adiabatic perturbations!
  Other possibles (isocurvature) modes (e.g. $\delta T = 0$, but $\delta n_B/n_B \neq 0$) are not found.
- $\Delta_{\mathscr{R}} = 5\times 10^{-5} \Rightarrow$ fixes model paramaters, e.g.:

$$V(\phi) = \frac{\beta}{4}\phi^4 \to \lambda \sim 10^{-13}$$

  With such a tiny coupling perturbations are obviously gaussian
  So far confirmed by observations
- That's why Higgs boson in the SM does not help!
  However, it can be exploited as inflaton if non-minimally coupled to gravity $\xi RH^\dagger H$

Critical point: where EW-vacuum becomes unstable





F.Bezrukov, M.Shaposhnikov (2009)
F.Bezrukov, D.G. (2011)
F.Bezrukov, M.Kalmykov, B.Kniehl, M.Shaposhnikov (2012)

G. Degrassi et al (2012)

$$m_h^{cr} > \left[129.0 + \frac{m_t - 172.9\,\text{GeV}}{1.1\,\text{GeV}} \times 2.2 - \frac{\alpha_s(M_Z) - 0.1181}{0.0007} \times 0.56\right]\,\text{GeV}$$

2203.17197

present theoretical uncertainties 0.5 GeV

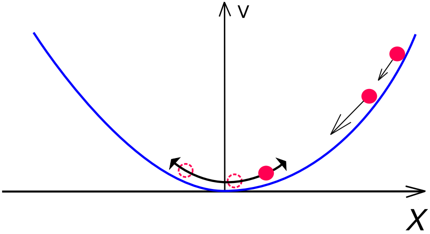
Important for inflation, when usually $h \sim H$

Inflation & Reheating: simple realization with Higgs

$$\ddot{X} + 3H\dot{X} + V'(X) = 0$$

$X_e > M_{Pl}$

generation of scale-invariant scalar (and tensor) perturbations from exponentially stretched quantum fluctuations of X



Chaotic inflation, A.Linde (1983)

$\delta\rho/\rho \sim 10^{-5}$ requires
$V = \beta X^4 : \beta \sim 10^{-13}$

reheating ? renormalizable?

the only choice: $\alpha H^\dagger H X^2$
"Higgs portal"

larger α larger T_{reh}

quantum corrections $\propto \alpha^2 \lesssim \beta$

No scale, no problem

Inflaton parameters and spectral parameters

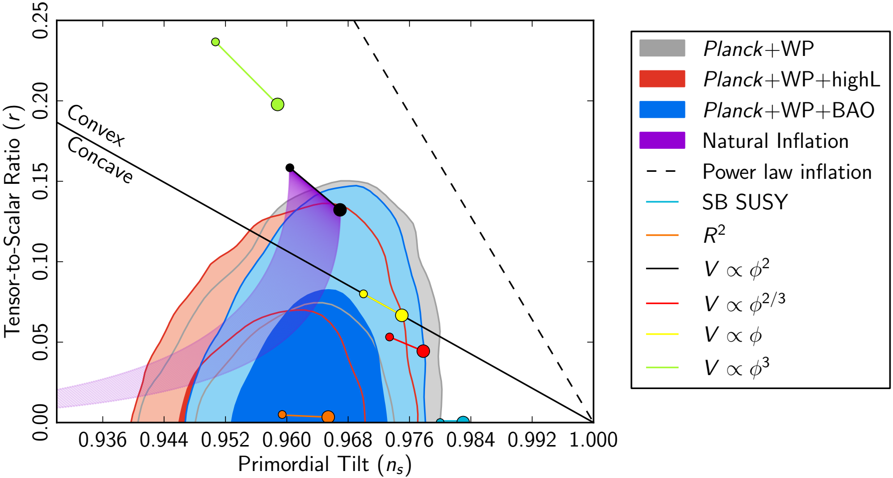
- In fact, spectra are a bit tilted, as H_{infl} slightly evolves

$$\mathcal{P}_{\mathcal{R}}(k) = A_{\mathcal{R}} \left(\frac{k}{k_*}\right)^{n_s - 1} , \qquad \mathcal{P}_T(k) = A_T \left(\frac{k}{k_*}\right)^{n_T} .$$

- Measure $\Delta_{\mathcal{R}}$ at present scales $q \simeq 0.002/\mathrm{Mpc}$, it fixes the number of e-foldings left N_e
- For tensor perturbations one introduces:

$$r \equiv \frac{\mathcal{P}_T}{\mathcal{P}_{\mathcal{R}}} = \frac{1}{\pi} \frac{M_{Pl}^2 V'^2}{V} = 16\,\varepsilon \;\; \to \;\; \frac{16}{N_e} \;\; \text{for} \;\; \beta\phi^4$$

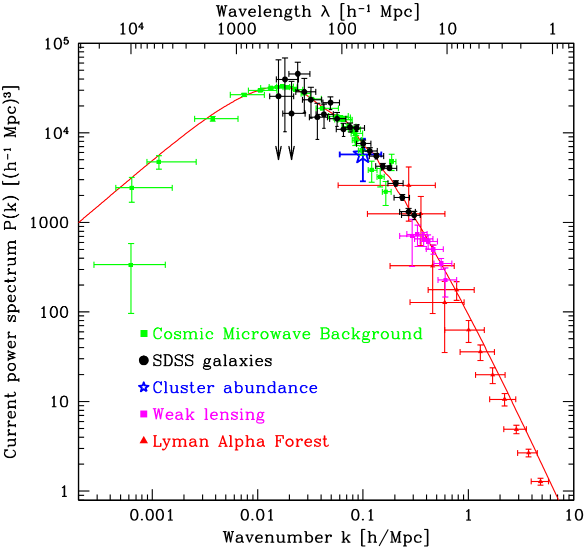
Recent analysis (Planck) of cosmlogical data



1303.5062

$N_e = 50 - 60$

Actually we observe rather narrow range



Observable range:

$$\frac{k_{max}}{k_{min}} \sim 10^5$$

$$\Delta N_e \simeq 10$$

Small scales cannot describe:
for a long time in nonlinear regime

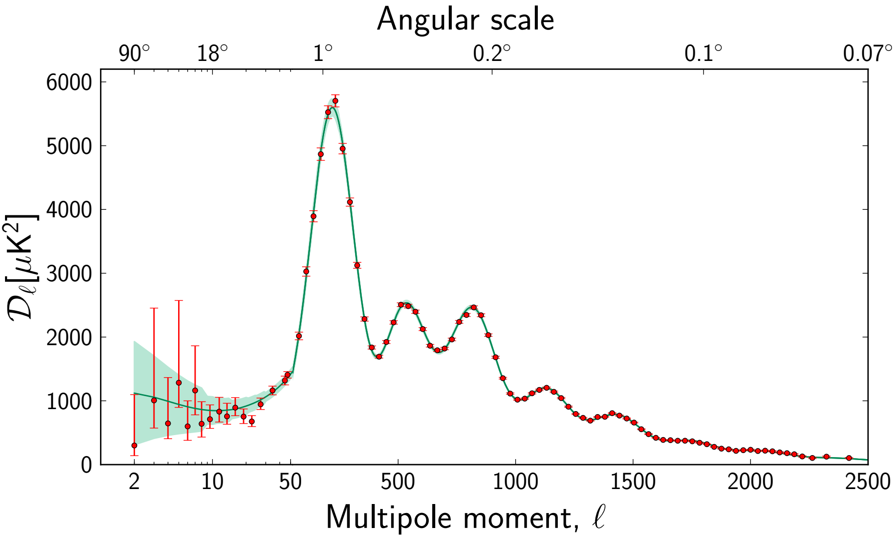
Mode evolution

- Amplitude remains constant, while superhorizon, e.g. $k/a < H$
- Subhorizon Inhomogeneities of DM start to grow at MD-stage, $\delta\rho_{CDM}/\rho_{CDM} \propto a$ from $T \approx 0.8\,\text{eV}$
  Smaller objects (first stars, dwarf galaxies) are first to form
- Subhorizon Inhomogeneities of baryons join those of DM only after recombination, $\delta\rho_{CDM}/\rho_{CDM} \propto a$ from $T_{rec} \approx 0.25\,\text{eV}$
- at recombination $\delta\rho_B/\rho_B \sim \delta T/T \sim 10^{-4}$ and would grow only by a factor $T_{rec}/T_0 \sim 10^3$ without DM
- Subhorizon Inhomogeneities of photons $\delta\rho_\gamma/\rho_\gamma$ oscillate with constant amplitude at RD and with decreasing amplitude at MD, thus we can measure $T_{RD/MD}/T_{rec}$
- Phase of oscillations decoupled after recombination depends on the wave-length, recombination time and sound speed

$$\delta\rho_\gamma/\rho_\gamma \propto \cos\left(k\int_0^{t_r} \frac{v_s\,dt}{a(t)}\right) = cos(kl_{sound})$$

- $$\delta T(\theta,\varphi) = \sum a_{lm} Y_{lm}(\theta,\varphi)\,, \qquad\qquad \langle a^*_{lm} a_{lm}\rangle = C_l \equiv 2\pi\mathscr{D}_l/(l(l+1))$$

CMB measurements (Planck) $H_0, \Omega_{DM}, \Omega_B, \Omega_\Lambda, \Delta_{\mathscr{R}}, n_s$



Other ways of testing inflation

- Curvature: the World is flat
  not convincing for many
- Relic tensor modes (gravitational waves)
  low-l B-mode: well below Galactic foreground
- preheating: $T_{reh} \to N_e$, GW ?
  tiny effects, $n_s, r = f(log(N_e))$, GW from clumps
- Direct tests: inflaton potential
  only in specific models with light inflaton
- Generic for many-field inflation are
  isocurvature modes, non-Gaussianity
- Exotic signatures
  primordial black holes, GW from oscillons, etc

Outline

Reheating

After inflation we must produce particles
to enter the radiation dominating stage
i.e. we must reheat the Universe

inflaton couples to SM

- perturbative... e.g. decays:

  $\phi \to hh$, reheating at $H = \Gamma$

- through oscillations induced by inflaton

  time-dependent external force $F(t)$ or mass $m(t)$

  — can be resonantly amplified !!

  — most efficient: tachionic, when $m^2(t) < 0$

Particle production I

equation of motion

$$\ddot{\phi}(t,\mathbf{x}) - \Delta\phi(t,\mathbf{x}) + m^2\phi(t,\mathbf{x}) = 0 \qquad \phi \propto e^{iEt+i\mathbf{kx}}$$

for particular 3-momenta looks as oscillator

$$\ddot{\phi}_k(t) + \left(\mathbf{k}^2 + m^2\right)\phi_k(t) = 0 \qquad \phi(t,\mathbf{x}) = \int d^3x\,\phi_k(t)e^{i\mathbf{kx}}$$

Quantum physics:

even in vacuum (no particles) $\qquad \phi_k = \phi_k^{vac}(t) \neq 0 \qquad$!!

Particle production II

In the expanding Universe

$$\ddot{\phi}_k(t) + 3H(t)\,\dot{\phi}_k(t) + \left(\frac{\mathbf{k}^2}{a^2(t)} + m^2\right)\phi_k(t) = 0$$

interaction with inflaton $X(t)$, e.g. $X^2\phi^2$:

$$\ddot{\phi}_k(t) + 3H(t)\,\dot{\phi}_k(t) + \left(\frac{\mathbf{k}^2}{a^2(t)} + m^2 + X^2(t)\right)\phi_k(t) = 0$$

oscillator with time-dependent frequency can be excited if
— $\Omega_X \gg \Omega_{\phi_k}$ high-frequency (this case)

among other generic options
— at zero crossings, that is $\Omega_X^{eff} \simeq 0$ large field X
— at tachyonic time slots with $\Omega_X^{eff\,2} < 0$

Natural completion with R^2

Y.Ema (2017), D.G., A.Tokareva (2018)

$\xi h^2 R$ induces R^2-term

hep-th/9510140

$$S_0 = \int d^4x\sqrt{-g}\left(-\frac{M_P^2+\xi h^2}{2}R + \frac{\beta}{4}R^2 + \frac{(\partial_\mu h)^2}{2} - \frac{\lambda}{4}(h^2-v^2)^2\right).$$

introduce a Lagrange multiplier L and auxiliary scalar $\mathscr{R}$

$$S = \int d^4x\sqrt{-g}\left(\frac{(\partial_\mu h)^2}{2} - \frac{\lambda}{4}(h^2-v^2)^2 - \frac{M_P^2+\xi h^2}{2}\mathscr{R} + \frac{\beta}{4}\mathscr{R}^2 - L\mathscr{R} + LR\right).$$

integrate out $\mathscr{R}$

$$S = \int d^4x\sqrt{-g}\left(\frac{(\partial_\mu h)^2}{2} - \frac{\lambda}{4}(h^2-v^2)^2 + LR - \frac{1}{\beta}(L + \frac{1}{2}\xi h^2 + \frac{1}{2}M_P^2)^2\right)$$

$$\xi \to \xi^2/\beta$$

with

$$\beta \gtrsim \frac{\xi^2}{4\pi}$$

everything here look healthy

Further transformations...

Y.Ema (2017)

introducing scalaron ϕ with $m = M_P/\sqrt{3\beta}$

$$g_{\mu\nu} \to \Omega^2 g_{\mu\nu}\,, \quad \Omega^2 \equiv \frac{2L}{M_P^2}\,, \;\; L \to \phi \equiv M_P\sqrt{\frac{2}{3}}\log\Omega^2\,.$$

and setting $M_P = 1/\sqrt{6}$

$$S = \int d^4x\sqrt{-g}\left(-\frac{R}{12} + \frac{1}{2}e^{-2\phi}(\partial h)^2 + \frac{1}{2}(\partial\phi)^2 - \frac{1}{4}e^{-4\phi}\left(\lambda h^4 + \frac{1}{36\beta}(e^{2\phi} - 1 - 6\xi h^2)^2\right)\right)$$

both gravity and scalar sector are weakly coupled up to M_P with $\beta \gtrsim \xi^2/(4\pi)$

And one more...

D.G., A.Tokareva (2018)

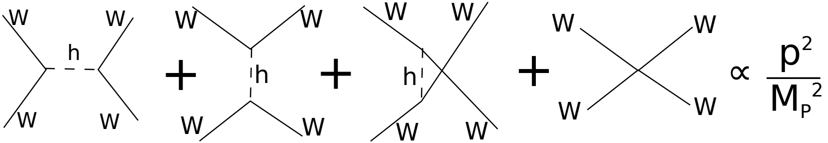
$$h = e^{\Phi}\tanh H,\; \phi = e^{\Phi}/\cosh H,$$

The scalar sector becomes

$$L = \frac{1}{2}\cosh^2 H(\partial\Phi)^2 + \frac{1}{2}(\partial H)^2 - \frac{\lambda}{4}\sinh^4 H - \frac{\lambda}{144\beta}\left(1 - e^{-2\Phi}\cosh^2 H - 6\xi\sinh^2 H\right)^2.$$

and the Higgs coupling to gauge bosons, e.g.,

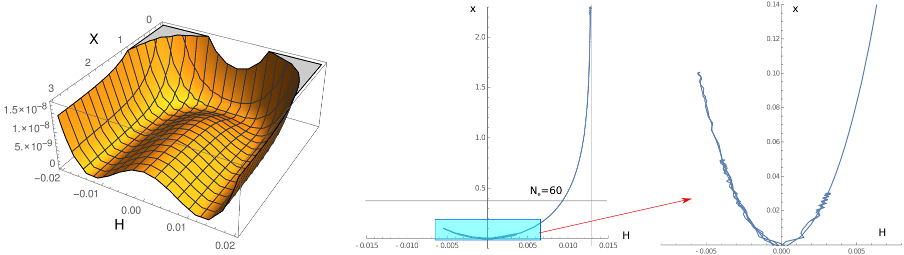
$$L_{gauge} = \frac{g^2 h^2}{4} e^{-2\phi} W^+_\mu W^-_\mu = \frac{g^2}{4}\sinh^2 H\; W^+_\mu W^-_\mu\,.$$



$$\mathscr{A} \sim \frac{g^2 p^2}{m_W^2}\left(\frac{4}{g^2}\left(\frac{dm_W(H)}{dH}\right)^2 - 1\right) \;\to \mathscr{A} \propto \frac{p^2}{M_P^2}$$

Cosmological spectra

D.G., A.Tokareva 1807.02392



Scalar perturbations: adiabatic

1701.07665

$$\beta + \frac{\xi^2}{\lambda} \simeq 2 \times 10^9$$

At small β like in the Higgs-inflation

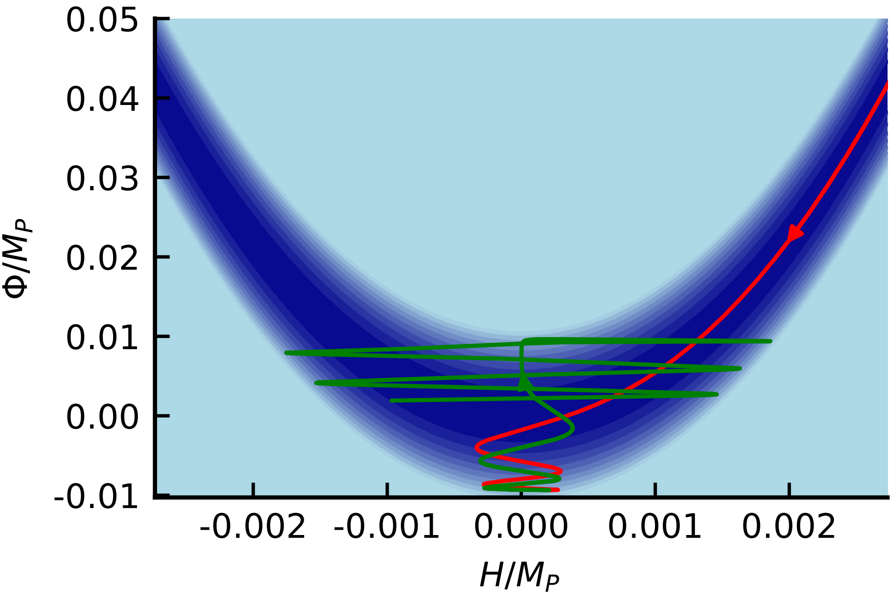
heavy scalaron is integrated out

$$\frac{\xi^2}{4\pi} < \beta < \frac{\xi^2}{\lambda} \quad \to \quad 5 \times 10^{13}\,\text{GeV} < m < 1.5 \times 10^{15}\,\text{GeV}$$

We can calculate observables at any energy scale up to Planck

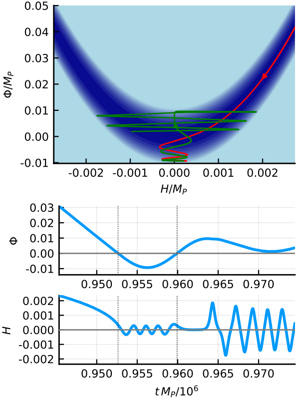
Reheating. . . all masses depend on oscillating Higgs

- Huge spikes do not reheat !! 1812.10099
- it is a highly nonlinear system
- ω^2 for W_L and Z_L rapidly oscillates and becomes negative for some time
- similar for one of the scalars (a mixture of Higgs and scalaron)
- we expect instant preheating, at least for a region in model parameter space F.Bezrukov, D.G., Ch.Shepherd, A.Tokareva (2019)
- but for precise number the backreaction must be taken into account

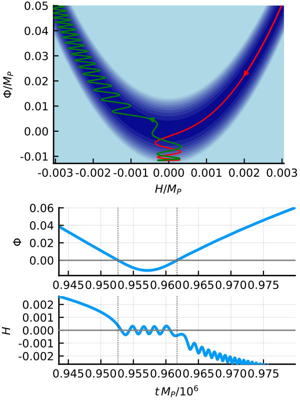

0.05
0.04
0.03
0.02
0.01
0.00
-0.01
Φ/M_P
-0.002
-0.001
0.000
0.001
0.002
H/M_P

Scalaron Φ and Higgs H evolution after inflation

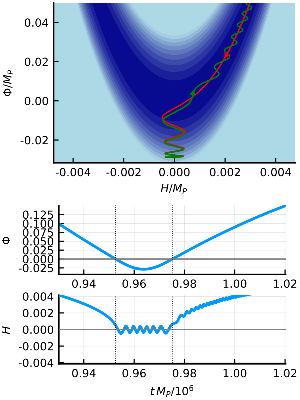
Homogeneous modes (mixing in kinetic sector), $\dot{\Phi} < 0$, $\dot{\Phi} > 0$



$\beta = 1.869 \times 10^6$



$\beta = 2.9 \times 10^6$



$\beta = 18 \times 10^6$

$$V(H,\Phi) = \frac{1}{4}\left(\lambda + \frac{\xi^2}{\beta}\right)H^4 + \frac{M_P^2}{6\beta}\Phi^2 - \frac{\xi M_P}{\sqrt{6}\beta}\Phi H^2 + \frac{7}{108\beta}\Phi^4 + \frac{\xi}{6\beta}\Phi^2 H^2 - \frac{M_P}{3\sqrt{6}\beta}\Phi^3$$

Linear equations for gauge bosons

Gauge bosons (e.g. $W^{\pm}$)

$$L_g^{(2)} = -\frac{1}{2}\left(\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+\right)\left(\partial_\lambda W_\rho^- - \partial_\rho W_\lambda^-\right) g^{\mu\lambda} g^{\nu\rho} + \frac{g^2 H_0^2}{4} W_\mu^+ W_\nu^- g^{\mu\nu},$$

transverse modes

$$\ddot{W}_k^T + 3\mathscr{H}\dot{W}_k^T + \frac{k^2}{a^2} W_k^T + m_T^2 W_k^T = 0, \qquad m_T \equiv \frac{g}{2} H_0$$

longitudinal modes

$$\ddot{W}_k^L + 3\mathscr{H}\dot{W}_k^L + \omega_W^2(\mathbf{k}) W_k^L = 0.$$

$$\omega_W^2(\mathbf{k}) = \frac{k^2}{a^2} + m_T^2 - \frac{k^2}{k^2 + a^2 m_T^2}\left(\dot{\mathscr{H}} + 2\mathscr{H}^2 + 3\mathscr{H}\frac{\dot{m}_T}{m_T} + \frac{\ddot{m}_T}{m_T} - \frac{3(\dot{m}_T + \mathscr{H} m_T)^2}{k^2/a^2 + m_T^2}\right).$$

for $k/a \gg m_T$ after inflation

$$\omega_W^2 = \frac{k^2}{a^2} + \frac{g^2}{4} H_0^2 + \frac{\xi}{3\beta}\Phi_0^2 + \left(\lambda + \frac{\xi^2}{\beta}\right) H_0^2 - \frac{\xi\sqrt{2}}{\beta\sqrt{3}} M_P \Phi_0,$$

Linear equations for scalaron and Higgs

A mixutre of the two scalars

$$m^2_{L,H} = \frac{1}{2}\left(V_{H_0H_0} + V_{\Phi_0\Phi_0}\right) \times \left(1 \pm \sqrt{1 - 4\frac{V_{\Phi_0\Phi_0}V_{H_0H_0} - V^2_{\Phi_0H_0}}{\left(V_{H_0H_0} + V_{\Phi_0\Phi_0}\right)^2}}\right).$$

after inflation can be approximated as

$$m^2_{H,L} \approx V_{H_0H_0} \approx 2\left(\lambda + \frac{\xi^2}{\beta}\right)H_0^2 + \left(\lambda + \frac{\xi^2}{\beta}\right)H_0^2 - \frac{\sqrt{2}\xi}{\sqrt{3}\beta}M_P\Phi_0$$

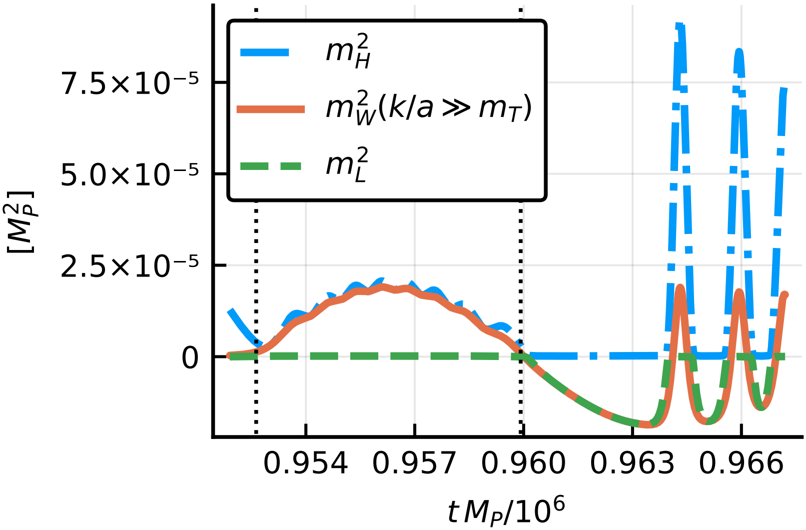
Then we calculate the Bogolubov coefficients from the field solutions $f_{\mathbf{k}}(t) = e^{-i\omega t}/\sqrt{2\omega(\mathbf{k})}$ at $t \to 0$, which gives for the number density

$$n_{\mathbf{k}} = \frac{1}{2}\left|\sqrt{\omega(\mathbf{k})}f_{\mathbf{k}} - \frac{i}{\sqrt{\omega(\mathbf{k})}}\dot{f}_{\mathbf{k}}\right|^2$$

and the physical energy

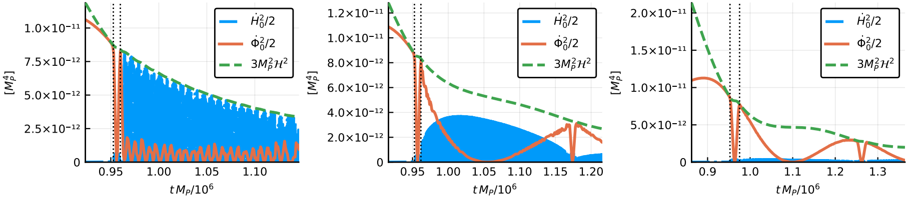
$$\rho = \int \frac{d^3\mathbf{k}}{(2\pi)^3a^3(t)}\omega(\mathbf{k})n_{\mathbf{k}}.$$

Numerical results: mass squared

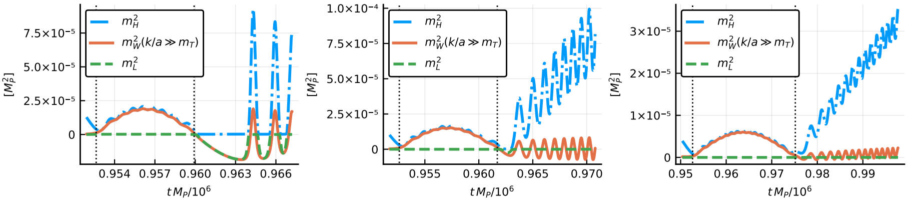


Numerical results for perturbations

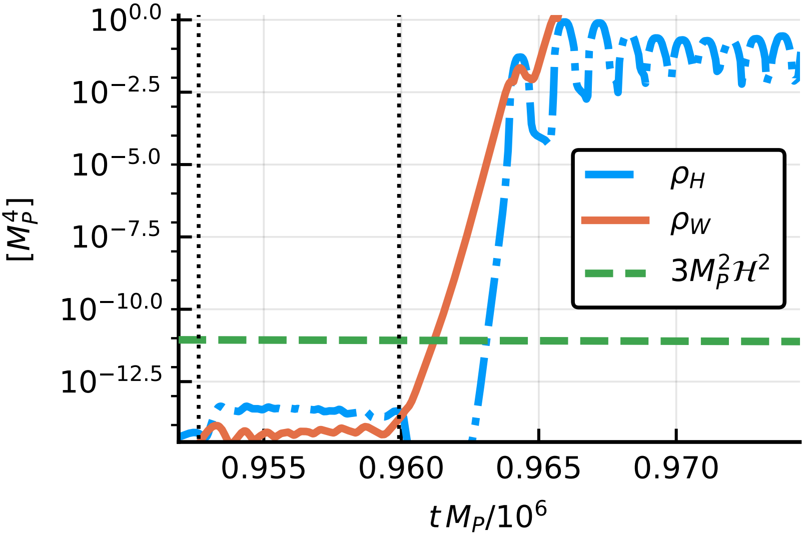
energy distribution over homogeneous modes



mass squared for the relevant perturbations

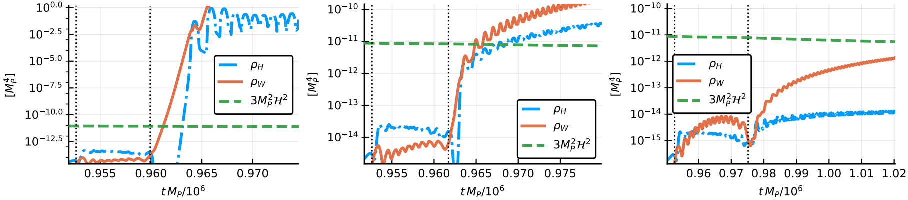


Numerical results: energy in perturbations

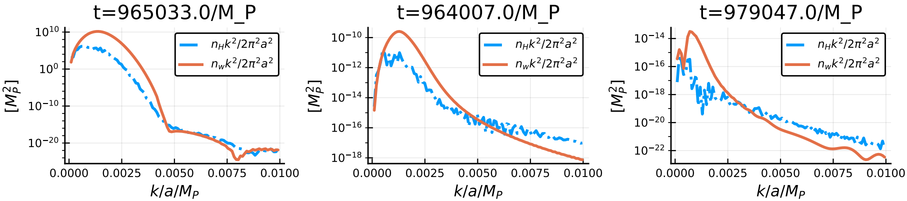


Spectra and energy density of produced particles

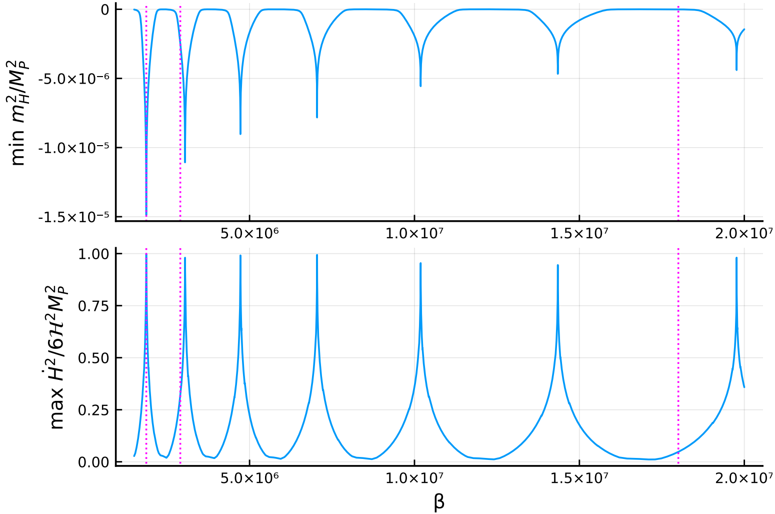
neglecting the adiabaticity conditions... and backreaction



spectra at a reference moment



The resonance positions and energy in Higgs between two zero crossings are correlated



Direct check of the inflation potential

– Higgs frequency is much and scalaron frequency is significantly higher than the expansion rate:
It seems that the reheating is instant (can be refined at NLO)

$$N_e = 59, \quad n_s = 0.97, \quad r = 0.0034.$$

– Higgs selfcoupling becomes canonical λ below the scalaron scale $\mu = M_P/\sqrt{3\beta}$

$$V(H,\Phi) = \frac{1}{4}\left(\lambda + \frac{\xi^2}{\beta}\right)H^4 + \frac{M_P^2}{6\beta}\Phi^2 - \frac{\xi M_P}{\sqrt{6}\beta}\Phi H^2 + \frac{7}{108\beta}\Phi^4 + \frac{\xi}{6\beta}\Phi^2 H^2 - \frac{M_P}{3\sqrt{6}\beta}\Phi^3$$

cancellation: $\xi M_P/\beta \times 1/\mu^2 \times \xi M_P/\beta \to \xi^2/\beta$