Lorentz-Invariant Path Integral Measure

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The measure for relativistic particle

$$w_\sigma(d\xi) = \exp\left(-\frac{1}{2\sigma^2}\int_0^T \left\{(\dot{\xi}^0)^2 + (\dot{\xi}^1)^2\right\} d\tau\right) d\xi$$

$$\exp\left(-\frac{1}{2\sigma^2}\int_0^T \left\{(\dot{\xi}^0)^2 - (\dot{\xi}^1)^2\right\} d\tau\right) d\xi =$$

$$= \exp\left(-\frac{1}{2\sigma^2}\int_0^T \left\{(\dot{\xi}^0)^2 + \alpha(\dot{\xi}^1)^2\right\} d\tau\right) d\xi \qquad \alpha = -1\,.$$

Lorentz-invariant measure

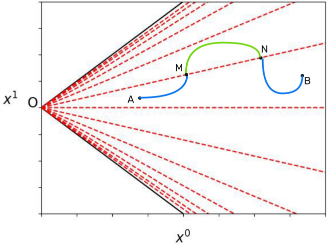
$$\mathrm{Cone} = \{(\xi^0, \xi^1), |\xi^1| < \xi^0, \xi^0 > 0\}$$

$$\tilde{w}_\sigma(d\xi) = \exp\left(-\frac{1}{2\sigma^2}\int_0^T \left\{(\dot{\xi}^0)^2 - (\dot{\xi}^1)^2 + 2\frac{(\dot{\xi}^0\xi^1 - \dot{\xi}^1\xi^0)^2}{(\xi^0)^2 - (\xi^1)^2}\right\}\right) d\xi =$$

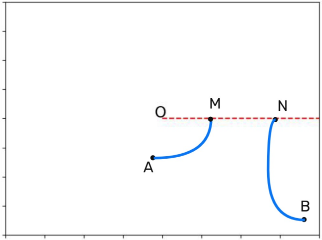
$$= \exp\left(-\frac{1}{2\sigma^2}\int_0^T \left\{\dot{r}^2 + r^2\dot{\theta}^2\right\} d\tau\right) d\xi$$

$$\xi^0 = r\cosh\theta\,, \qquad \xi^1 = r\sinh\theta\,.$$

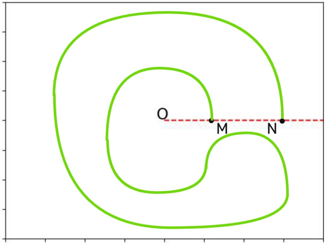
Cone and Cover



(a)



(b)



(c)