



1 Introduction

There are a large number of experiments investigating both neutrino oscillations and their interactions. In both cases, it is important to theoretically investigate neutrino scattering on various targets [1–3], since scattering processes are either a tool for detecting neutrino fluxes as well as studying fundamental interactions of neutrinos. Also, the beyond Standard-Model theories discuss properties of right-handed and sterile neutrinos [4, 5]. In this work, we present analytical and numerical results for the effects of neutrino electromagnetic properties, spin polarization, and possible sterile neutrino states. The neutrino electromagnetic properties emerge in different extensions of the Standard Model, and they include [6]: millicharges, charge radii, electric, magnetic and anapole moments.

2 Active neutrinos

We consider the process where an ultrarelativistic neutrino with 4-momentum $k^\mu = (E_\nu, \mathbf{k})$ originates from a source (reactor, accelerator, Sun, etc.) and elastically scatters on a particle in a detector at energy-momentum transfer $q = (T, \mathbf{q})$. If the neutrino has a nonzero magnetic moment, its spin-flavor oscillations may take place due to interactions with a magnetic field in the source and with interstellar and/or intergalactic magnetic fields. In the most general case, the neutrino state in the detector can be mixed in both flavor/mass space and spin space. We can describe the neutrino state before scattering by a statistical operator acting in the Hilbert space of neutrino mass and spin states as

$$\rho_{ij} = \frac{1}{2} \mathbb{k} \left(\hat{\rho}_{ij} - \zeta_{ij}^\parallel \gamma_5 + (\boldsymbol{\zeta}_{ij}^\perp \cdot \boldsymbol{\gamma}_\perp) \gamma_5 \right), \quad (1)$$

where $\hat{\rho}_{ij} = \frac{1}{2E_\nu} \text{tr}(\rho_{ij} \gamma^0)$ is a reduced density matrix in the neutrino mass space. $\zeta_{ij}^\parallel$ and, with $\{\mathbf{e}_x, \mathbf{e}_y, \mathbf{k}/E_\nu\}$ forming a 3-vector basis, $\boldsymbol{\zeta}_{ij}^\perp$ are the matrices of longitudinal ($\parallel$) and transverse ($\perp$) components, with respect to the neutrino momentum $\mathbf{k}$, corresponding to the diagonal ($i = j$) and transition ($i \neq j$) values of the spin operator $\boldsymbol{\Sigma}$ averaged over the spin states of the neutrino in its rest frame. If there is a state with zero neutrino mass, then the diagonal and transition $\zeta^\perp$ components that involve this state vanish.

We assume that the target particle is free and at rest in the lab frame. The matrix element of the transition $\nu_\ell(L) + N \rightarrow \nu_j + N$, where N is either a nucleon or a nucleus (the electron case can be found in our recent work [3]), due to weak interaction is given by

$$\mathcal{M}^{(w)} = -\frac{G_F}{\sqrt{2}} \bar{u}_{k',r'}^{(\nu_n)} \gamma^\mu (1 - \gamma^5) \delta^{ni} u_{k,r}^{(\nu_i)} J_\mu^{(NC)}(q), \quad (2)$$

where $J_\lambda^{(NC)}(q)$ is the weak neutral current of the particle, $\bar{u}_{k',r'}^{(\nu_n)} = u_{k',r'}^{(\nu_n)\dagger} \gamma^0$, where $u_{k,r}^{(\nu_i)}$ is the bispinor amplitude of the massive neutrino state $|\nu_i\rangle$ with 4-momentum $\mathbf{k}$ and spin state r . The matrix element due to electromagnetic interaction is

$$\mathcal{M}^{(\gamma)} = \frac{4\pi\alpha}{q^2} \bar{u}_{k',r'}^{(\nu_n)} \Lambda_\mu^{(EM;\nu)ni}(q) u_{k,r}^{(\nu_i)} J_\mu^{(EM)}(q), \quad (3)$$

where $J_\mu^{(EM)}(q)$ is the EM current of the target particle and $\Lambda_\mu^{(EM;\nu)ni}(q)$ is the neutrino EM vertex.

In what follows, for illustrative purposes, we assume that the target particle is a nucleon. The cases of a target electron and a target nucleus can be found in our recent work [3]. The nucleon currents can be expanded as follows:

$$\begin{aligned} J_\lambda^{(NC)}(q) &= \bar{u}_\nu^{(N)}(p') \Lambda_\lambda^{(NC;N)}(-q) u_s^{(N)}(p), \\ J_\lambda^{(EM)}(q) &= \bar{u}_\nu^{(N)}(p') \Lambda_\lambda^{(EM;N)}(-q) u_s^{(N)}(p), \end{aligned} \quad (4)$$

where $\Lambda_\lambda^{(NC;N)}(q)$ and $\Lambda_\lambda^{(EM;N)}(-q)$ are the nucleon NC weak and EM vertices, respectively. We consider the following vertices:

$$\begin{aligned} \Lambda_\mu^{(EM;\nu)fi}(q) &= (\gamma_\mu - q_\mu \not{q}/q^2) [f_Q^{fi}(q^2) + f_A^{fi}(q^2) q^2 \gamma_5 - i\sigma_{\mu\nu} q^\nu [f_M^{fi}(q^2) + i f_E^{fi}(q^2) \gamma_5], \\ \Lambda_\mu^{(EM;N)}(q) &= \gamma_\mu F_Q^N(q^2) - \frac{i}{2m_N} \sigma_{\mu\nu} q^\nu F_M^N(q^2) \\ &\quad + \frac{1}{2m_N} \sigma_{\mu\nu} q^\nu \gamma_5 F_E^N(q^2) - (q^2 \gamma_\mu - q_\mu \not{q}) \gamma_5 \frac{F_A^N(q^2)}{m_N^2}, \\ \Lambda_\mu^{(NC;N)}(q) &= \gamma_\mu F_1^N(q^2) - \frac{i}{2m_N} \sigma_{\mu\nu} q^\nu F_2^N(q^2) - \gamma_\mu \gamma_5 G_A^N(q^2) + \frac{1}{m_N} G_P^N(q^2) q_\mu \gamma_5. \end{aligned} \quad (5)$$

In the case of Dirac neutrinos, when evaluating the cross section, we neglect the neutrino masses. Since the final massive state of the neutrino is not resolved in the detector, the differential cross section as a function of the polar (θ) and azimuthal (φ) angles of the recoil nucleon in terms of the Mandelstam variables $s = (k + p)^2 = m_N^2 + 2E_\nu m_N$ and $t = (k - k')^2 = -\frac{4m_N^2 E_\nu^2 \cos^2 \theta}{(E_\nu + m_N)^2 - E_\nu^2 \cos^2 \theta}$ is given by

$$\frac{d\sigma}{d\Omega} = \frac{d^2\sigma}{\sin\theta d\theta d\varphi} = \frac{|\mathcal{M}|^2 t^2}{64\pi^2 (s - m_N^2)^2 m_N^2} \left(1 - \frac{4m_N^2}{t} \right)^{3/2}, \quad (6)$$

where averaging over initial and summing over final spin polarizations of the nucleon and summing over final neutrino polarizations and mass states are assumed. The cross section can be split into the following parts: $\frac{d\sigma}{d\Omega} = \frac{d\sigma^L}{d\Omega} + \frac{d\sigma^R}{d\Omega} + \frac{d\sigma^\perp}{d\Omega}$, where $\frac{d\sigma^K}{d\Omega}$ are cross sections with projection of the initial neutrino state on the left ($K = L$) and right ($K = R$) helicity states, respectively. The cross section $\frac{d\sigma^\perp}{d\Omega}$ describes the contribution from the interference of the helicity states and depends on the transverse neutrino spin components ζ_{ij}^x and ζ_{ij}^y . It is important to note that in the case of right-handed neutrino scattering cross section only EM interactions can contribute. The cross section $\frac{d\sigma}{d\Omega}$ after integrating over φ and expressing $\cos\theta$ in terms of energy transfer T determines the shape of the recoil-nucleon spectrum, which can be measured in experiment. Since the formulae for the cross section are cumbersome [2], we present here only numerical results.

Obviously, manifestations of neutrino EM properties should be much more pronounced in neutrino-proton scattering than in neutrino-neutron scattering. In order to illustrate the characteristic effects of these properties, in the following we present the results of numerical calculations of the differential cross sections for elastic neutrino-proton scattering.

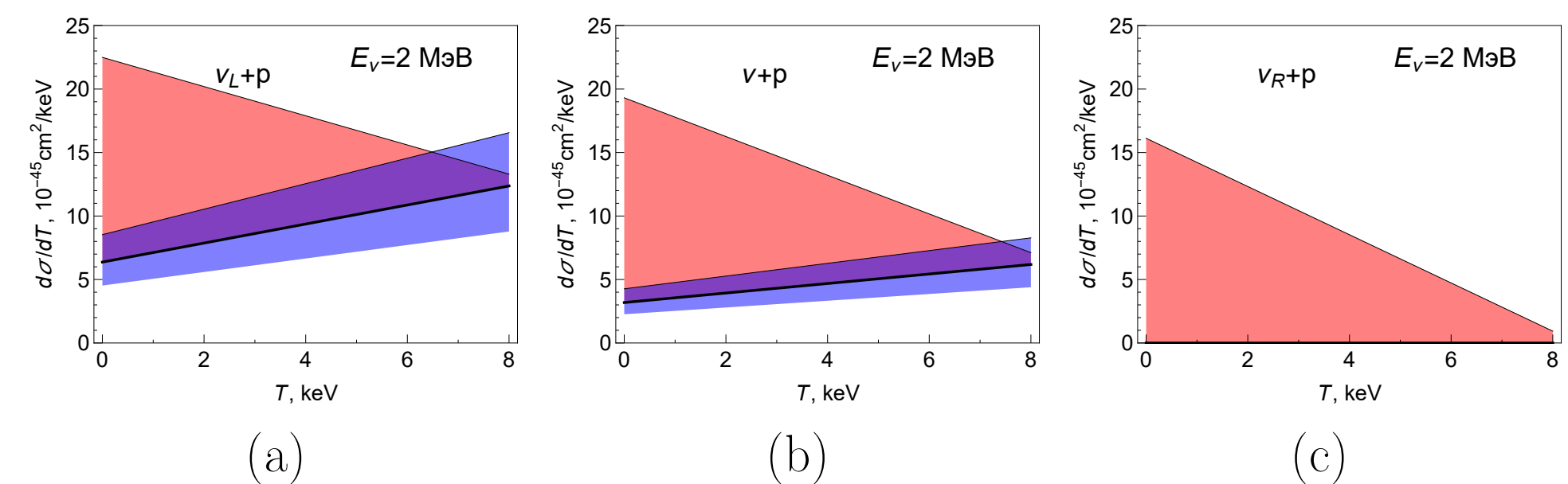


Figure 1: The differential cross sections of elastic neutrino-proton scattering for incident (a) left-handed, (b) unpolarized and (c) right-handed neutrinos at $E_\nu = 2$ MeV.

We restrict ourselves to the case of EM charge and magnetic form factors of a nucleon, accounting for the relation between the nucleon neutral weak and EM form factors

$$\begin{aligned} F_{1,2}^p(q^2) &= \left(\frac{1}{2} - 2\sin^2\theta_W \right) F_{Q,M}^p(q^2) - \frac{1}{2} F_{Q,M}^n(q^2) - \frac{1}{2} F_{1,2}^S(q^2), \\ F_{1,2}^n(q^2) &= \left(\frac{1}{2} - 2\sin^2\theta_W \right) F_{Q,M}^n(q^2) - \frac{1}{2} F_{Q,M}^p(q^2) - \frac{1}{2} F_{1,2}^S(q^2), \\ G_{A,P}^N(q^2) &= \frac{7}{2} G_{A,P}^a(q^2) - \frac{1}{2} G_{A,P}^s(q^2), \end{aligned} \quad (7)$$

where $F_{1,2}^S$, $G_{A,P}^S$ are strange form factors of the nucleon. We use the parameterization that can be found in [7] (and references therein), and for the strange form factors see [8].

We present the numerical results for incident left-handed, unpolarized, and right-handed Dirac neutrino states, accounting for the nucleon strange form-factor contribution and the neutrino transition charge radii (Fig. 1) with limits according to current constraints [9]:

$$0 \leq |\langle r_{e\mu}^2 \rangle|, |\langle r_{e\tau}^2 \rangle|, |\langle r_{\mu\tau}^2 \rangle| < 3 \times 10^{-31}. \quad (8)$$

Note that right Dirac neutrinos can interact electromagnetically.

3 Sterile neutrino

Let us consider the process where an active flavor neutrino ν_ℓ scatters from the proton and becomes a fourth massive sterile neutrino ν_4 due to weak or electromagnetic interaction. Since there is a sterile neutrino, the mixing matrix U is now a 3×4 matrix and is no longer unitary, because $UU^\dagger = \mathbb{1}_{3 \times 3}$, but $U^\dagger U \neq \mathbb{1}_{4 \times 4}$. Thus, the weak NC interaction is not diagonal in mass basis. In case of EM interaction, we consider transition active-to-sterile charge radii.

The cross section differential with respect to the energy transfer T for the process $\nu_l + N \rightarrow \nu_4 + N$ is given by

$$\begin{aligned} \frac{d\sigma^L}{dT} &= \frac{d\sigma_0^L}{dT} + \frac{d\sigma_{m_4}^L}{dT}, \\ \frac{d\sigma_0^L}{dT} &= \frac{G_F^2 m_N}{2\pi} \left[2 \left(1 + \frac{st}{(s - m_N^2)^2} \right) \left(\tilde{C}_V^L + \tilde{C}_A^L - \frac{t}{4m_N^2} \tilde{C}_M^K \right) \right. \\ &\quad - \frac{4m_N^2 t}{(s - m_N^2)^2} \left(\tilde{C}_A^L - \frac{t}{4m_N^2} \tilde{C}_M^L \right) + \frac{t^2}{(s - m_N^2)^2} \left(\tilde{C}_V^L + \tilde{C}_A^L - 2\text{Re} \tilde{C}_{V\&M}^L \right) \\ &\quad \left. + \frac{2t}{s - m_N^2} \left(2 + \frac{t}{s - m_N^2} \right) \text{Re} (\tilde{C}_{V\&A}^L - \tilde{C}_{A\&M}^L) \right], \\ \frac{d\sigma_{m_4}^L}{dT} &= \frac{G_F^2 m_N m_4^2}{2\pi (s - m_N^2)^2} \left[- (2s + t) \tilde{C}_V^L + \tilde{C}_A^L (4m_N^2 - 2s - t) + 2t \text{Re} (\tilde{C}_{A\&M}^L - \tilde{C}_{V\&A}^L) \right. \\ &\quad - \frac{\tilde{C}_M^L}{4m_N^2} \left(2m_N^2 (m_4^2 + t) + \frac{t}{2} (m_4^2 - 4s - t) \right) + \text{Re} \tilde{C}_{V\&M}^L (m_4^2 + t) \\ &\quad \left. + (t - m_4^2) \left(\frac{t}{2m_N^2} \tilde{C}_P^L + 2\text{Re} \tilde{C}_{P\&A}^L \right) \right], \end{aligned} \quad (9)$$

where

$$\begin{aligned} \tilde{C}_V^L &= [(-F_1^N U^\dagger U + F_Q^N Q^L) \rho^L (-F_1^N U^\dagger U + F_Q^N Q^L)]_{44}, \\ \tilde{C}_A^L &= \left[\left(U^\dagger U G_A^N - \frac{t F_A^N Q^L}{m_N^2} \right) \rho^L \left(U^\dagger U G_A^N - \frac{t F_A^N Q^L}{m_N^2} \right) \right]_{44}, \\ \tilde{C}_{V\&A}^L &= \left[(-F_1^N U^\dagger U + F_Q^N Q^L) \rho^L \left(U^\dagger U G_A^N - \frac{t F_A^N Q^L}{m_N^2} \right) \right]_{44}, \\ \tilde{C}_M^L &= [(U^\dagger U F_2^N - F_M^N Q^L) \rho^L (U^\dagger U F_2^N - F_M^N Q^L)]_{44}, \\ \tilde{C}_{V\&M}^L &= [(-F_1^N U^\dagger U + F_Q^N Q^L) \rho^L (U^\dagger U F_2^N - F_M^N Q^L)]_{44}, \\ \tilde{C}_{A\&M}^L &= \left[\left(U^\dagger U G_A^N - \frac{t F_A^N Q^L}{m_N^2} \right) \rho^L (U^\dagger U F_2^N - F_M^N Q^L) \right]_{44}, \\ \tilde{C}_P^L &= [(U^\dagger U G_P^N - 2F_A^N Q^L) \rho^L (U^\dagger U G_P^N - 2F_A^N Q^L)]_{44}, \\ \tilde{C}_{P\&A}^L &= \left[(U^\dagger U G_P^N - 2F_A^N Q^L) \rho^L \left(U^\dagger U G_A^N - \frac{t F_A^N Q^L}{m_N^2} \right) \right]_{44}, \\ Q^L &= \frac{2\sqrt{2}\pi\alpha}{G_F t} (f^Q - t f^A). \end{aligned} \quad (10)$$

The coefficients $\tilde{C}$ are similar to the coefficients C in the case of active neutrinos [2]. Here $[\dots]_{44}$ means an appropriate component in the mass basis of the result of matrix multiplication in square brackets. With the initial flavor neutrino ν_ℓ , i.e., $\rho_{ij}^L = U_{\ell i}^* U_{\ell j}$ (with a zero baseline), the contribution of weak interaction to the cross section is suppressed by a factor $[U^\dagger U \rho^L U^\dagger U]_{44} \approx |U_{\ell 4}|^2$.

Figure 2 shows the effects of scattering of ν_ℓ into ν_4 on a proton. A purely weak contribution is suppressed by $|U_{\ell 4}|^2 = 5 \times 10^{-5}$ [10]. In such a case, near the energy threshold of the process, the mass of the 4th neutrino also shapes the cross section, i.e., the influence of $\frac{d\sigma_{m_4}^L}{dT}$ also becomes visible. Due to the heavy neutrino mass, the pseudoscalar nucleon form factor is no longer suppressed in the cross section. Finally, we also consider the contribution from the transition charge radius $\langle r_{14}^2 \rangle = i3 \times 10^{-33} \text{ cm}^2$. The purely imaginary value is due to the Majorana neutrino nature [6] in the minimal $3 + 1$ scenario. Also, we take its absolute value by two orders less than the limits (8) since for EM interaction there is no suppression by the mixing matrix.

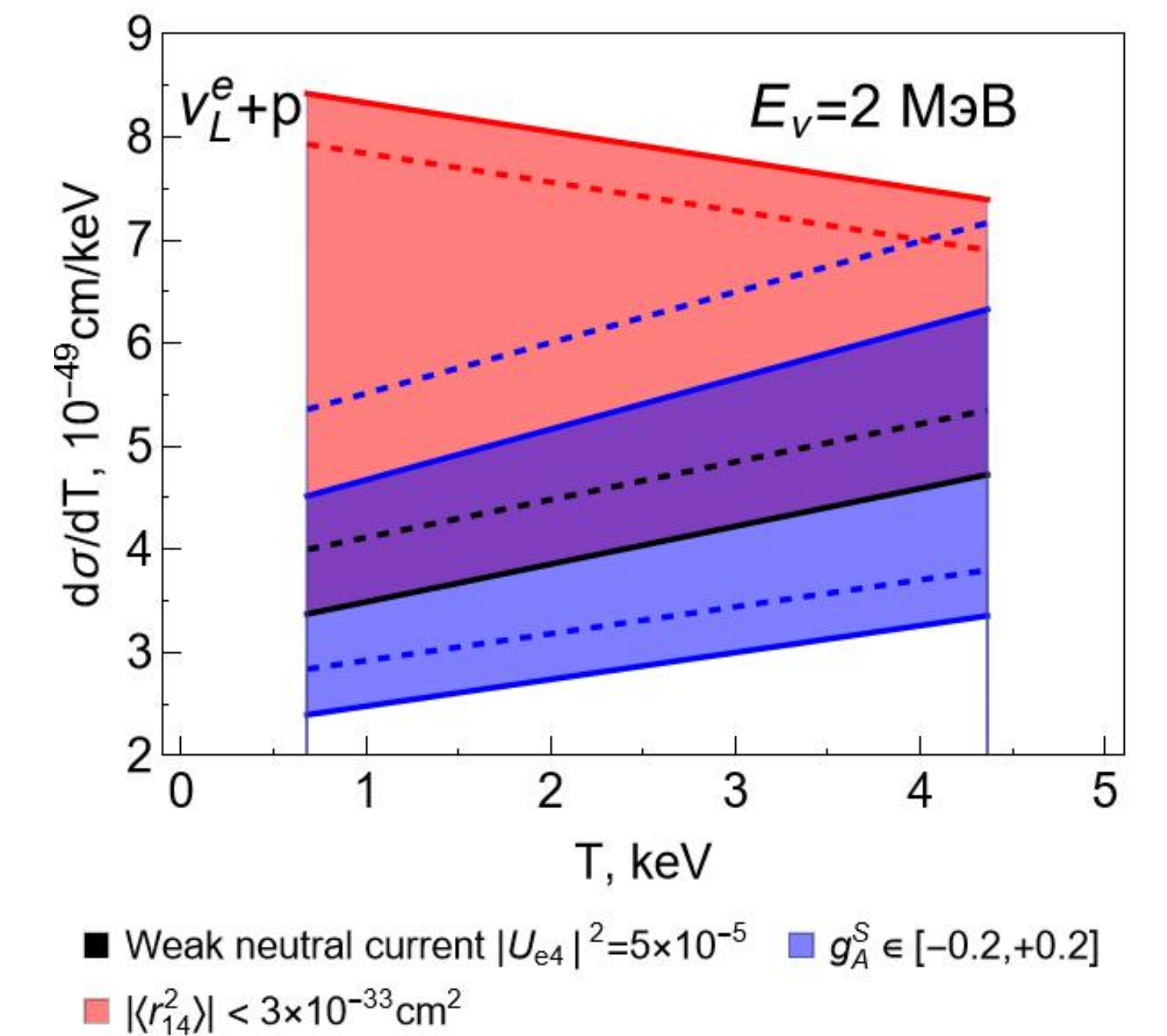


Figure 2: The differential cross sections of $\nu_l + p \rightarrow \nu_4 + p$ scattering for incident neutrinos at $E_\nu = 2$ MeV and $m_4 = 1.8$ MeV. Dashed (solid) lines correspond to cross sections with (without) term $\frac{d\sigma_{m_4}^L}{dT}$ that involves the mass of the 4th neutrino state.

4 Summary

Elastic neutrino-nucleon scattering has been theoretically considered, taking into account the EM interactions of massive neutrinos and their initial polarization. Thus, the processes under consideration have two channels: through the exchange of a Z^0 boson and a photon. The strange-quark contribution to the nucleon form factors that can have a similar effect as that due to the neutrino EM properties is taken into account. The derived cross sections contain information on the neutrino polarization and both the neutrino EM form factors and the nucleon form factors. This feature allows the expressions derived to be used in various studies. Among them are neutrino experiments with short and long baselines, the study of neutrino interactions and oscillations in matter, the registration of neutrinos from supernova explosions using elastic neutrino-proton scattering [11], and the search for the EM characteristics of neutrinos. In addition, neutral current interactions can be used to probe the existence of sterile neutrinos. Moreover, active-to-sterile EM properties can have a larger relative contribution to the scattering cross section.

The results of this work contribute to the development of a systematic approach to studying the properties of neutrinos in their elastic scattering on complex targets (nuclei, atoms, and condensed matter).

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