New viable models of inflation and PBH formation

Inflationary dynamics

The action for canonical single-field models of inflation reads

$$\mathcal{S} = \int \mathrm{d}^4x\sqrt{-g}\left(\frac{1}{2}R - \frac{1}{2}\left(\partial_\mu\phi\right)^2 - V(\phi)\right) \ , \quad c = \hbar = M_{\mathrm{Pl}} = 1 \ , \tag{1}$$

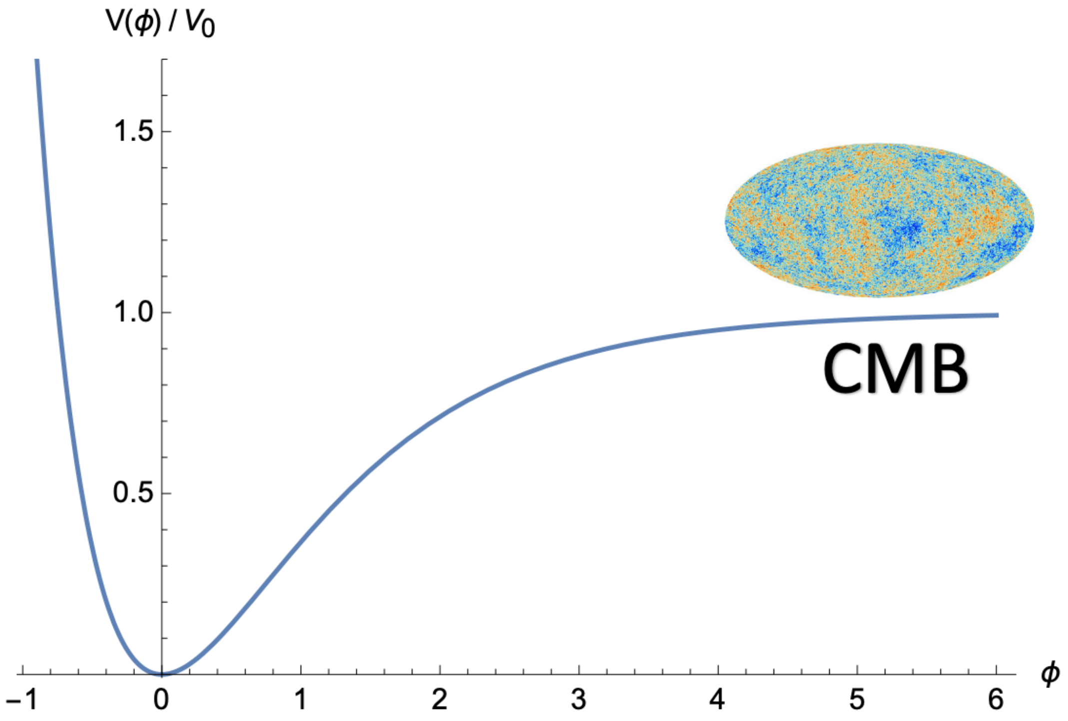
where ϕ is the inflaton field, and $V(\phi)$ is its potential. The Hubble rate, $H \equiv \dot{a}/a$, and ϕ satisfy the equations

$$\ddot{\phi} + 3H\dot{\phi} + V_\phi = 0 \ , \quad 3H^2 = \frac{1}{2}\dot{\phi}^2 + V(\phi) \ , \tag{2}$$

and the first and the second Hubble flow parameters are defined by

$$\epsilon \equiv -\frac{\dot{H}}{H^2} = \frac{1}{2}\frac{\dot{\phi}^2}{H^2} \ , \quad \eta \equiv \frac{\dot{\epsilon}}{H\epsilon} \ . \tag{3}$$

Inflationary dynamics I



slow-roll

$$|\ddot{\phi}| \ll 3H|\dot{\phi}|$$

$$3H\dot{\phi} + V_\phi \simeq 0$$

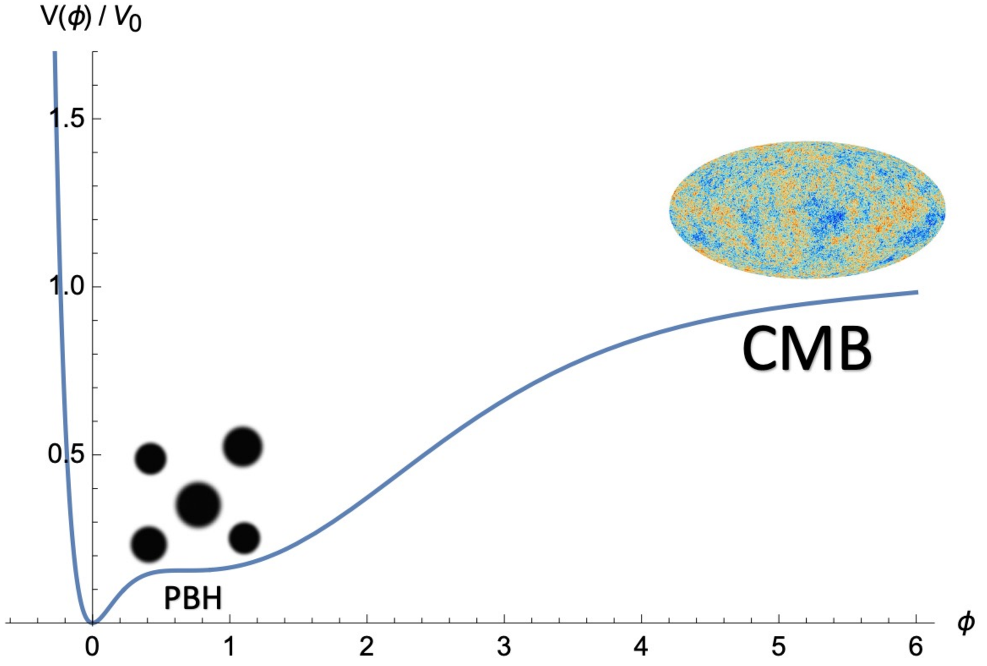
$$3H^2 \simeq V$$

$$\epsilon, |\eta| \ll 1$$

Inflationary dynamics II

ultra-slow-roll

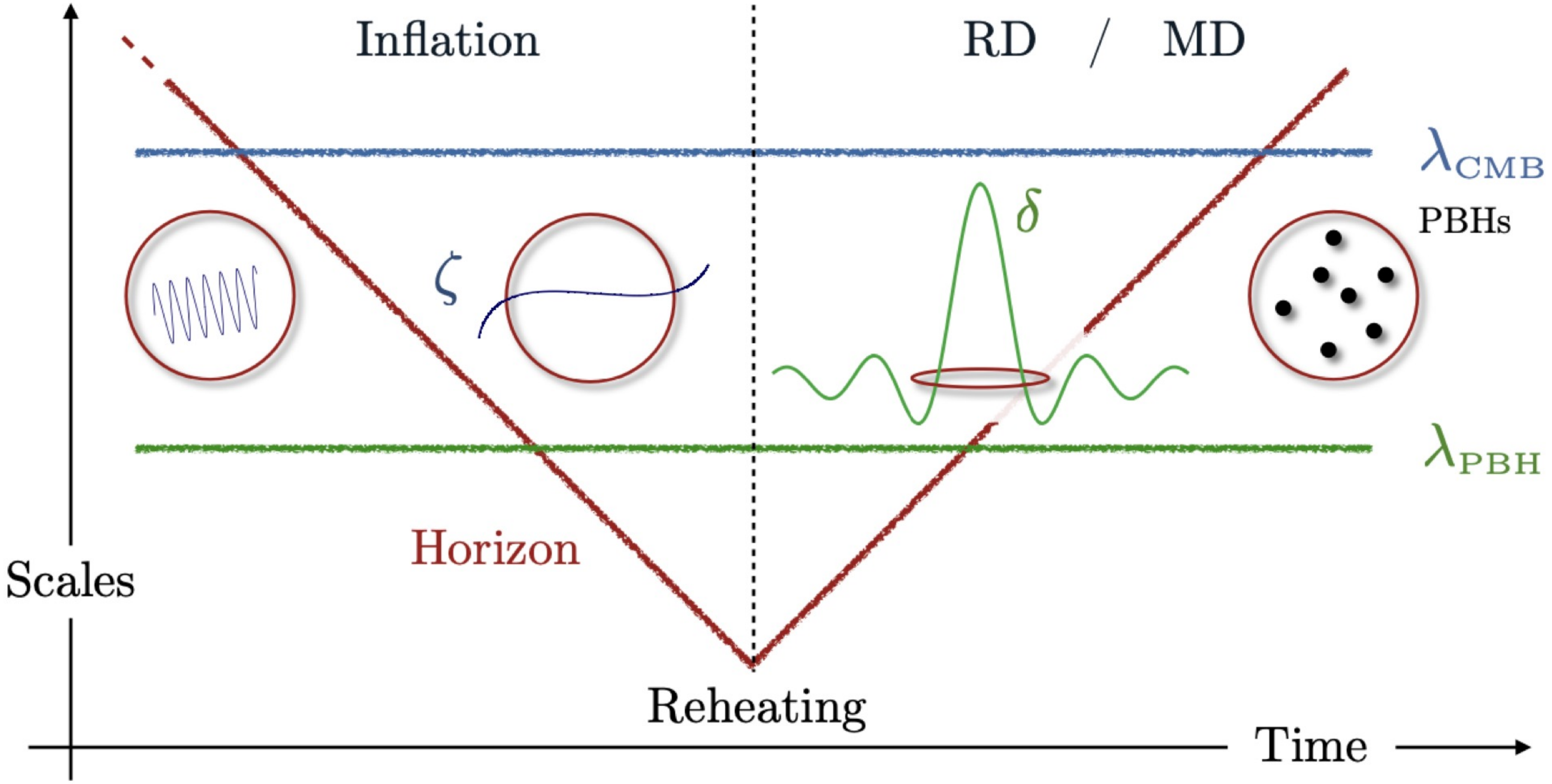
$$V_\phi \simeq 0$$
$$\ddot{\phi} + 3H\dot{\phi} \simeq 0$$
$$3H^2 \simeq V$$
$$\dot{\phi} \propto a^{-3}$$
$$\epsilon \propto a^{-6}\ ,\ \eta \simeq -6$$



slow-roll

$$|\ddot{\phi}| \ll 3H|\dot{\phi}|$$
$$3H\dot{\phi} + V_\phi \simeq 0$$
$$3H^2 \simeq V$$
$$\epsilon, |\eta| \ll 1$$

The curvature perturbation in the comoving gauge, $\zeta_k(t)$, can be expressed in terms of the Mukhanov–Sasaki variable, v_k, as $\zeta_k \equiv v_k/z$, where $z \equiv a\dot{\phi}/H$. Thus, $\zeta_k \propto a^3$.


Inflation
RD / MD
λ_{CMB}
PBHs
δ
ζ
λ_{PBH}
Horizon
Scales
Reheating
Time

Our model

The basic α-attractor models are divided into two types depending upon the global shape of the inflaton scalar potential,

$$\text{E-type:}\ V \sim \left(1 - \exp\left(-\sqrt{\frac{2}{3\alpha}}\frac{\phi}{M_{\rm Pl}}\right)\right)^2, \quad \text{and} \quad \text{T-type:}\ V \sim \tanh^2\frac{\phi/M_{\rm Pl}}{\sqrt{6\alpha}}. \tag{4}$$

The E-type potential can be modified to include PBH production [DF, Ketov, 2023] as

$$V(\phi) = \frac{3}{4}M^2 M_{\rm Pl}^2\left[1 - y - \theta y^{-2} + y^2(\beta - \gamma y)\right]^2, \quad y = \exp\left(-\sqrt{\frac{2}{3\alpha}}\phi/M_{\rm Pl}\right). \tag{5}$$

This potential can be rewritten in terms of the new dimensionless parameters

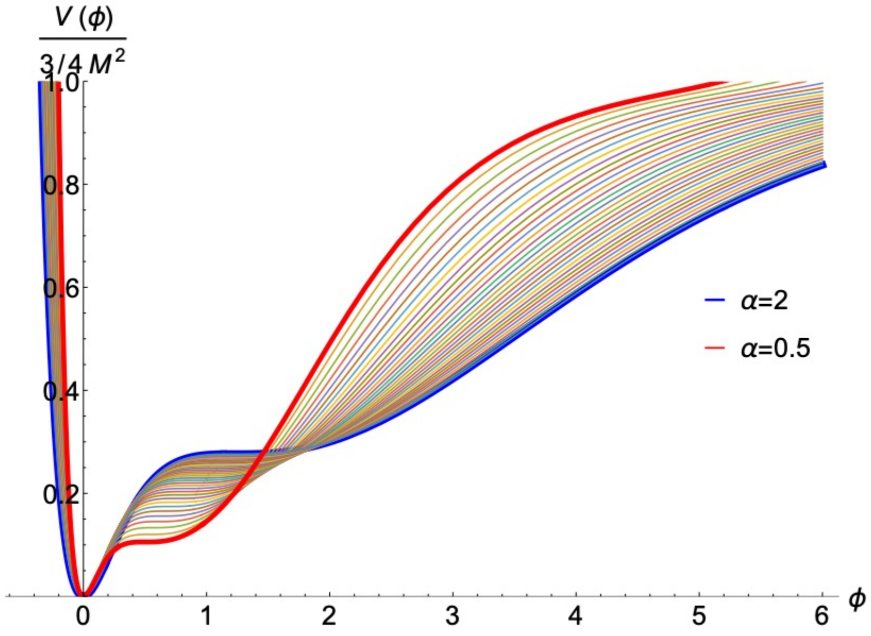
$$\beta = \frac{\exp\left(\sqrt{\frac{2}{3\alpha}}\phi_i\right)}{1-\sigma^2} \quad \text{and} \quad \gamma = \frac{\exp\left(2\sqrt{\frac{2}{3\alpha}}\phi_i\right)}{3(1-\sigma^2)}. \tag{6}$$

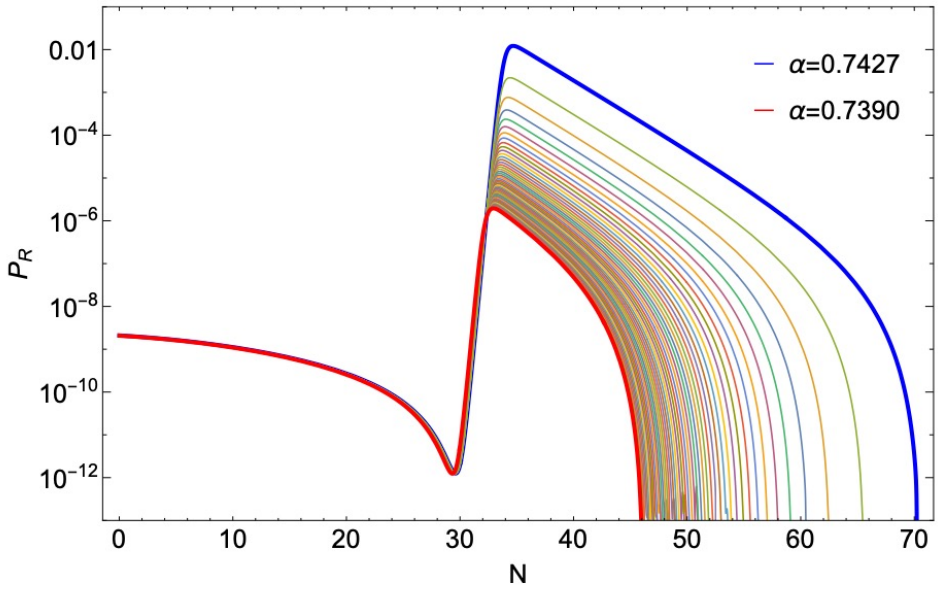
Two extrema are symmetrically located around the inflection point y_i as $y_{\rm ext}^{\pm} = y_i\,(1 \pm \sigma)$.

The power spectrum of scalar perturbations in the slow-roll approximation is

$$\mathcal{P}_R = \frac{H^2}{8\pi^2 \epsilon} \, . \tag{7}$$

It is of primary interest when calculating the observable predictions of an inflation model.





$$M_{\mathrm{PBH}}(k) \simeq 10^{20} \left(\frac{7 \cdot 10^{12}}{k \cdot \mathrm{Mpc}} \right)^2 \mathrm{g}$$

$$f_{\mathrm{GW}} \approx \left(\frac{M_{\mathrm{PBH}}}{10^{16} \ \mathrm{g}} \right)^{-1/2} \ \mathrm{Hz}$$

Reconstruction of inflation potential from GW signal

GW signal $\Omega_{\rm GW}(k)$ → **Power spectrum** $\mathcal{P}_\zeta(k)$ → **Scalar potential** $V(\phi)$

The present GW energy density computed in the second order is given by

$$\frac{\Omega_{\rm GW}(k)}{\Omega_r} = \frac{c_g}{72} \int_{-\frac{1}{\sqrt{3}}}^{\frac{1}{\sqrt{3}}} \mathrm{d}\, d \int_{\frac{1}{\sqrt{3}}}^{\infty} \mathrm{d}\, s \left[\frac{\left(s^2 - \frac{1}{3}\right)\left(d^2 - \frac{1}{3}\right)}{s^2 + d^2} \right]^2 \mathcal{P}_\zeta(kx)\, \mathcal{P}_\zeta(ky)\, (I_c^2 + I_s^2)\ ,$$

where $x = \frac{\sqrt{3}}{2}(s+d)$, and $y = \frac{\sqrt{3}}{2}(s-d)$, and the functions I_c and I_s are given by

$$I_s = -36 \frac{s^2 + d^2 - 2}{(s^2 - d^2)^2} \left[\frac{s^2 + d^2 - 2}{s^2 - d^2} \log \left| \frac{d^2 - 1}{s^2 - 1} \right| + 2 \right],\ I_c = -36\pi \frac{(s^2 + d^2 - 2)^2}{(s^2 - d^2)^3} \theta(s-1)\ ,$$

see [Espinosa, Racco, Riotto, 2018], and [Kohri & Terada, 2018].

The procedure of reconstruction is ambiguous. Nevertheless, it is possible to systematically identify the power spectra corresponding to a specific GW energy density curve [DF, Kühnel, Stamou, 2024], [LISA Cosmology Working Group, 2025].

Reconstruction of scalar potential from GW signal

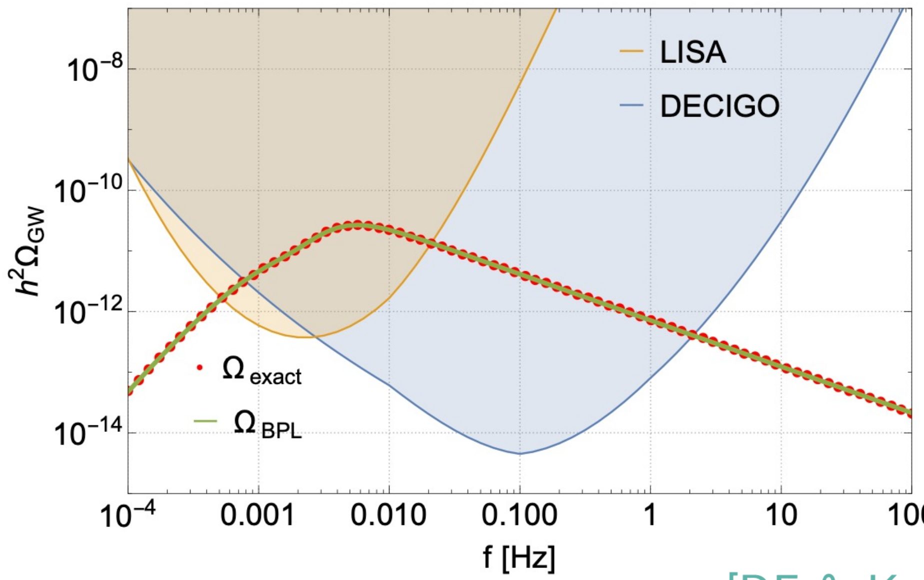
A curvature perturbation during inflation, $\zeta_k(N)$, can be expressed in terms of v_k as $\zeta_k \equiv v_k/z$, where $z \equiv a \cdot \phi'$. The variable v_k obeys the Mukhanov-Sasaki (MS) equation

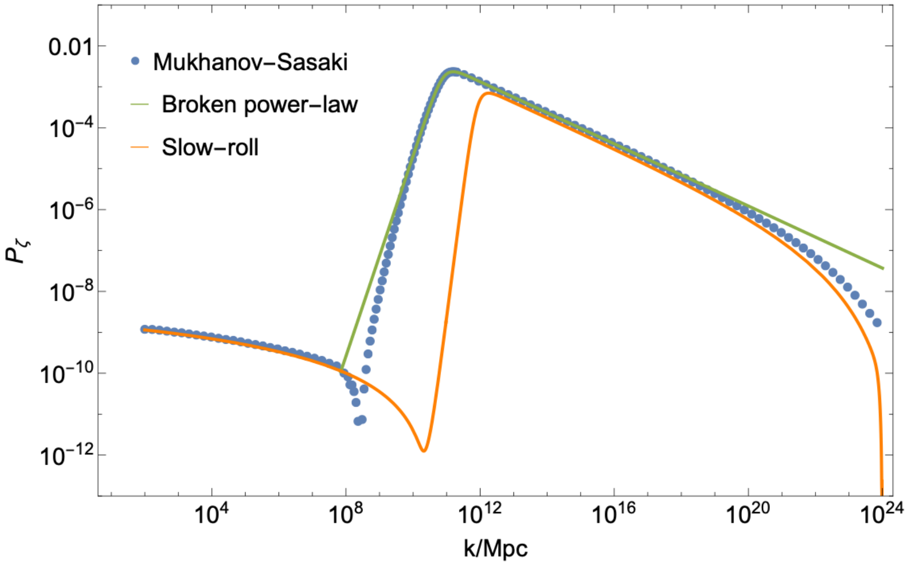
$$v_k'' + (1-\epsilon)\, v_k' + \left[\frac{k^2}{a^2H^2} + (1+\eta/2)(\epsilon - \eta/2 - 2) - \eta'/2\right] v_k = 0 \quad , \quad \mathcal{P}_\zeta(k) = \frac{k^3}{2\pi^2}\left|\frac{v_k}{z}\right|^2$$

$$\phi_{\rm sol}[N,\vec{\theta}] \longrightarrow v[k,\vec{\theta}] \longrightarrow \mathcal{P}_\zeta[k_i;\vec{\theta}] \longrightarrow S(\vec{\theta}) = \sum_i \left[\mathcal{P}_\zeta(k_i;\vec{\theta}) - \mathcal{P}_\zeta^{\rm rec}(k_i)\right]^2$$

$$\phi_{\rm sol}[N,\vec{\theta}] \searrow \mathcal{P}_R = \frac{H^2}{8\pi^2\epsilon} \longrightarrow \vec{\theta}\text{in} \nearrow S(\vec{\theta})$$

$$\mathcal{P}_{\rm BPL} = A\frac{\alpha_1+\beta_1}{\beta_1(k/k_*)^{-\alpha_1} + \alpha_1(k/k_*)^{\beta_1}}$$





[DF & Ketov, 2025]

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1. **D. Frolovsky**, S. Ketov, Are single-field models of inflation and PBH production ruled out by ACT observations?, arXiv.2505.17514
2. **D. Frolovsky**, S. Ketov, One-loop Corrections to the E-type α-attractor Models of Inflation and Primordial Black Hole Production, Physical Review D, 2025, 111, 083533
3. **D. Frolovsky**, F. Kühnel, I. Stamou, Reconstructing Primordial Black Hole Power Spectra from Gravitational Waves, Physical Review D, 2025, 111, 043538
4. **D. Frolovsky**, S. Ketov, Dilaton-axion Modular Inflation in Supergravity, International Journal of Modern Physics D, 2024, 33(14), 234008
5. **D. Frolovsky**, S. Ketov, Production of Primordial Black Holes in Improved E-Models of Inflation, Universe, 2023, 9(6), 294
6. **D. Frolovsky**, S. Ketov, Fitting Power Spectrum of Scalar Perturbations for Primordial Black Hole Production during Inflation, Astronomy, 2023, 2(1), 47–57
7. **D. Frolovsky**, S. Ketov, S. Saburov, E-models of Inflation and Primordial Black Holes, Frontiers in Physics, 2022, 10, 1005333
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9. V. Abakumova, **D. Frolovsky**, H.-C. Herbig, S. Lyakhovich, Gauge Symmetry of Linearised Nordström Gravity and the Dual Spin Two Field Theory, The European Physical Journal C, 82(9), 780