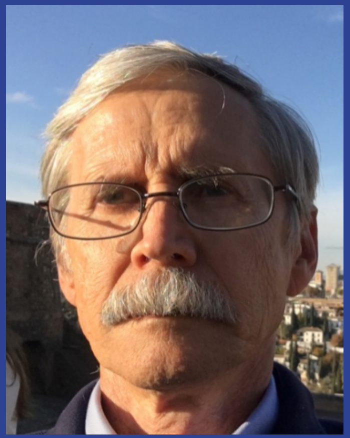
Galaxy Formation in Modern Cosmology

BASIS
Theoretical Physics
and Mathematics
Advancement Foundation

Sochi, July 2019



Anatoly Klypin,
University of Virginia and New Mexico State University

Andrey Kravtsov,
University of Chicago

Daniel Ceverino,
Cosmic Dawn Center, University of Copenhagen

Arrow of time

Main events in the history of the Universe

Energy/Temp	Time	Event/Epoch
10^{19}GeV	10^{-43}sec	Planck Time
		Inflation
10^{14}GeV	10^{-35}sec	end of Inflation. Reheating. Beginning of Big Bang
	10^{-34}sec	end of grand unification. Baryogenesis: formation of matter-antimatter asymmetry
300GeV	10^{-12}sec	end of electroweak unification
1GeV	10^{-5}sec	Normal physics. Composition of the Universe: n, p, e^-,e^+, γ, ν
1MeV	1sec	Neutrino decoupling. Neutrino do not interact with the rest of matter
0.5MeV		Electron-positron annihilation. Composition: n, p, e^-, γ, ν
0.1MeV	100sec	Big Bang Nucleosynthesis: formation of elements He,D, Li
10^5K	10^3yrs	Equality of matter and radiation: $\rho_{matter} = \rho_{rel.particles}$
3000K =0.3eV	10^5yrs	Recombination and Decoupling. Composition: H,He, γ, ν
	1Gyr (z=10)	First galaxies. QSO quickly form.
	z=3	Galaxy formation
	z=1-2	Formation of clusters and superclusters. Acceleration of the Universe.
	13Gyrs	Now

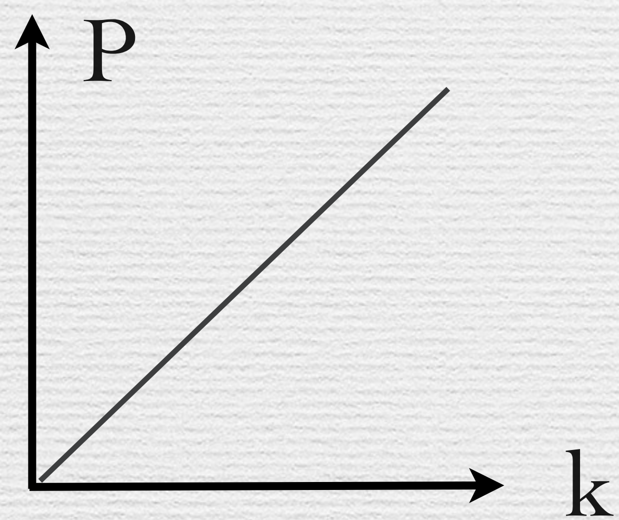
Probing different epochs with observations

Epoch	Phenomenon	Test
Inflation	Spectrum of perturbation on very long scales	•Large-scale CMB anisotropies •Large-scale spectrum of perturbation in distribution of galaxies
Moment of equality	Position of maximum in the spectrum of perturbations	Distribution of galaxies: Spectrum, sizes of large voids, Superclusters.
BBN	abundance of light elements: He, D, Li	ISM, stellar atmospheres, spectra of high-z galaxies
Recombination	Small-scale structure of CMB	CMB anisotropies on armin -degree scales
Acceleration of the Universe	Distances depend on the rate of expansion	Distances to SNI
	Dark matter	•Rotation curves of galaxies •Possible annihilation signal from centers of galaxies •X-ray emission from clusters of galaxies •Lensing of galaxies

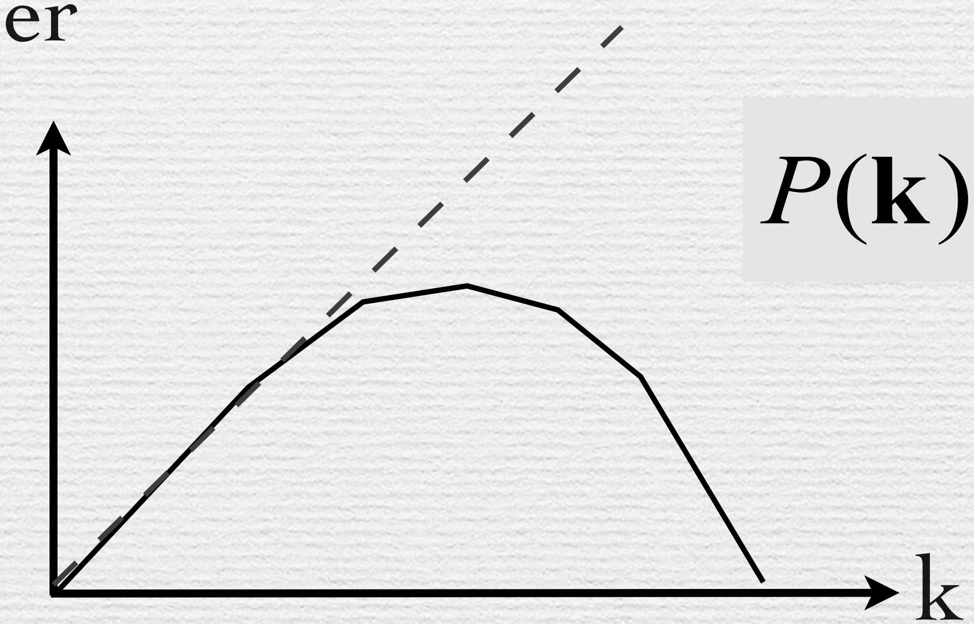
Evolution of perturbations at early times: linear growth

Inflation provides very a simple spectrum of fluctuations: gaussian fluctuations in metrics (=gravitational potential):
$(\Delta\phi)^2 \propto$ constant when averaged over spheres of radius R.

This gives the power spectrum of fluctuations in the density
$P(k) \propto k$, where k is a wavenumber



After Inflation



$$P(\mathbf{k}) = \left|\delta(\mathbf{k})\right|^2$$

After moment of equality

The Universe is not uniform. We have ignored this when we talked about FRW metric and when we discuss physics of early Universe such as Big Bang Nucleosynthesis or neutrino freeze-out (when neutrino decouple from the rest of the matter). There is a reason why we treated the Universe as homogeneous: the deviations from homogeneity are small and they were even smaller in the past.

There numerous issues related with perturbations. Big Bang itself cannot explain how the fluctuations formed. There is a natural source of fluctuations: statistical fluctuations in a medium, which consists of discrete particles. The amplitude of those fluctuations is roughly 1/sqrt(N), where N is the number of particles in some given volume. There are two problems with those fluctuations. First, their amplitude is very small. For example, consider a cluster of galaxies with mass about 1e15 Solar mass. Calculate the number of protons and take square root of it. This is what we expect from statistical mechanics. Second, the fluctuations (small or very small) grow relatively slow. This is due to the expansion of the Universe. In the absence of expansion the fluctuations grow exponentially:

$$\delta \propto e^{t/t_{dyn}}, \qquad t_{dyn} \simeq \frac{1}{\sqrt{4\pi G\rho}}$$

Here ρ is the density and t_{dyn} is the dynamical time scale. Unfortunately, the fluctuations grow much slower: only as a power-law $\delta \propto a^{\gamma}$, $\gamma \simeq 1-2$ and statistical fluctuations do not play any role as an origin of fluctuations. Thus, we need something else. *So far, the only explanation for the origin of fluctuations is coming from Inflation.*

Regardless their origin, fluctuations can be decomposed into MODES (components). In addition, we need to pay attention to different physical components: perturbations in dark matter, in gas, or radiation are different and they evolve differently.

Modes:

This is curvature ≡ adiabatic perturbations

$$\rho(\vec{x}) = \rho_{background} + \delta\rho(\vec{x})$$

$$\delta \equiv \frac{\delta\rho}{\rho_{backgr}}$$

In this mode the metric is perturbed and The amplitude of perturbations initially is the same for all different mass component.

$$\ddot{\delta} + 2\frac{\dot{a}}{a}\dot{\delta} = 4\pi G\rho_m\delta$$

Different modes of perturbations

Vorticity: dies out due to expansion: v ~ 1/a

Perturbations from Big Bang do not have rotational component

Distance to the horizon Question: what fraction of the Universe can be possibly in causal contact? We need to find the proper distance at z=0 for a point, from which we receive light for the first time. This will be the distance to the horizon.

We chose a frame, which is most convenient for integration. We are at the origin and the point, from which we receive the light is along the radius.

We start with FRW:

$$ds^2 = c^2 dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right]$$

Fix time and find proper distance to an object with coordinate distance r_H

$$d_H(t) = a(t)\int_0^{r_H} \frac{dr}{\sqrt{1-kr^2}}$$

In order to take the integral, we need to know the coordinate distance to the point, from which we receive the light for the first time in the history of the Universe. We find this by putting ds=0 into FRW and integrating it from t=0 till present:

$$ds = 0 \rightarrow c\,dt = a(t)\frac{dr}{\sqrt{1-kr^2}} \rightarrow \int_0^t \frac{c\,dt}{a(t)} = \int_0^{r_H} \frac{dr}{\sqrt{1-kr^2}}$$

Thus

$$d_H(t) = a(t)\int_0^t \frac{c\,dt}{a(t)}$$

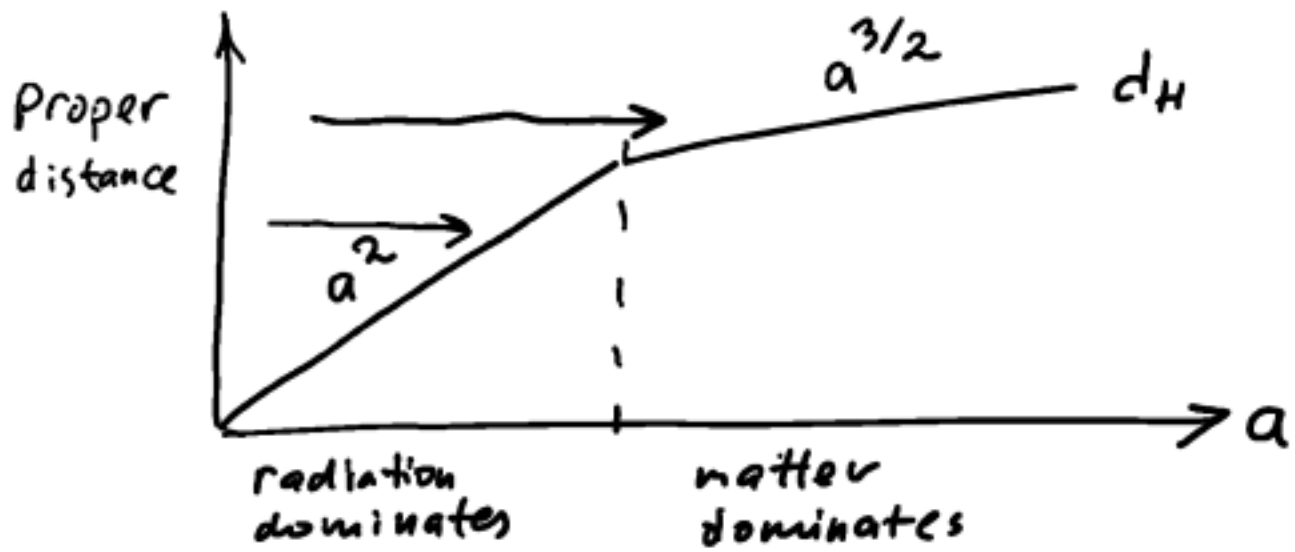
Thus

$$d_H(t) = a(t)\int_0^t \frac{c\,dt}{a(t)}$$

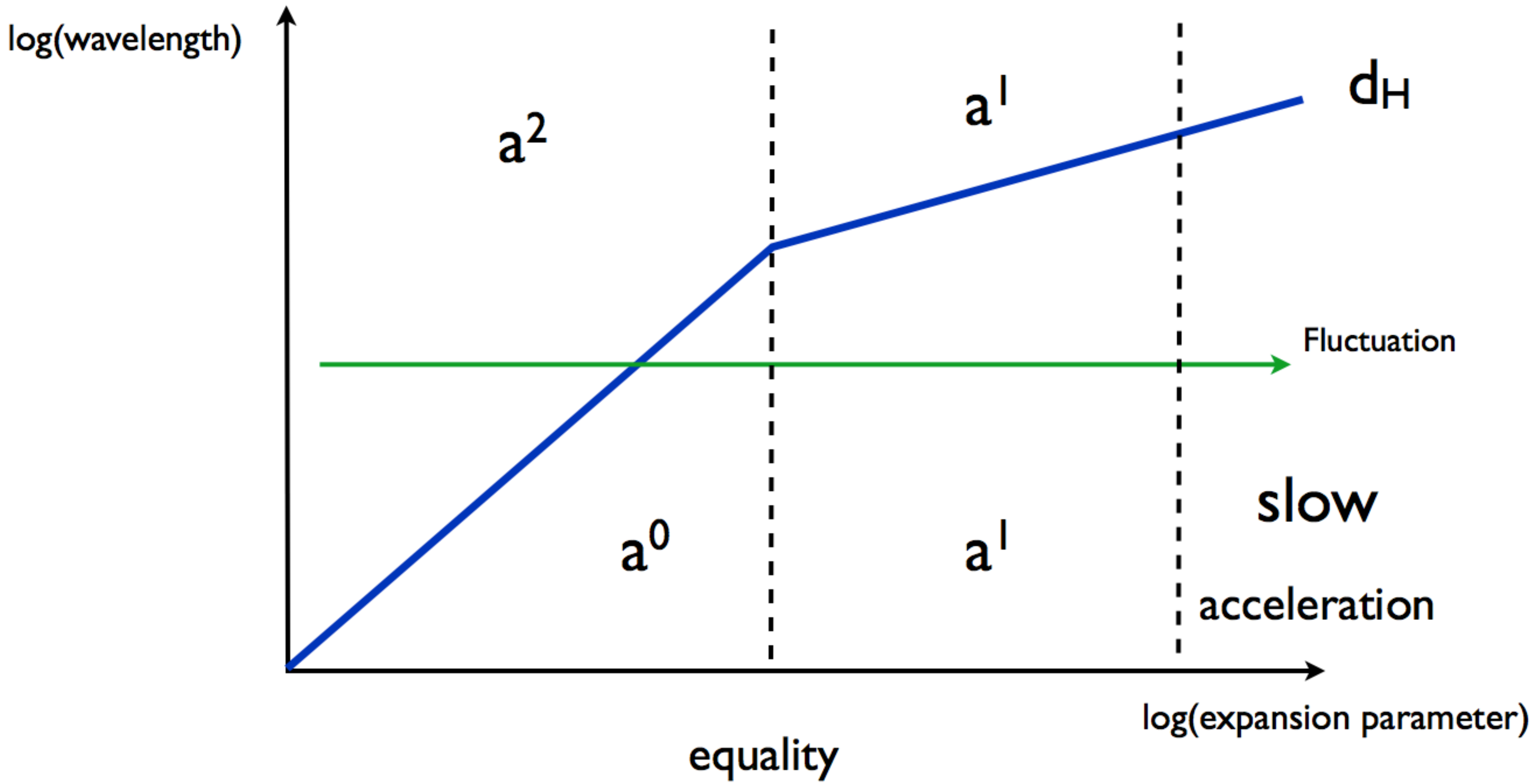
For flat universe dominated by non-relativistic particles $a(t) \propto t^{2/3}$
In this case

$$d_H(t) = \frac{2c}{H_0} a^{3/2} = 3ct$$

For flat universe dominated by relativistic particles $a(t) \propto t^{1/2}$
And $d_H \propto a^2 \quad (d_H = 2ct)$



Important notion: distance to the horizon grows faster than wavelengths of waves, which expand together with the Universe. Thus, free waves were outside of the horizon at early moment and cascade inside the horizon at later moments.

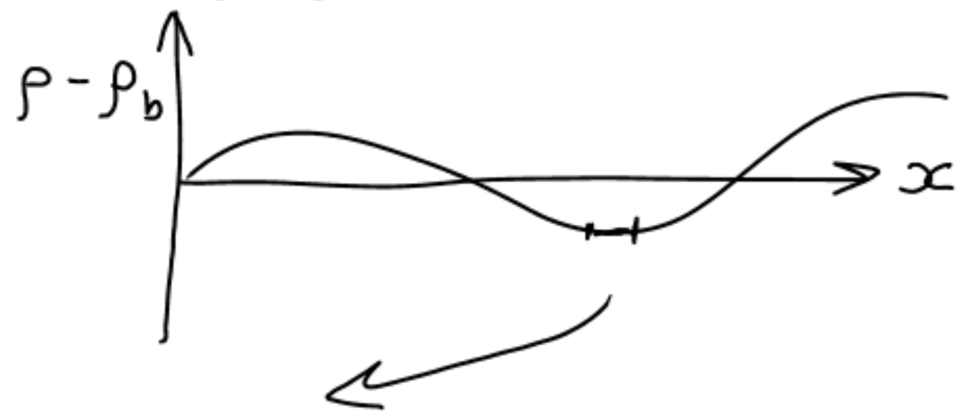


Chessboard of growth of adiabatic perturbations

Very long waves

Adiabatic modes with wavelength much longer than the distance to the horizon. There are two modes: growing and decaying. Here is the derivation for the growing mode:

$$\boxed{\lambda \gg ct}$$



Each fragment of the wave has size of the horizon at that moment of time. Thus, different parts of the long wave cannot "communicate" with other parts. Each fragment evolves independently as a Friedmann universe with slightly different density, but with the same Hubble constant:

$$\left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G}{3}\rho_b \quad , \quad \rho_b = \text{unperturbed density}$$

$$\left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{K}{a^2} \quad , \quad \rho = \text{perturbed density}$$

$$\rho = \rho_b + \delta\rho$$

Because the Hubble constant is the same (this selects the fastest growing mode), We get:

$$\frac{8\pi G}{3}\rho - \frac{K}{a^2} = \frac{8\pi G}{3}\rho_b \Rightarrow$$

$$\delta = \frac{\delta\rho}{\rho} = \frac{K}{a^2}\frac{3}{8\pi G}\frac{1}{\rho_b} \propto \frac{1}{a^2\rho_b}$$

For matter-dominated Universe we get:

$$\rho_b \propto a^{-3} \Rightarrow \qquad \delta \propto a \propto t^{2/3}$$

For radiation-dominated Universe:

$$\rho_b \propto a^{-4} \Rightarrow \qquad \delta \propto a^2 \propto t$$

Case: **waves inside the horizon, relativistic particles dominate**
Growth of perturbations in non-relativistic matter. Fluctuations in the relativistic matter are wiped out by the free streaming.

$$H^2 = \frac{8\pi G}{3}\left(\rho_m + \rho_\gamma\right)$$

$$\rho_m \propto a^{-3}, \quad \rho_\gamma \propto a^{-4}$$

$$\ddot{\delta} + 2\frac{\dot{a}}{a}\dot{\delta} = 4\pi G \rho_m \delta$$

$$\delta = \frac{\rho_m - \langle \rho_m \rangle}{\langle \rho_m \rangle}$$

Note that delta is the density contrast in matter, not in the total density

Introduce new variable:
Change variable t->y

$$y \equiv \frac{\rho_m}{\rho_\gamma} = \frac{a}{a_{eq}}$$

The equation for the growth rate takes the form:

$$\frac{d^2\delta}{dy^2} + \frac{2+3y}{2y(1+y)}\frac{d\delta}{dy} - \frac{3\delta}{2y(1+y)} = 0$$

The growing solution of this equation can be found by trying: $\ddot{\delta} = 0$
This gives:

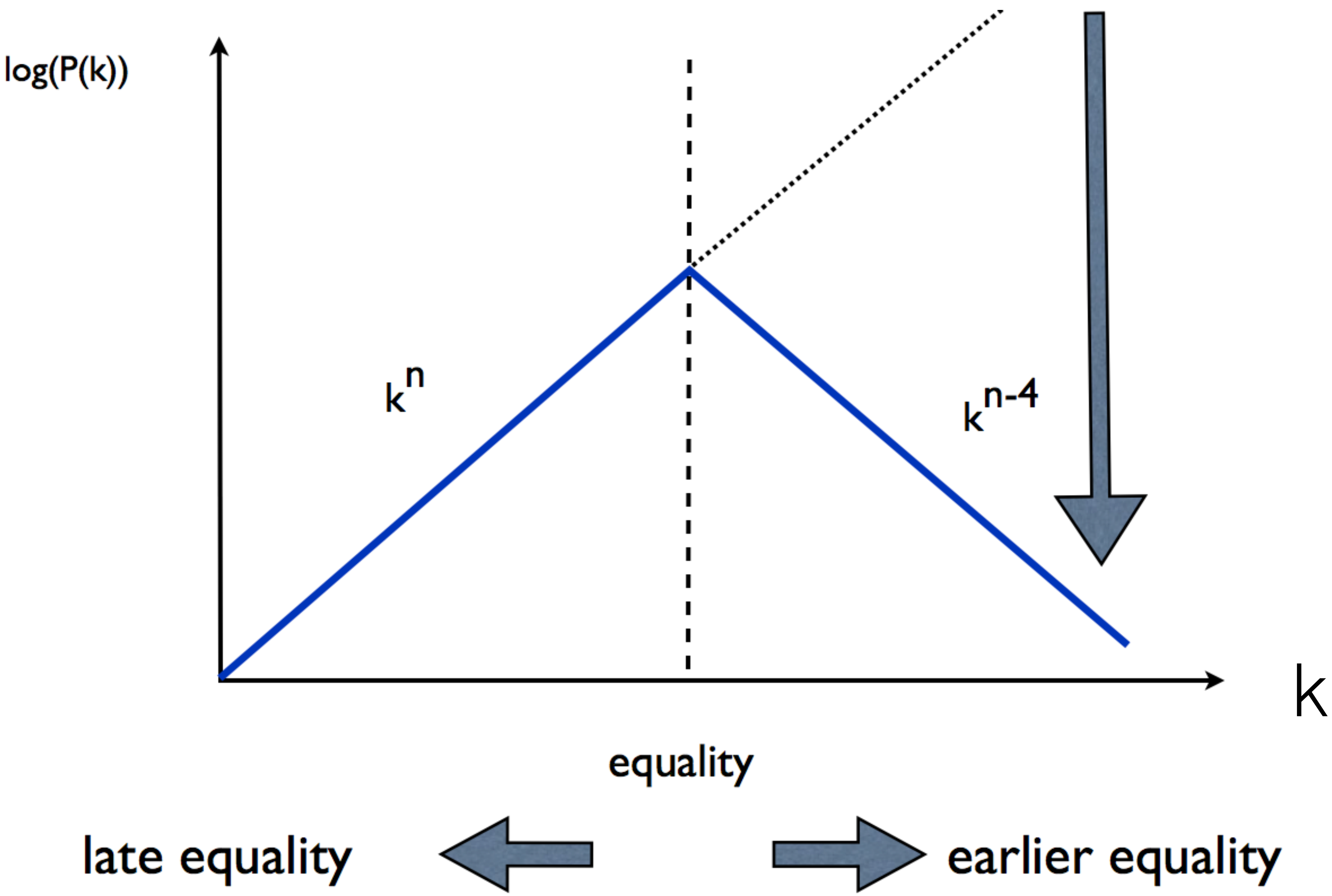
$$\delta_{grow} = 1 + \frac{3}{2}y = 1 + \frac{3}{2}\frac{a}{a_{eq}}$$

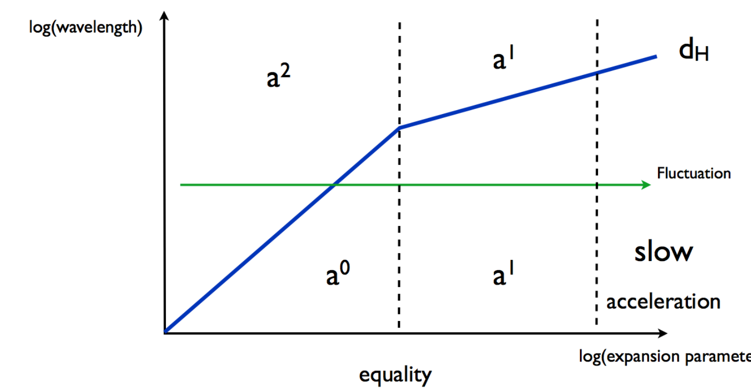
We define the **power spectrum** as

Here the averaging is done over all
Waves with given k and over the whole space

$$P(k)=\langle |\delta_{\vec{k}}|^2 \rangle$$

Correlation function is defined as $$\xi(r)=\langle \delta(\vec{x})\,\delta(\vec{x}+\vec{r}) \rangle$$





Chessboard of growth of adiabatic perturbations

Power spectrum evolution

Case: ***flat Universe with cosmological constant***

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho + \frac{\Lambda c^2}{3}$$

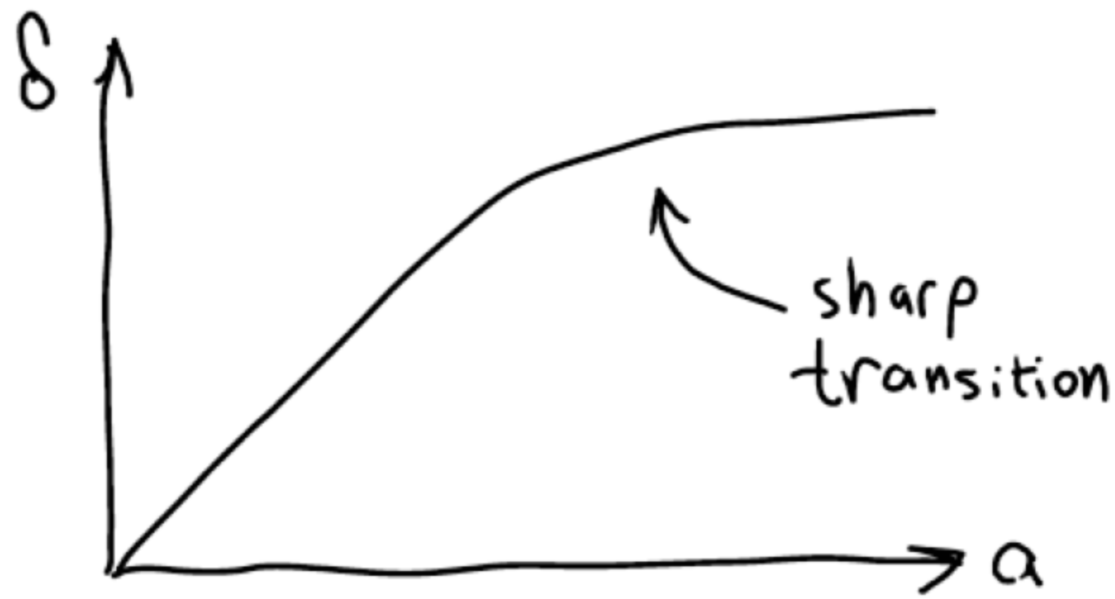
Solution for the growth of small perturbations is

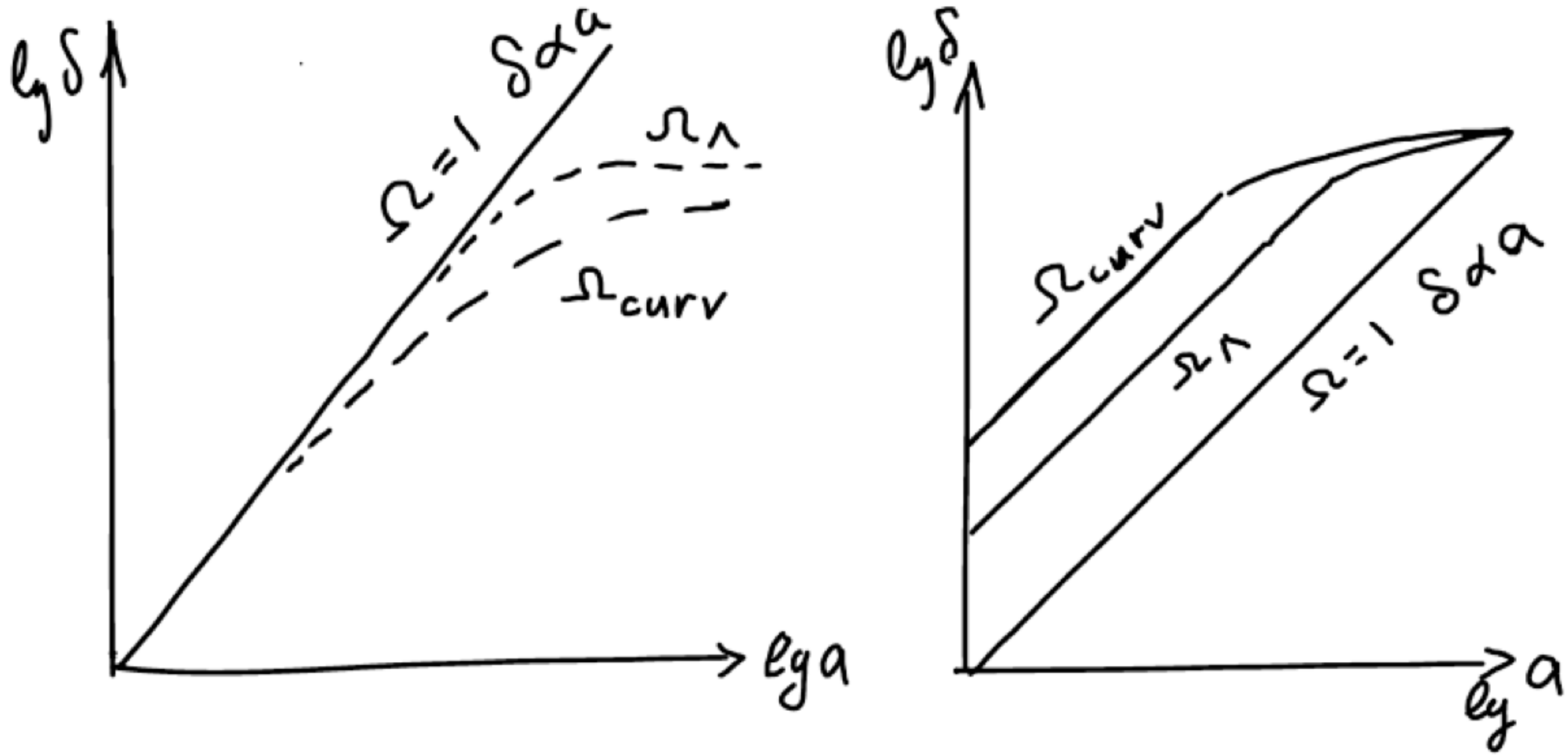
$$\delta = \frac{1}{X_0}\frac{\sqrt{1+x^3}}{x^{3/2}}\int_0^x \frac{x^{3/2}\,dx}{[1+x^3]^{3/2}}$$

$$\Omega_\Lambda = \Omega_{\Lambda,0}\frac{H_0^2}{H^2}$$

$$x = x_0 a = \left(\frac{\Omega_{\Lambda,0}}{\Omega_0}\right)^{1/3} a$$

$$x_0 = \left(\frac{\Omega_{\Lambda,0}}{\Omega_0}\right)^{1/3}$$

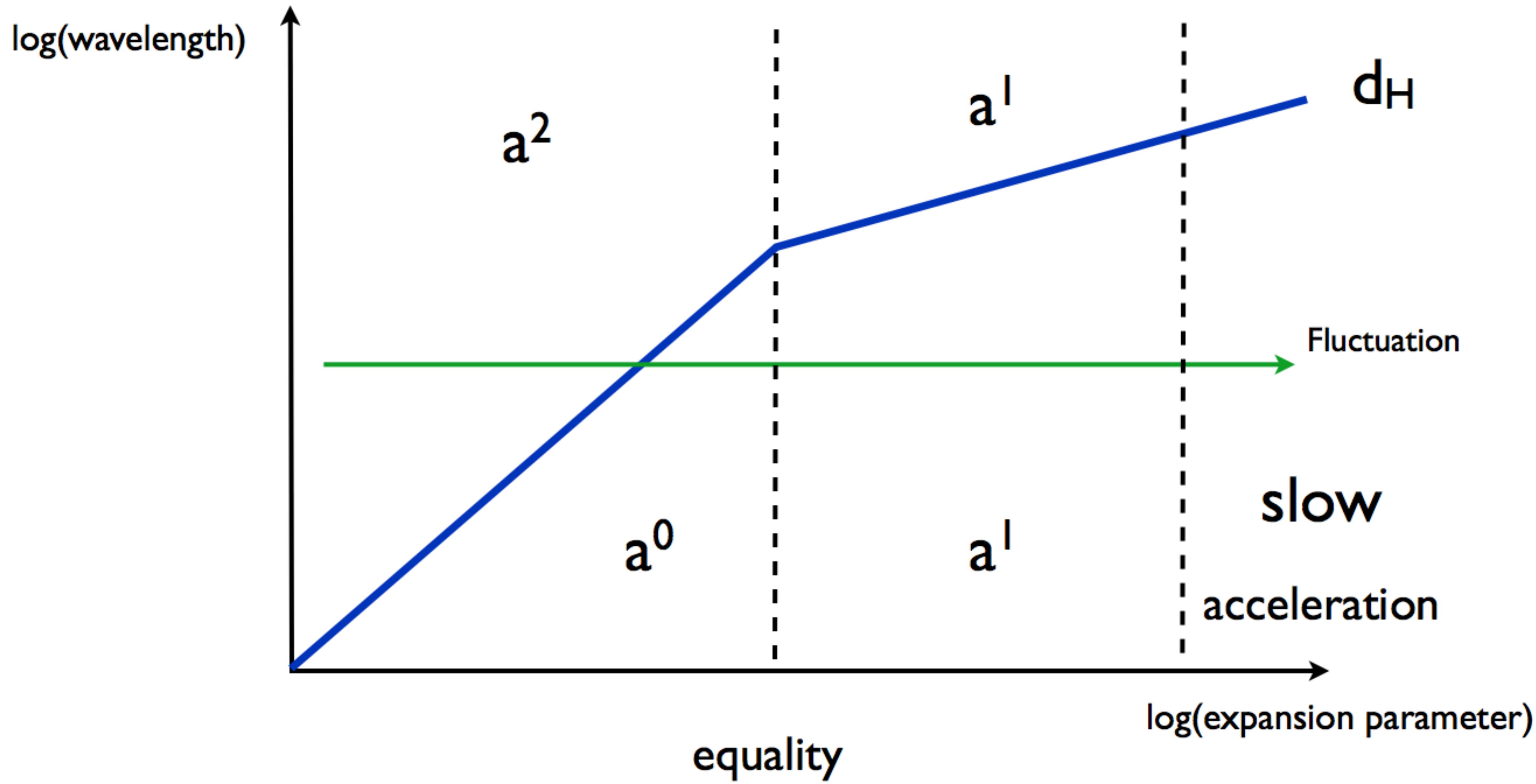




Three models: 1) Flat, matter only
2) Flat +Cosmological constant
3) Open, no cosmological constant

Left panel: the same amplitude of fluctuations at early times

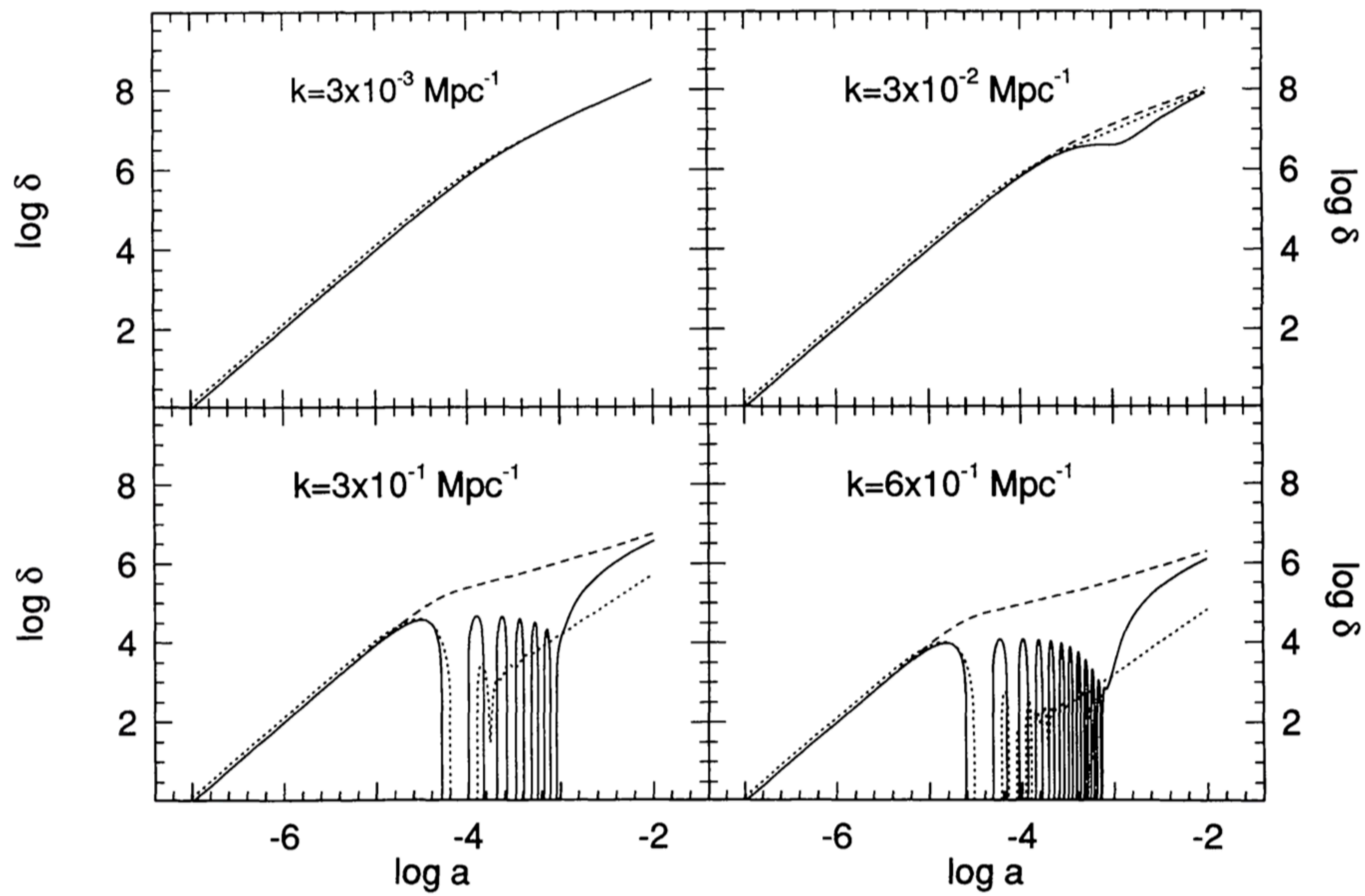
Right panel: The same amplitude at z=0



Chessboard of growth of adiabatic perturbations

Evolution of adiabatic CDM and baryons from earlytimes through recombination.

Solid lines are the baryon perturbations, dashed lines the CDM perturbations, and dotted lines the massive neutrino perturbations. Perturbations on four size scales are shown; each scale is normalized to have the same initial amplitude,



Holtzman 1989

Power Spectrum

Consider a field of density perturbations in a volume V

$$\delta(\vec{x}) = \frac{\rho(\vec{x}) - \rho_b}{\rho_b}$$

The average of the density contrast is equal to zero:

$$\langle \delta(\vec{x}) \rangle = 0$$

Let's find the dispersion of the density contrast:
Decompose the density contrast into the Fourier spectrum

$$\langle \delta^2(\vec{x}) \rangle$$

$$\delta(\vec{x}) = \frac{V}{(2\pi)^3} \int d^3k \; \delta_{\vec{k}} \, e^{-i\vec{k}\vec{x}}$$

$$\delta_{\vec{k}} = \frac{1}{V} \int d^3x \; \delta(\vec{x}) \, e^{i\vec{k}\cdot\vec{x}}$$

Find the dispersion:

$$\langle \delta^2(\vec{x}) \rangle = \frac{1}{V}\int d^3x\, \delta^2(\vec{x}) = \frac{1}{V}\int d^3x\, \delta(x)\,\delta(x) =$$

$$= \int \frac{d^3x}{V}\, V\int \frac{d^3k}{(2\pi)^3}\,\delta_{\vec{k}}\, e^{-i\vec{k}\vec{x}}\; V\int \frac{d^3k'}{(2\pi)^2}\,\delta^*_{\vec{k}'}\, e^{i\vec{k}'\vec{x}} = \qquad \Bigg\|\; \delta_{-\vec{k}} = \delta^*_{\vec{k}}$$

$$= \int \frac{d^3x}{V}\, V^2 \int \frac{d^3k\, d^3k'}{(2\pi)^6}\,\delta_{\vec{k}}\,\delta^*_{\vec{k}'}\, e^{-i(\vec{k}-\vec{k}')\vec{x}}$$

Change the order of integration:

$$\langle \delta^2 \rangle = \int \frac{d^3k\, d^3k'}{(2\pi)^6}\,\delta_{\vec{k}}\,\delta^*_{\vec{k}'}\, \underbrace{V^2 \int \frac{d^3x}{V}\, e^{-i(\vec{k}-\vec{k}')\vec{x}}}_{\delta(\vec{k}-\vec{k}')}$$

$$= V\int \frac{d^3k}{(2\pi)^3}\,|\delta_{\vec{k}}|^2 = \frac{V}{(2\pi)^3}\int_0^\infty 4\pi k^2 dk\, |\delta_k|^2$$

Thus, we get:

$$\langle \delta^2 \rangle = \frac{V}{2\pi^2}\int_0^\infty k^2 \delta_k^2\, dk$$

We define the **power spectrum** as

Here the averaging is done over all
Waves with given k and over the whole space

$$P(k) = \langle |\delta_{\vec{k}}|^2 \rangle$$

Correlation function is defined as

$$\xi(r) = \langle \delta(\vec{x})\,\delta(\vec{x}+\vec{r}) \rangle$$

The averaging is done for the whole
volume and over angles of vector $\vec{r}$

$$\xi(r) = \frac{1}{4\pi}\int d\Omega \int \frac{d^3x}{V} \int \frac{V d^3k}{(2\pi)^3}\,\delta_{\vec{k}}\, e^{-i\vec{k}\vec{x}} \int \frac{V d^3k'}{(2\pi)^3}\,\delta^*_{\vec{k}'}\, e^{i\vec{k}'(\vec{x}+\vec{r})} =$$

$$= \frac{1}{4\pi}\int d(\cos\theta)\,d\varphi \int \frac{V d^3k}{(2\pi)^3}\,\delta_{\vec{k}}\, e^{i\vec{k}\vec{r}}\,\delta^*_{\vec{k}} =$$

$$= \int \frac{V d^3k}{(2\pi)^3}\,\delta_{\vec{k}}\,\delta^*_{\vec{k}}\,\frac{1}{4\pi}\int d(\cos\theta)\,d\varphi\, e^{i\vec{k}\vec{r}}$$

imaginary

$$e^{i\vec{k}\vec{r}} = \cos(kr\cos\theta) + i\,\cancel{\sin(kr\cos\theta)}$$

$$\frac{1}{4\pi}\int_{-1}^{1} dx \int_0^{2\pi} d\varphi\, \cos(krx) = \frac{\sin(kr)}{kr}$$

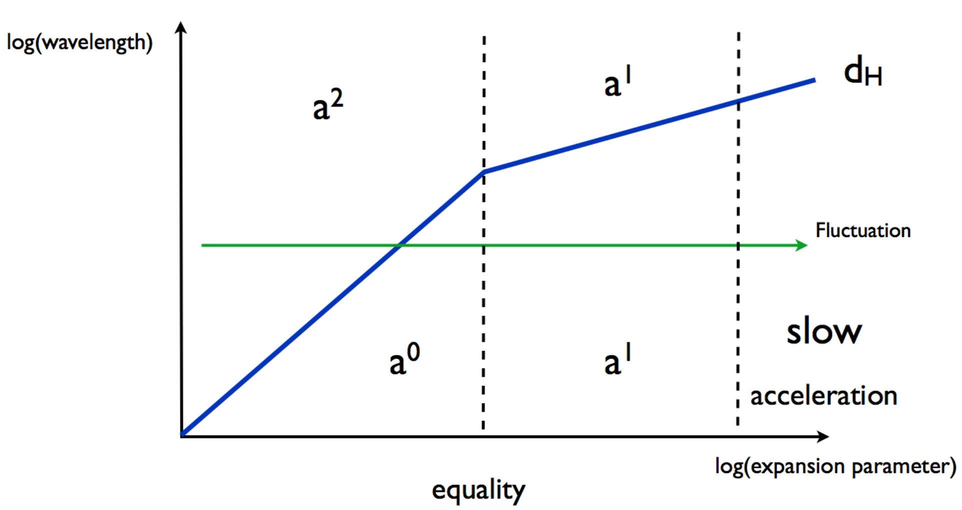
Thus, we get the relation between the correlation function and the power spectrum:

$$\xi(r) = \frac{1}{2\pi^2} \int_0^\infty k^2 dk \, \frac{\sin kr}{kr} P(k)$$

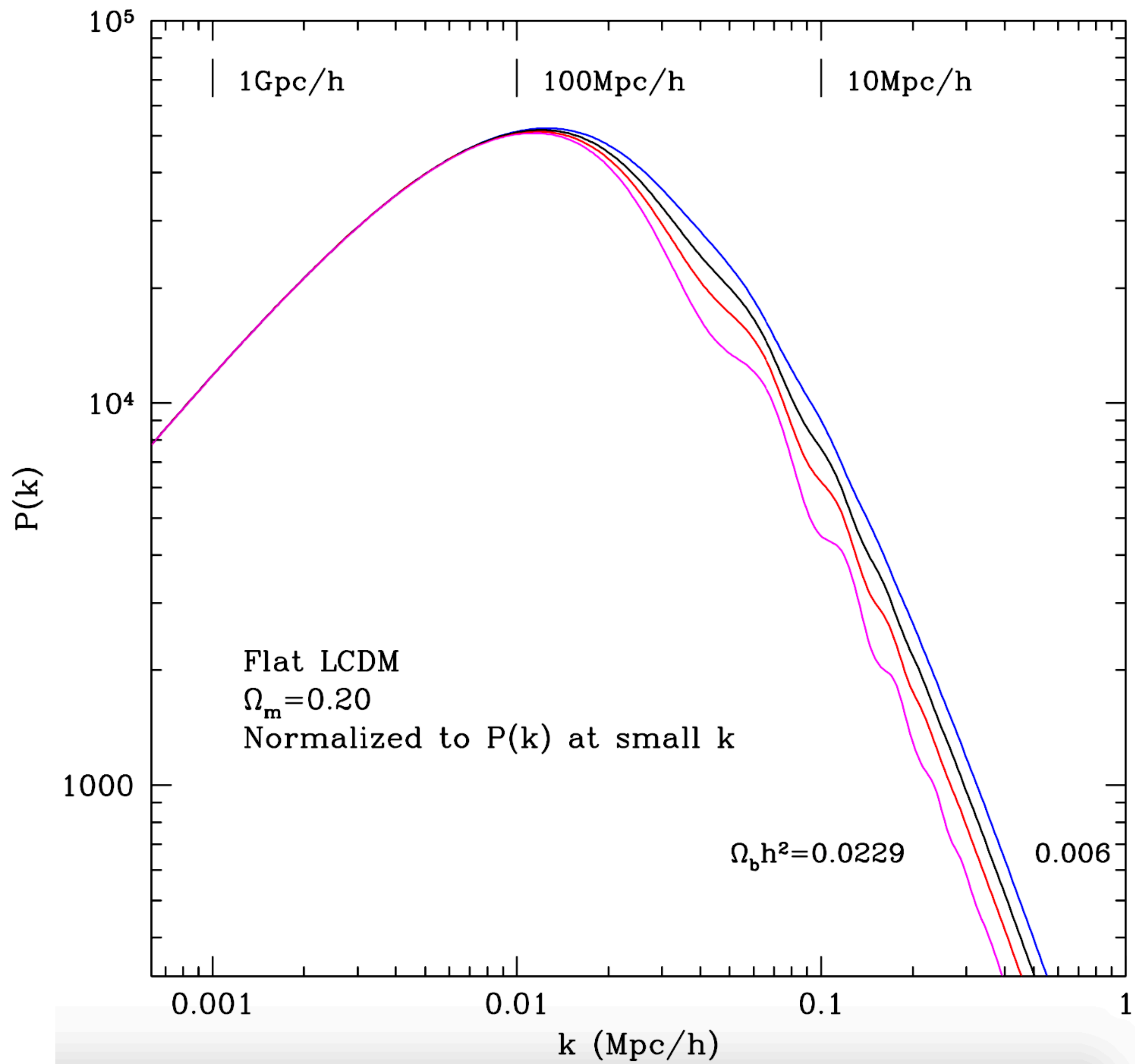
There is an inverse relation:

$$P(k) = 4\pi \int r^2 dr \, \xi(r) \frac{\sin kr}{kr}$$

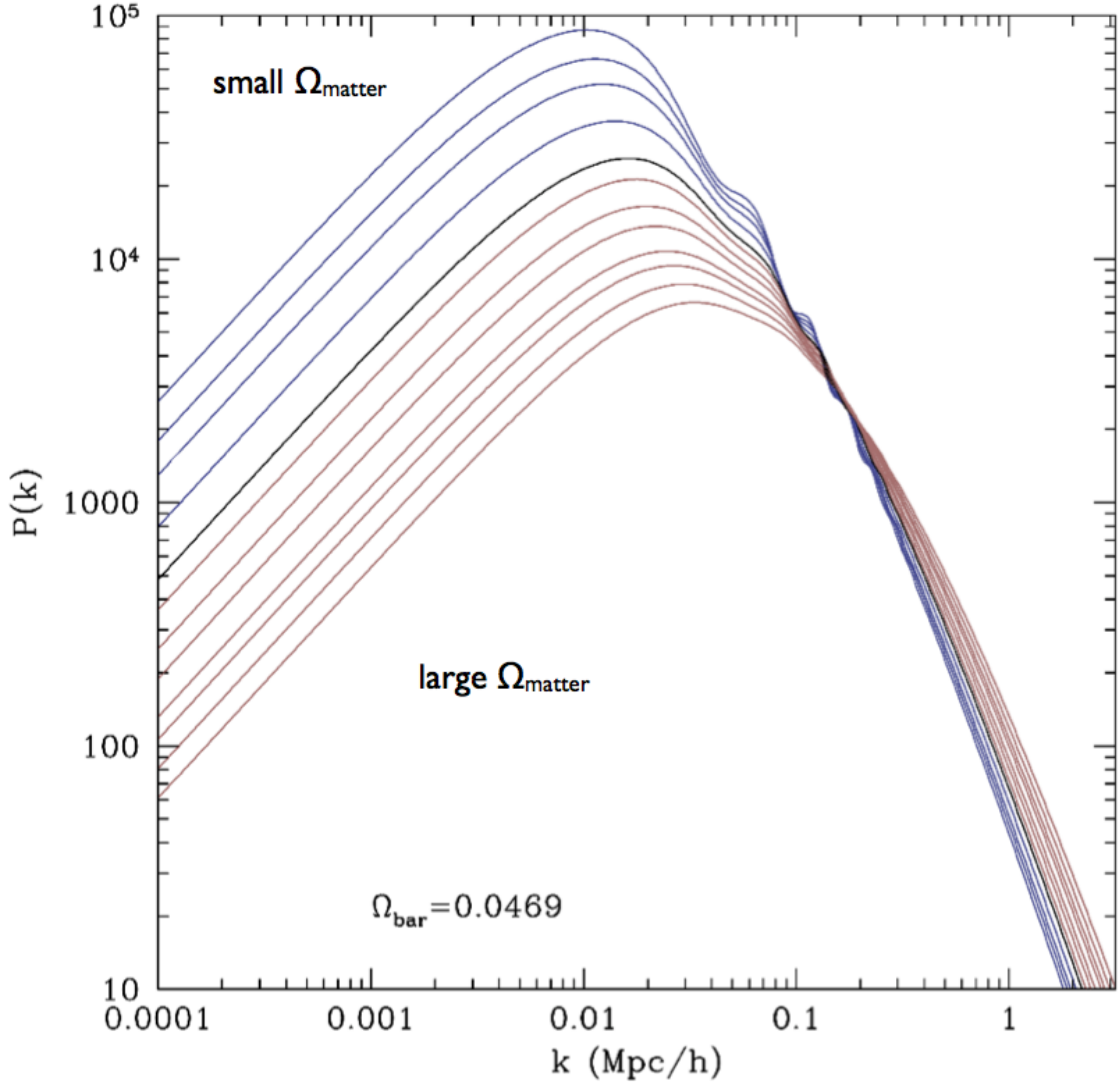
Power Spectrum



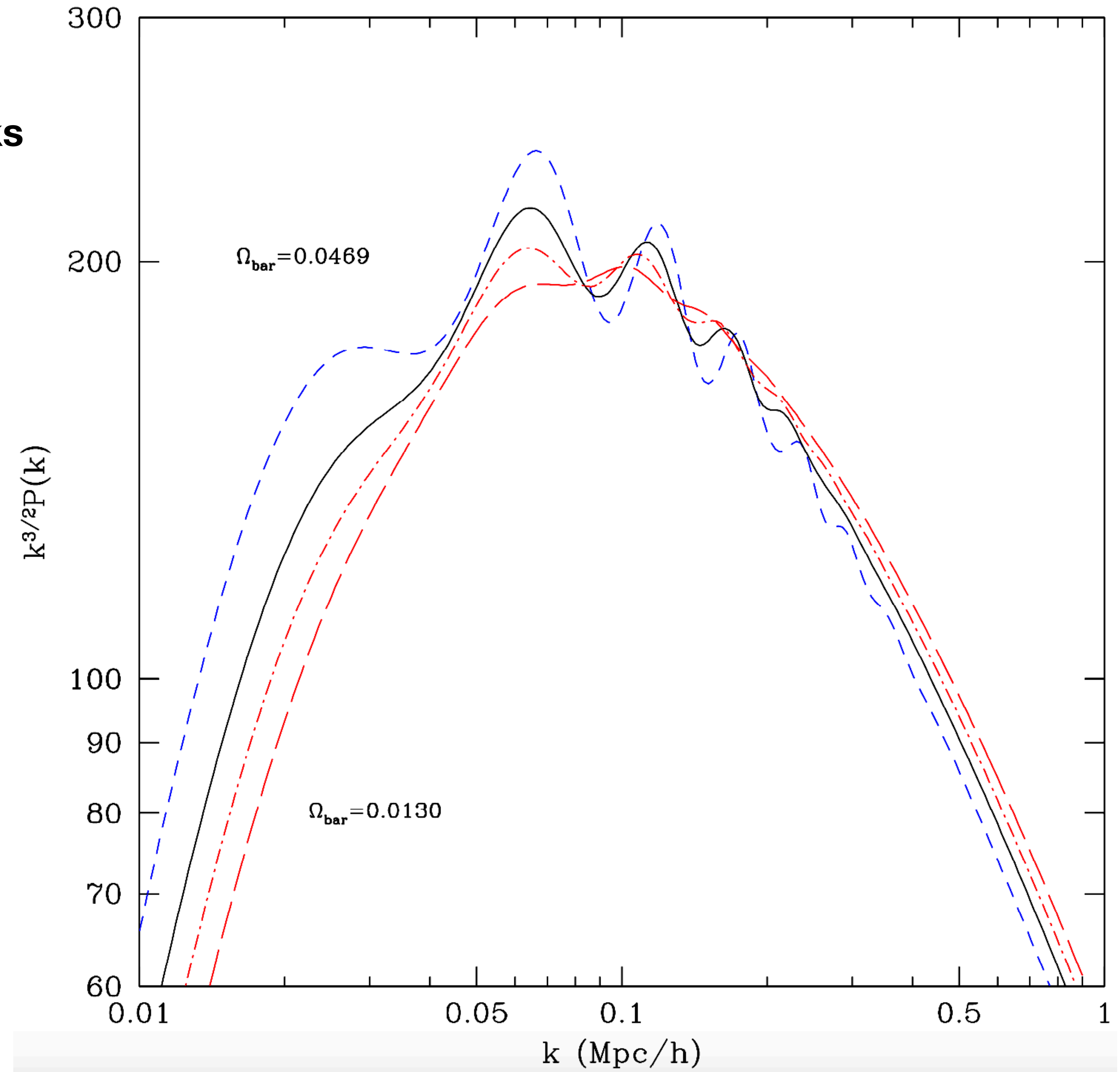
Chessboard of growth of adiabatic perturbations



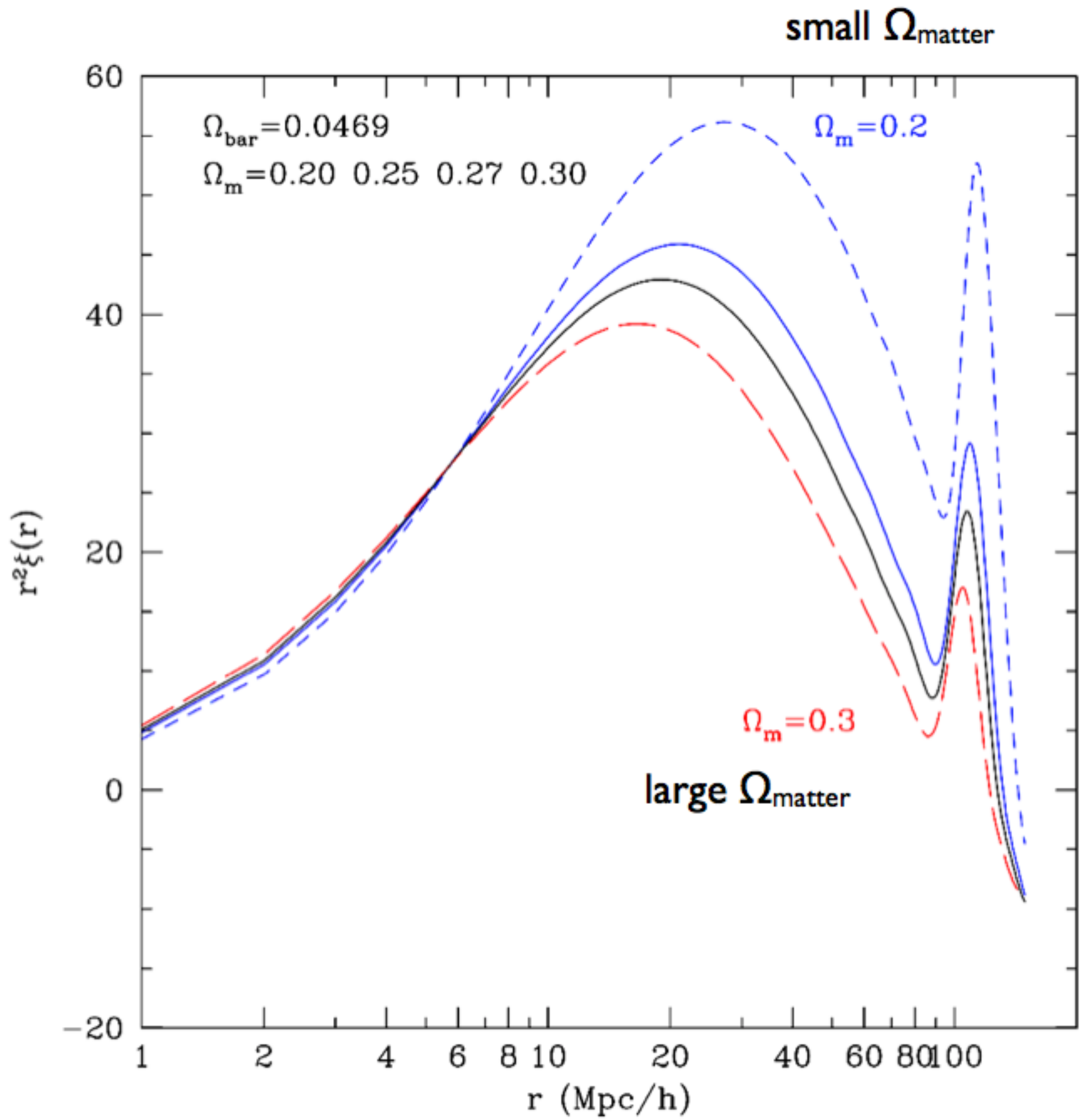
Dependance of P(k) on Ω_{matter}
Amplitude of fluctuations and $\Omega_{baryons}$ are fixed.



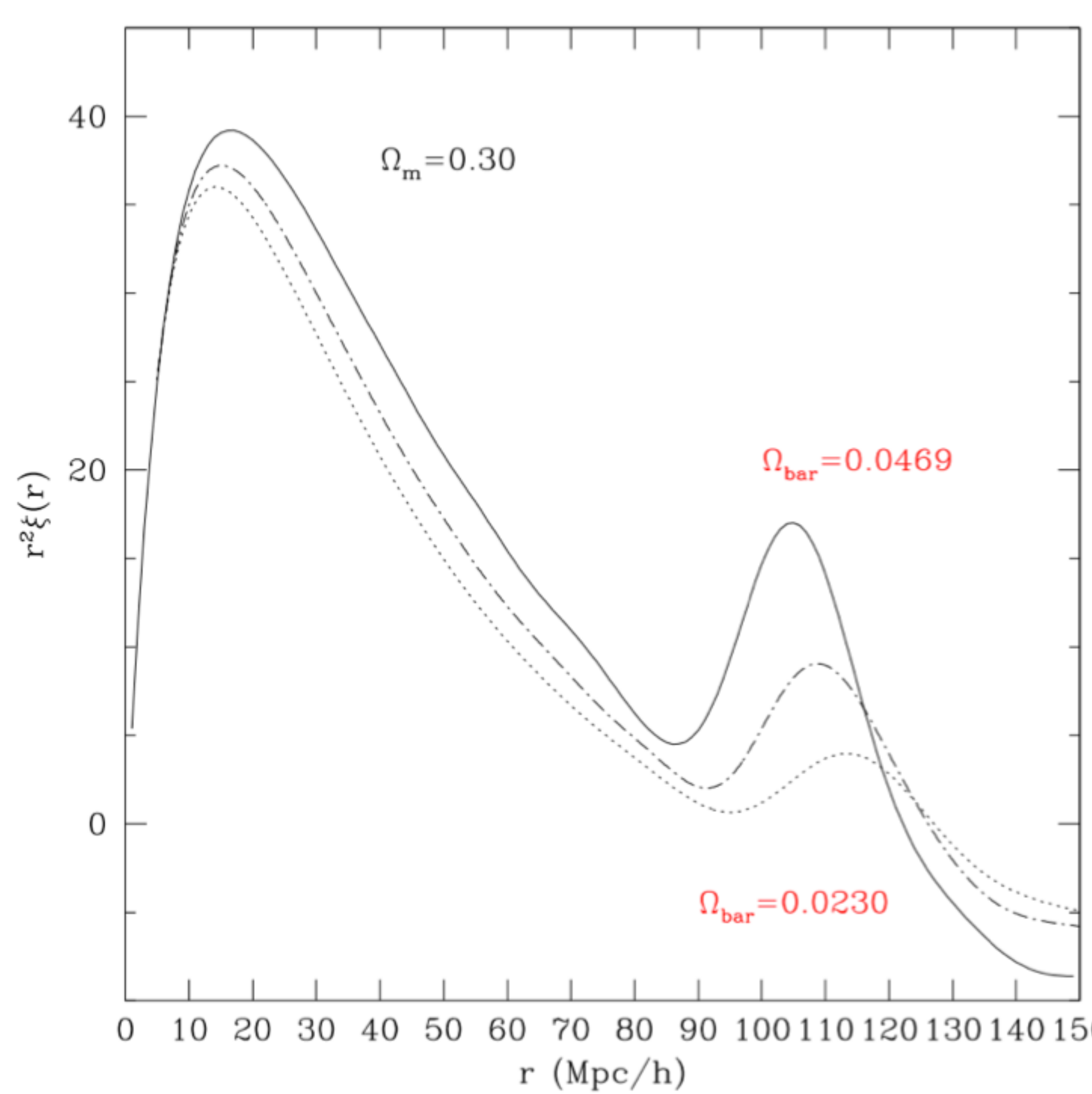
Zoom-in on the domain of BAO peaks



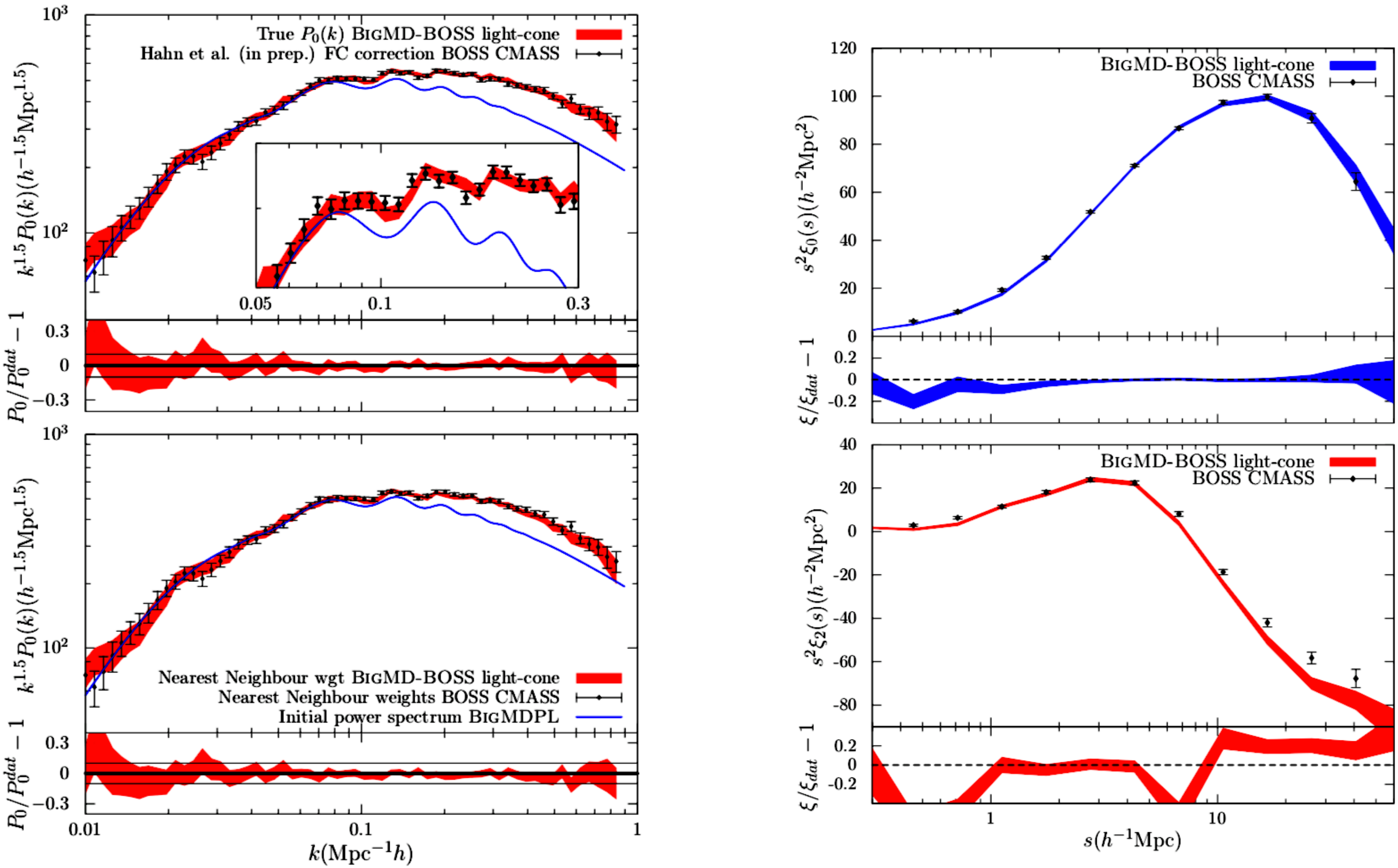
Dependance of Correlation function on Ω_{matter}
Amplitude of fluctuations is fixed at 5Mpc/h



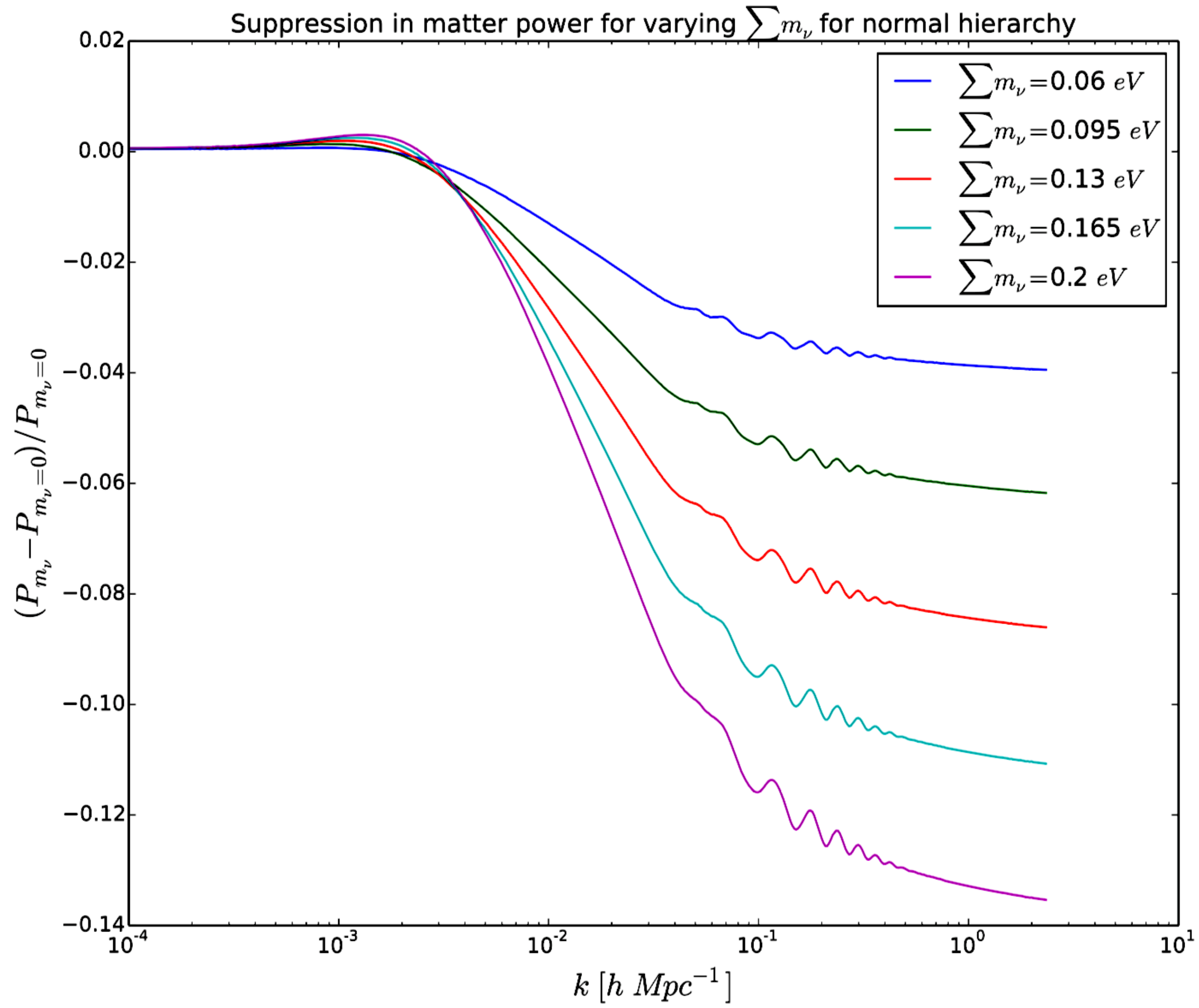
Dependance of Correlation function on $\Omega_{mbaryon}$
Amplitude of fluctuations is fixed at 5Mpc/h



At the end of the day the LCDM explains both P(k) and correlation function of galaxies



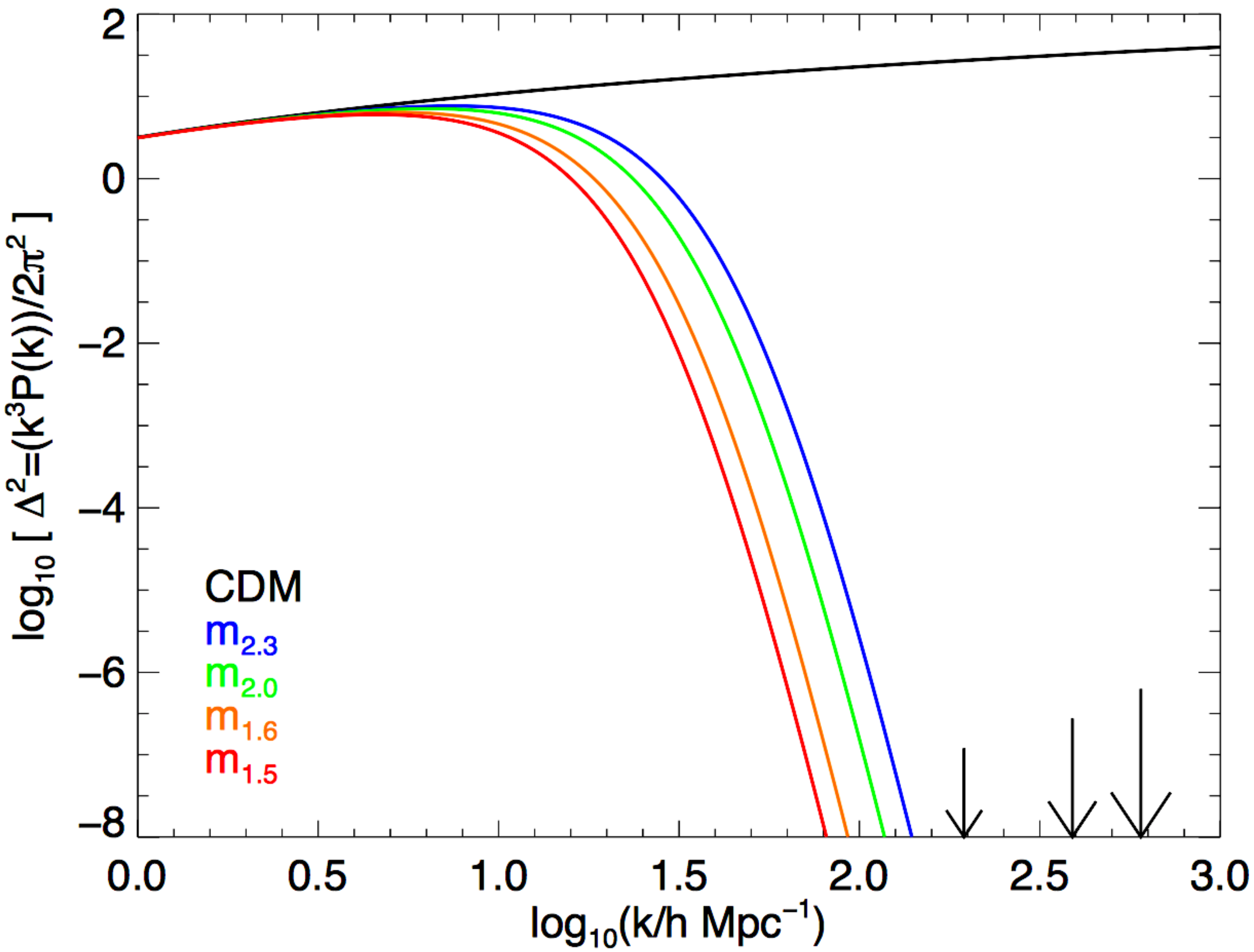
Effects of massive neutrinos on power spectrum



Dawson etal (eBOSS) 2017

Warm dark matter

Simulation	$m_{\rm WDM}$[keV]
CDM-W7	–
$m_{2.3}$	2.322
$m_{2.0}$	2.001
$m_{1.6}$	1.637
$m_{1.5}$	1.456



Lovell et al 2014

Warm dark matter

Random ("thermal") velocities of dark matter particles suppress fluctuations in dark matter: free-streaming effect. Details depend on particular particle model of wdm candidates.

the wavenumber at which the linear WDM suppression reaches 50% in terms of matter power, $k_{1/2}$, w.r.t. the ΛCDM case can be approximated as:

$$k_{1/2} \sim 6.5 \frac{h}{\mathrm{Mpc}} \left(\frac{m_{\mathrm{WDM}}}{1\mathrm{keV}}\right)^{1.11} \left(\frac{\Omega_{DM}}{0.25}\right)^{-0.11} \left(\frac{h}{0.7}\right)^{1.22},$$

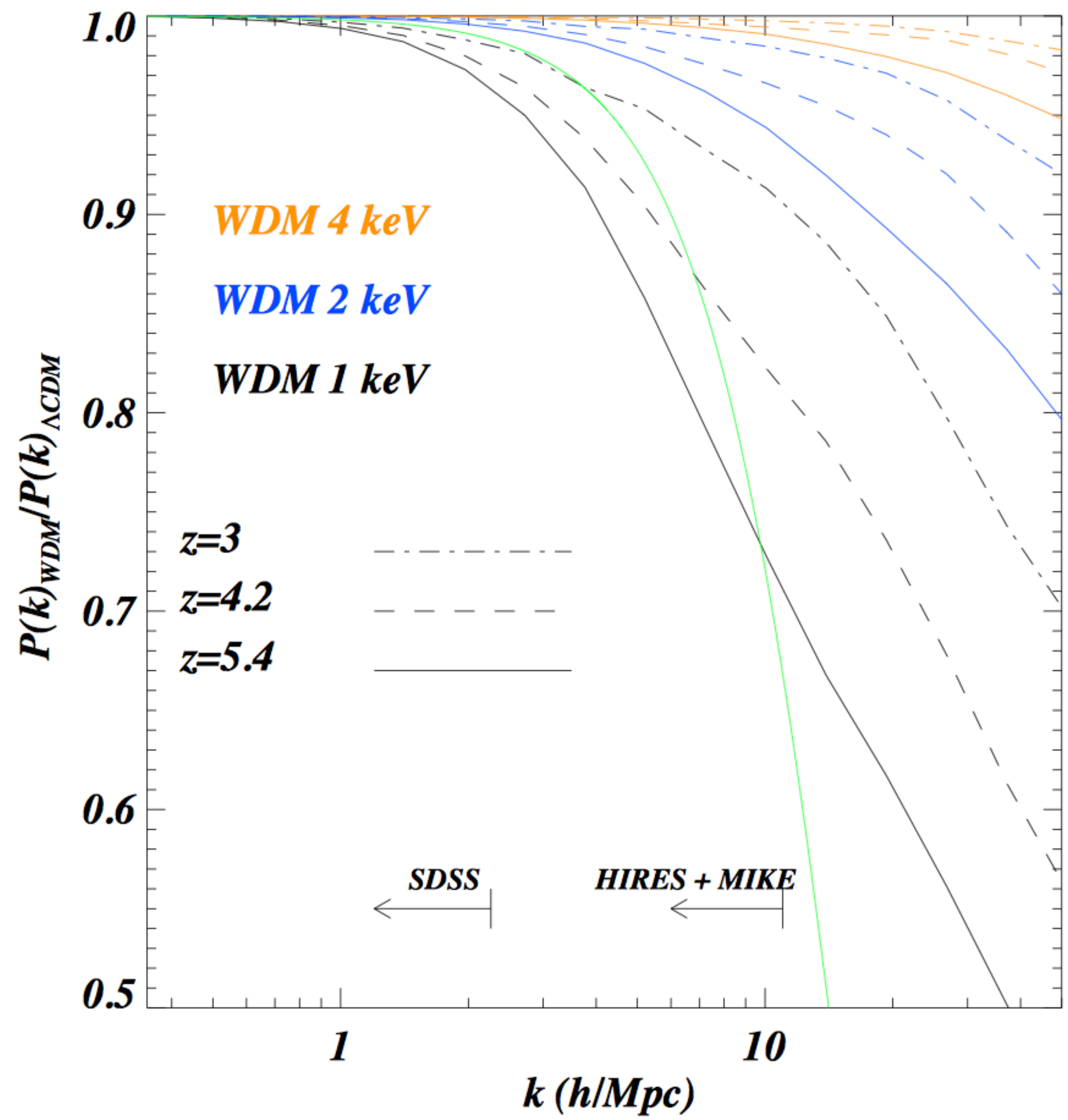


FIG. 1: Ratio between the 3D non-linear matter power spectrum of 3 different WDM models (1, 2 and 4 keV, black, blue and orange curves) at 3 different redshifts (z = 3, 4.2, 5.4, represented by the dot-dashed, dashed and continuous curves) and the corresponding ΛCDM model. The green curve represents the linear redshift independent suppression in terms of matter power for a $m_{\mathrm{WDM}} = 2$ keV model obtained using

Viel et al 2013

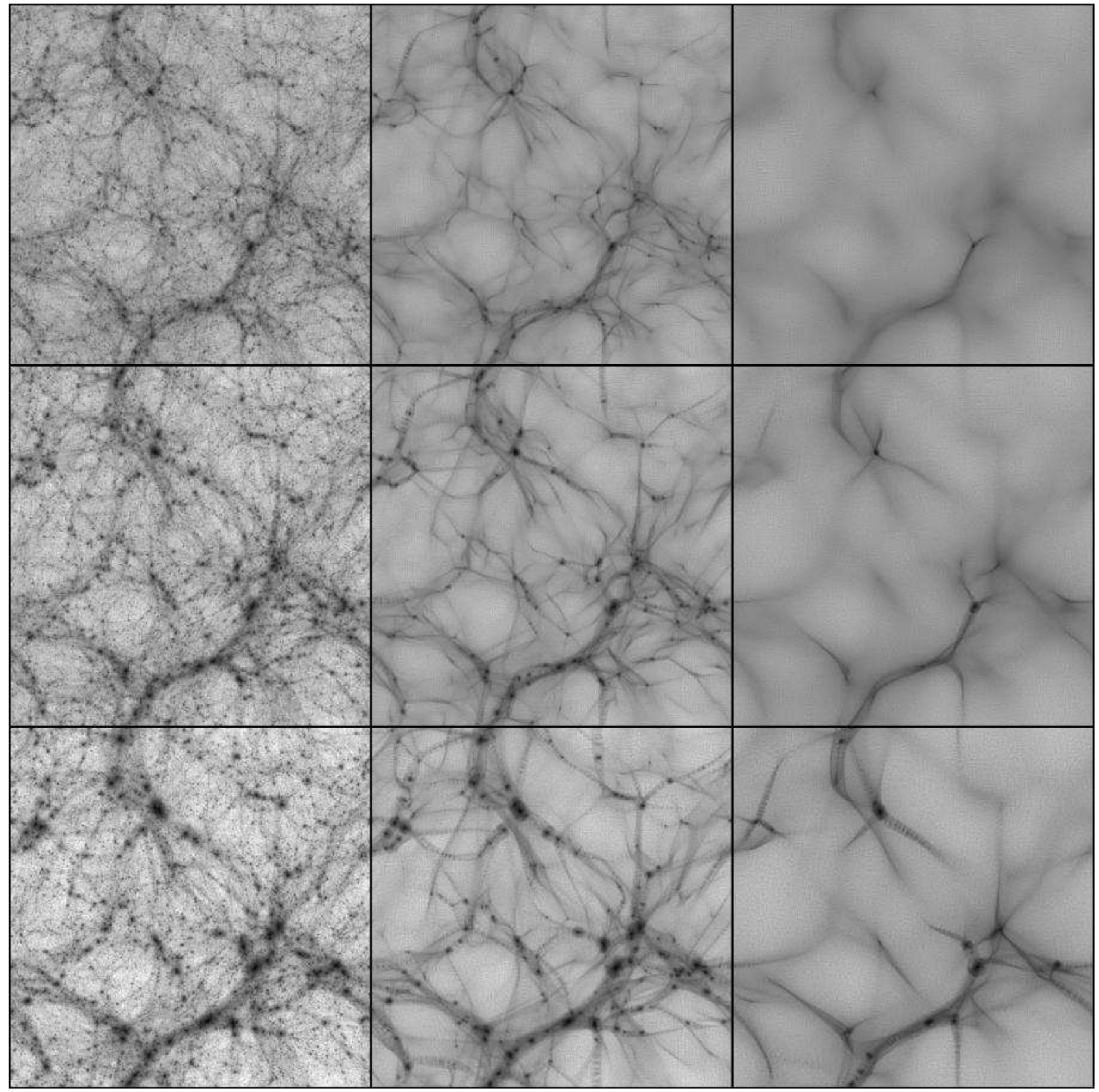
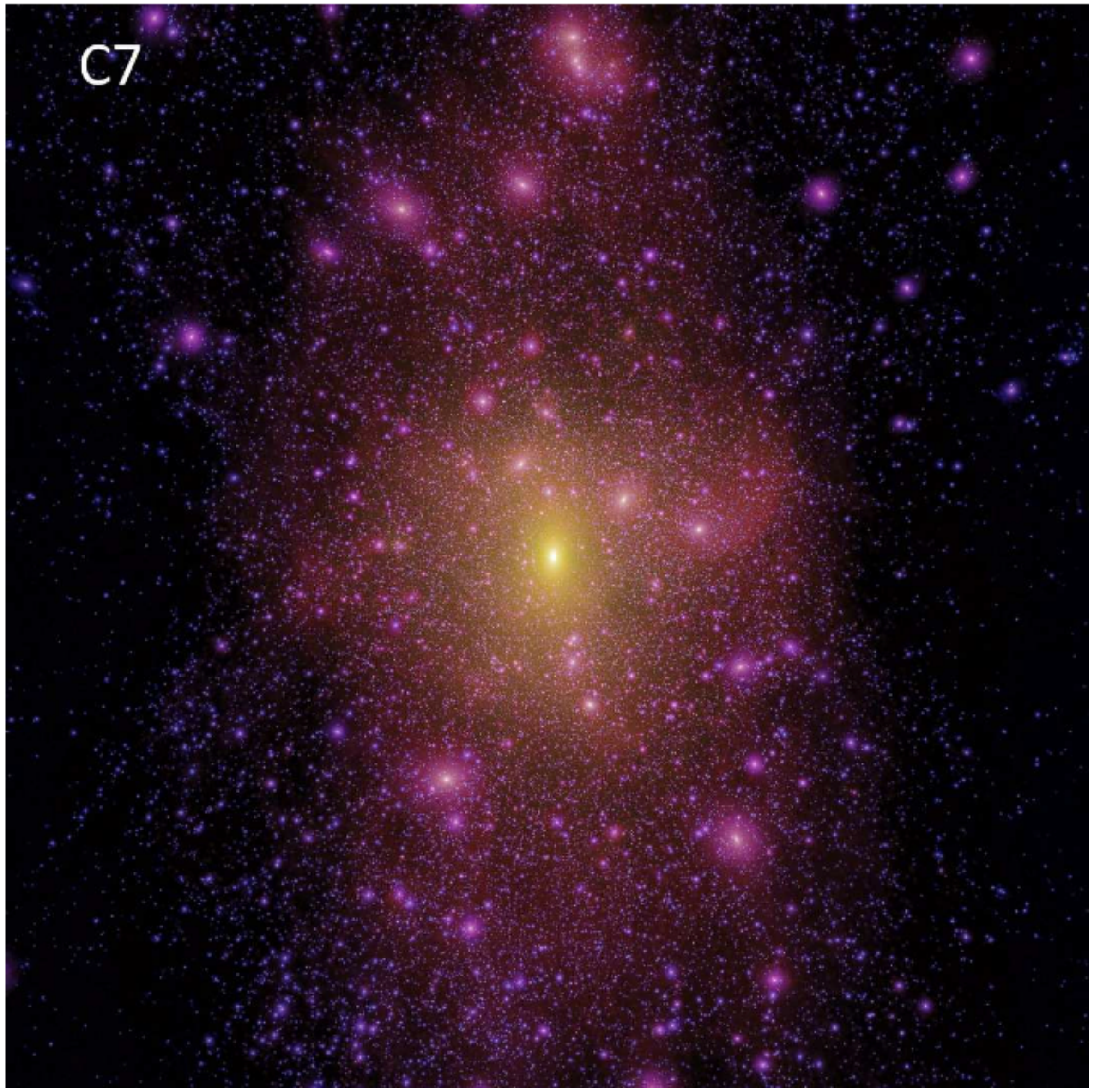


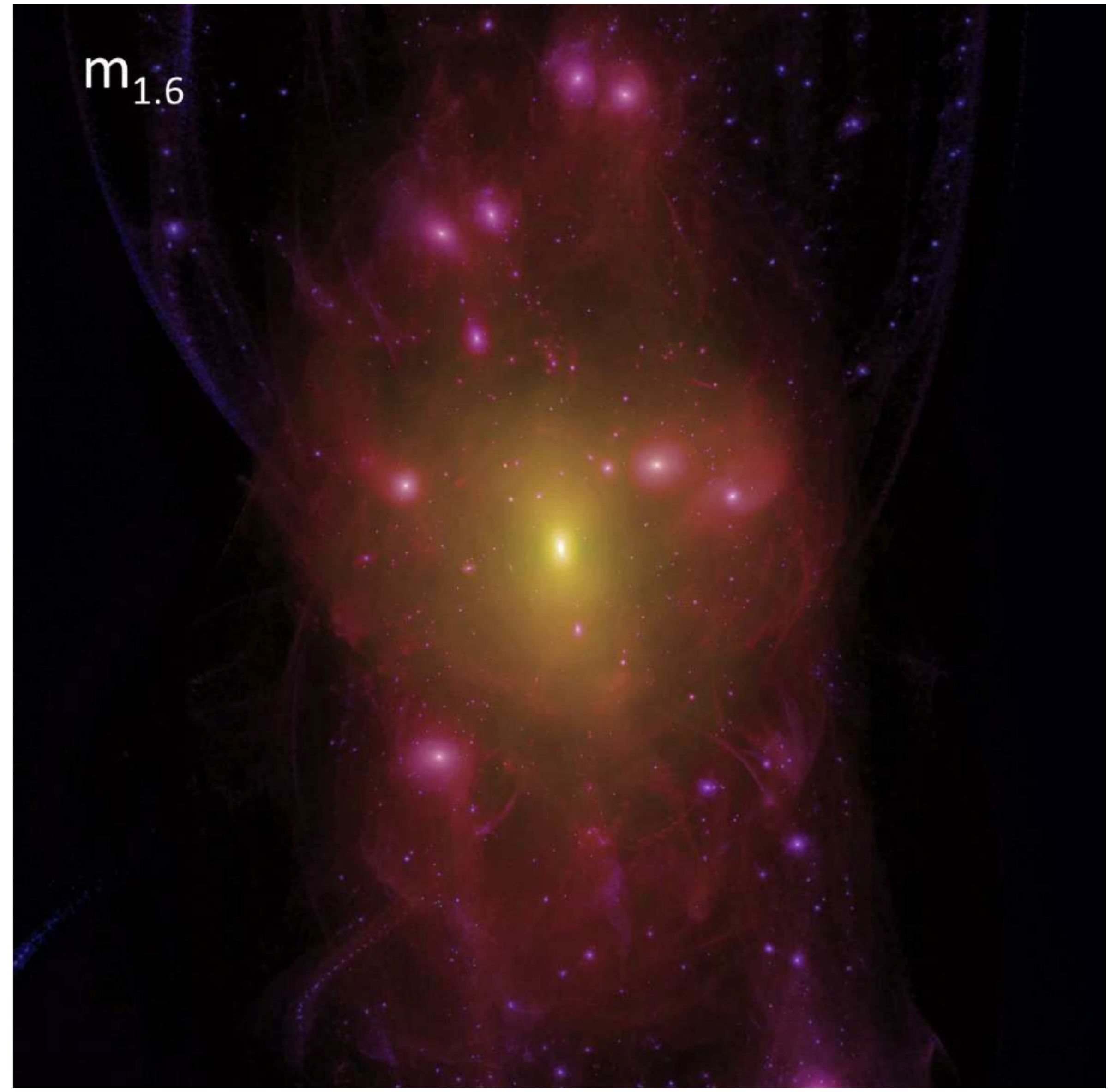
Fig. 4.— Projected density of 20 h^{-1}Mpc boxes, on a logarithmic scale of surface density. Left to right: ΛCDM, m_X=350 eV and m_X=175 eV ΛWDM. Top to bottom: redshift Z =3, 2, and 1. A simulation with $m_X \sim 1$ keV would have an appearance intermediate between the left and central columns. (A higher resolution version of this Figure is available at the web site referred to in the introduction.)

Bode et al 2001

LCDM

WDM





Lovell et al 2014

Abundance of galaxies in WDM vs. LCDM

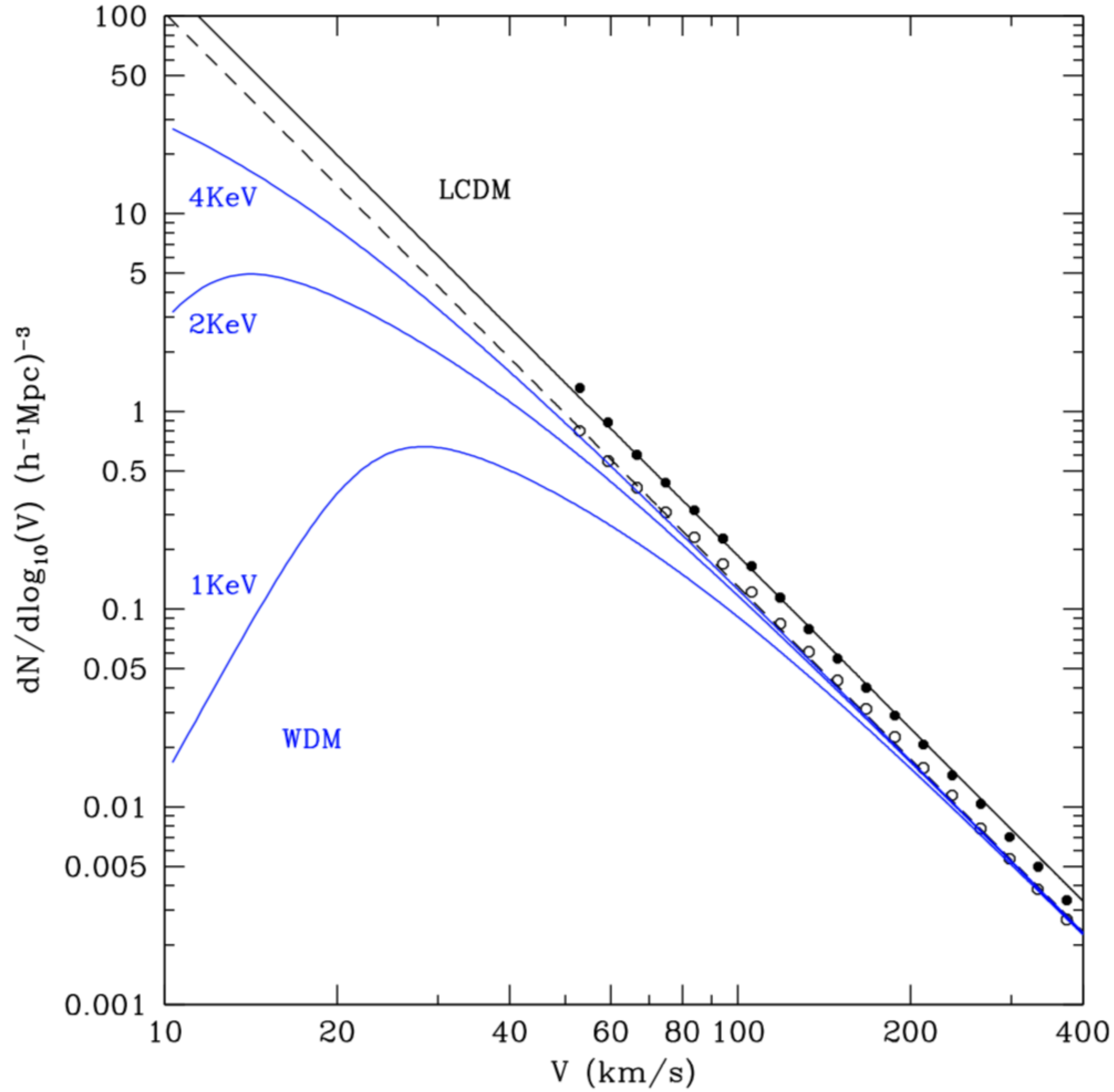


Figure 6. Circular VF for haloes in the ΛCDM and WDM models. Open circles are results from the Bolshoi simulation (Klypin et al. 2011) for *WMAP7* cosmology. The dashed line is the power-law approximation. Filled circles are for the BolshoiP simulation (Klypin et al. 2014) for the *Planck* cosmology. The top solid line shows a power-law fit for this cosmology. Other curves are analytical fits for the WDM model with *WMAP7* cosmological

Klypin et al. 2015