Critical non-Abelian Vortex String and 2D $\mathcal{N} = 2$ Black Hole

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Problem

Main goal: to describe the spectrum of

$$\mathcal{N} = 2 \ \mathrm{U}(4) \text{ SQCD with } N_f = 8 \text{ flavours of quarks}$$

Ingredients

- Confining non-Abelian vortex string
- Well-known spectrum of $\mathcal{N} = 2$ U(2) SQCD with $N_f = 4$

$\mathcal{N} = 2$ U(N) SQCD with N_f flavours of quarks

$\mathcal{N} = 2$ U(N) SQCD with $N_f \geq N$ flavours of quarks has solitonic non-Abelian vortex strings.
$N_f = 2N$ case provides the conformal invariance of the world-sheet effective $\mathbb{WCP}(N, N)$ model.

When $N = 2$, $N_f = 4$, this string becomes critical superstring — **its spectrum can be interpreted as the spectrum of the original $\mathcal{N} = 2$ U(2), $N_f = 4$ SQCD**.

$\mathcal{N} = 2$ SQCD with U(2) and $N_f = 4$

$$
\begin{array}{ccccccc}
\text{4D } \mathcal{N} = 2 \text{ SQCD} & \rightarrow & \text{NA-string} & \rightarrow & \mathbb{WCP}(2,2) & \rightarrow & \mathcal{N} = 2 \text{ Liouville} \\
\uparrow & & & & & & \downarrow \\
\text{spectrum} & \Leftarrow & \multicolumn{5}{c}{\mathcal{N} = 2 \text{ SL}(2,R)/\text{U}(1) \text{ coset model (2D Black Hole)}}
\end{array}
$$

Each transition in this scheme represents an equivalence/duality of a certain kind:

- Equivalence: $\mathbb{WCP}(2,2) \longleftrightarrow \mathcal{N} = 2$ Liouville theory
- Mirror duality: $\mathcal{N} = 2$ Liouville theory $\longleftrightarrow$ 2D Black hole

Following this scheme the spectrum of $\mathcal{N} = 2$ SQCD with U(2) and $N_f = 4$ was described.

From U(2), $N_f = 4$ to U(4), $N_f = 8$

We want to apply the similar scheme to $\mathcal{N} = 2$ U(4) SQCD with $N_f = 8$. Consider the special choice of quark masses

$$\{m_A\}_{A=1}^{8} = \{\underbrace{0\,,\ldots\,,0}_{2}\,,\underbrace{M\,,\ldots\,,M}_{2}\,,\underbrace{0\,,\ldots\,,0}_{2}\,,\underbrace{M\,,\ldots\,,M}_{2}\}$$

$$\boxed{\mathcal{N} = 2 \text{ SQCD } \mathrm{U}(2),\ N_f = 4 \xleftrightarrow{\infty\leftarrow M\rightarrow 0} \mathcal{N} = 2 \text{ SQCD } \mathrm{U}(4),\ N_f = 8}$$

$\mathcal{N}=2$ SQCD with U(4) and $N_f=8$

Wrong:

$$\text{4D } \mathcal{N}=2 \text{ SQCD} \quad \rightarrow \quad \text{NA-string} \quad \nrightarrow \quad \mathbb{WCP}(4,4)$$

Right:

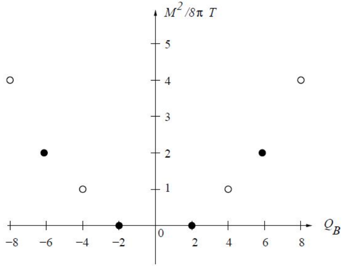
$$\begin{array}{ccccc} \text{4D } \mathcal{N}=2 \text{ SQCD} & \rightarrow & \text{NA-string} & \rightarrow & M\text{-deformed } \mathcal{N}=2 \text{ Liouville} \\ \uparrow & & & & \downarrow \\ \text{spectrum} & \Leftarrow & \multicolumn{3}{c}{\mathcal{N}=2 \text{ SL}(2,R)/\text{U}(1) \text{ coset model (2D Black Hole)}} \end{array}$$

We show that mass deformed $\mathcal{N}=2$ Liouville theory has the geometry of a "trumpet" and actually is T-dual to the same 2D black hole.

- U(2), $N_f=4$: undeformed Liouville $\xleftrightarrow{\text{Mirror dual}}$ 2D BH
- U(4), $N_f=8$: M-deformed Liouville $\xleftrightarrow{\text{T-dual}}$ 2D BH

We use this T-duality to describe the spectrum of the original $\mathcal{N}=2$ SQCD with U(4) and $N_f=8$

Spectrum



- $j=-\frac{1}{2}$ — spin-0 states: $\frac{M^2}{8\pi T}=2m^2-\frac{1}{2}, |m|=\frac{1}{2},\frac{3}{2}\ldots$
- $j=-1$ — spin-2 states: $\frac{M^2}{8\pi T}=2m^2, |m|=1,2,\ldots$

Conclusions

- We demonstrate that mass-deformed $\mathcal{N} = 2$ Liouville theory has the geometry of a "trumpet" and is T-dual to the same 2D BH, as an undeformed theory (with different value of dilaton).
- **Spectrum doesn't change** when we interpolate to $\mathcal{N} = 2$ U(4) SQCD with $N_f = 8$.
  The only effect — increasing of the multiplicity of states.
- The growth of multiplicity of states manifests itself as the growth of the black hole mass when we reduce M (i. e. we introduce more quark flavours into original SQCD with U(2) and $N_f = 4$)
- This non-obvious result, obtained from the string theory, we explain with field theory arguments on the SQCD side.