Inflation and reheating in the early Universe
Lecture #4
Inflation

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from gravity and cosmology
to physics of condensed matter”
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Outline

1. Inflation
   - Problems of the Big Bang Theory
   - Inflationary stage

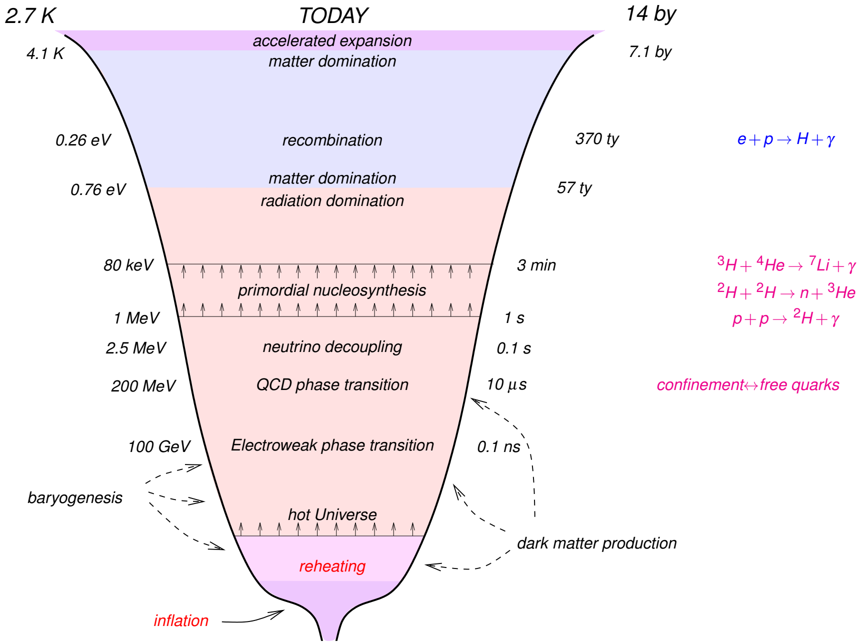
Outline

Big Bang within GR and SM: problems

- Dark Matter
- Baryogenesis
- Horizon, Enthropy, Flatness, ... problems
  $l_{H_0}/l_{\mathrm{H},r}(t_0) \sim \sqrt{1+z_r} \simeq 30$
- Singularity at the beginning
- Heavy relics
- Initial fluctuations $\delta T/T \sim \delta\rho/\rho \sim 10^{-4}$, scale-invariant
- Dark Energy $0 \neq \Lambda \ll M_{Pl}^4 \; M_W^4 \; \Lambda_{QCD}^4$ etc ?
- Coincidence problems:

$$\Omega_B \sim \Omega_{DM} \sim \Omega_\Lambda \; ,$$
$$\eta_B = n_B/n_\gamma \sim (\delta T/T)^2 \; ,$$
$$T_d^n \sim (m_n - m_p) \; ,$$
$$\ldots$$

- ΛCDM tensions: lack of dwarfs? cusps? (recall: reionization @ $z=20$)


2.7 K
TODAY
14 by
accelerated expansion
4.1 K
matter domination
7.1 by
0.26 eV
recombination
370 ty
$e+p\to H+\gamma$
0.76 eV
matter domination
radiation domination
57 ty
80 keV
3 min
$^3H+{}^4He\to{}^7Li+\gamma$
primordial nucleosynthesis
$^2H+{}^2H\to n+{}^3He$
1 MeV
1 s
$p+p\to{}^2H+\gamma$
2.5 MeV
neutrino decoupling
0.1 s
200 MeV
QCD phase transition
10 μs
confinement↔free quarks
100 GeV
Electroweak phase transition
0.1 ns
baryogenesis
hot Universe
dark matter production
reheating
inflation

Initial singularity problem (Bang!)

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3} G\rho\,, \qquad p = w\rho\,,\;\; w > -\frac{1}{3} \qquad (?)$$

dust: $p = 0$ — singular at $t = t_s$

$$\rho = \frac{\text{const}}{a^3}\,, \quad a(t) = const \cdot (t - t_s)^{2/3}\,, \quad \rho(t) = \frac{\text{const}}{(t - t_s)^2}$$

$$t_s = 0\,, \quad H(t) = \frac{\dot{a}}{a}(t) = \frac{2}{3t}\,, \quad \rho = \frac{3}{8\pi G} H^2 = \frac{1}{6\pi G} \frac{1}{t^2}$$

radiation: $p = \frac{1}{3}\rho$ — singular at $t = t_s$

$$\rho = \frac{\text{const}}{a^4}\,, \quad a(t) = const \cdot (t - t_s)^{1/2}\,, \quad \rho(t) = \frac{\text{const}}{(t - t_s)^2}$$

$$t_s = 0\,, \quad H(t) = \frac{\dot{a}}{a}(t) = \frac{1}{2t}\,, \quad \rho = \frac{3}{8\pi G} H^2 = \frac{3}{32\pi G} \frac{1}{t^2}$$

Entropy problem

$$\nabla_\mu T^{\mu 0} = 0 \;\longrightarrow\; \dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = 0$$

for equation of state

$$p = p(\rho)$$

of the primordial plasma we obtain

$$-3d(\ln a) = \frac{d\rho}{p+\rho} = d(\ln s)$$

entropy is conserved in a comoving volume

$$sa^3 = \text{const}$$

For the visible part of the Universe: $S \sim s_{\gamma,0} \cdot l_H^3 \sim 10^{88}$

At the “Bang” for the Planck-size volume: $S_{BB} \sim s_{\gamma,0} \cdot l_{Pl}^3 \sim 100$

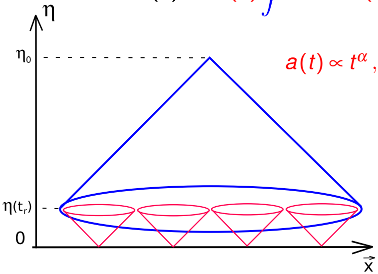
Horizon problem $l_H(t)$

a distance covered by photon emitted at $t = 0$

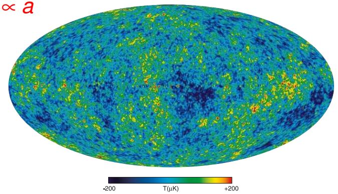
size of the causally connected part, that is the visible part of the Universe (“inside horison”)

$ds^2 = dt^2 - a^2(t)\,dx^2 = a^2(\eta)\left(d\eta^2 - dx^2\right)$ $\qquad ds^2 = 0$

$$l_H(t) = a(t)\int dx = a(t)\int d\eta = a(t)\int_0^t \frac{c\,dt'}{a(t')} \propto t \quad \propto 1/H(t)$$



$a(t) \propto t^\alpha\,,\ 0 < \alpha < 1\,,\ L_{phys} \propto a$



$$l_{H_0}/l_{H,r}(t_0) \sim l_{H_0}/l_{H,r}(t_r)\,a(t_r)/a_0 \sim H_r/H_0\,a(t_r)/a_0 \sim \sqrt{1+z_r} \simeq 30$$

Flatness problem

- Take non-flat 3-dim manifold (general case)
- Curvature contribution to the total energy density behaves as $\rho_{curv}(t) \propto 1/a^2(t)$
- Then at present:

$$0.01 > \Omega_{curv} = \frac{\rho_{curv}(t_0)}{\rho_c} \sim 10^{-4} \times \frac{\rho_{curv}(t_0)}{\rho_{rad}(t_0)} = 10^{-4} \times \frac{a^2(t_0)}{a^2(t_*)} \frac{\rho_{curv}(t_*)}{\rho_{rad}(t_*)}$$

$$\sim 10^{-4} \times \frac{T_*^2}{T_0^2} \frac{\rho_{curv}(T_*)}{\rho_{tot}(T_*)}$$

- For hypothetical Planck epoch $T_* \sim M_{Pl} \sim 10^{19}$ GeV one gets

$$0.01 > \Omega_{curv} \sim 10^{60} \times \frac{\rho_{curv}(M_{Pl})}{\rho_{tot}(M_{Pl})}$$

Heavy relics problem (monopole problem)

- Let's introduce new stable particle X of mass M_X
- Imagine: at moment t_X they appear in the early Universe with small velocities (e.g. nonrelativistic) and small density $n_X(t_X) \ll n_{rad}(t_X)$
- Since $n_X \propto a^{-3} \propto n_{rad}$ then $n_X(t)/n_{rad}(t) \simeq \text{const}$

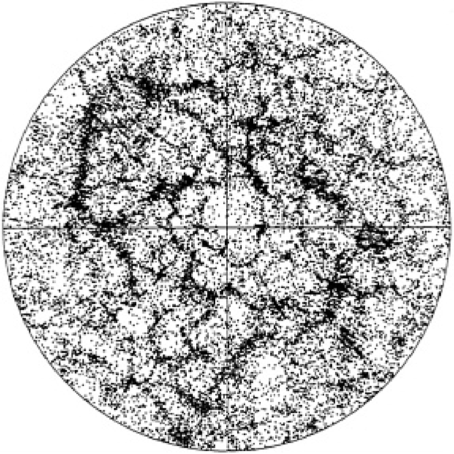
$$\frac{\rho_X(t)}{\rho_{rad}(t)} \sim \frac{M_X}{T(t)} \cdot \frac{n_X(t_X)}{n_{rad}(t_X)} \propto a(t)$$

- Radiation dominates at least while $1\,\text{eV} \lesssim T \lesssim 3\,\text{MeV}$
- Therefore even for $M_X = 10\,\text{TeV}$ we must require $n_X(t_X)/n_{rad}(t_X) \ll 10^{-12}$!!!
- In some SM extenstions it is difficult to avoid heavy relics production: gravitational production, $M_X \sim H$, phase transitions...

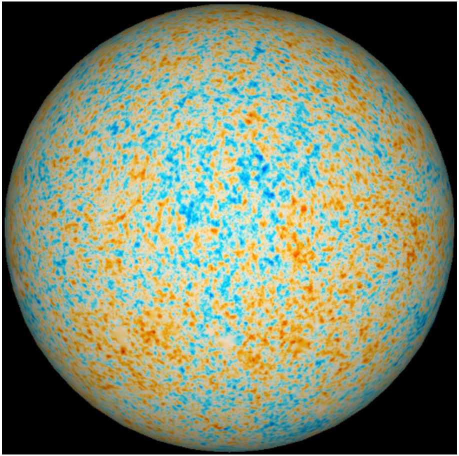
Example: monopoles, produced "one per horizon volume", $n_X(t_X) = 1/l_H^3(t_X) = H^3(t_X)$; Then for its present contribution:

$$\Omega_X = \frac{\rho_X}{\rho_c} \sim 10^{17} \times \frac{M_X}{10^{16}\,\text{GeV}} \left(\frac{T_X}{10^{16}\,\text{GeV}}\right)^3 \sqrt{\frac{g_*}{100}}$$

Inhomogeneous Universe



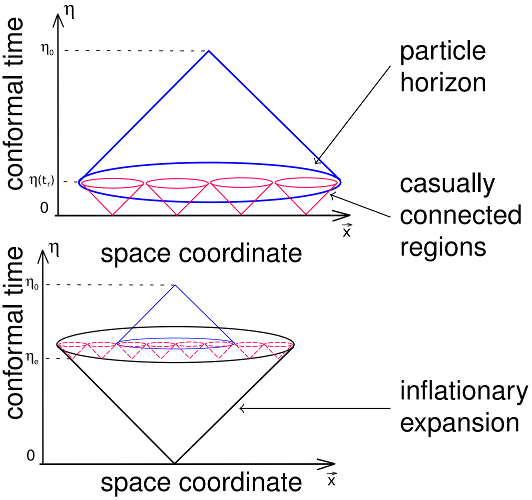
Large Scale Structure



CMB anisotropy

Inflationary solution of Hot Big Bang problems

- no initial singularity in dS space
- all scales grow exponentially, including the radius of the 3-sphere
  the Universe becomes exponentially flat
- any two particles are at exponentially large distances
  no heavy relics
  no traces of previous epochs!
- no particles in post-inflationary Universe
  to solve entropy problem we need post-inflationary reheating



Inflation: general remarks

- Simplest variant

$$H^2 = \frac{8\pi}{3M_{Pl}^2}\rho_\Lambda = \text{const}\,,\ \Rightarrow\ a(t) \propto e^{Ht}$$

is not suitable: inflation must not last for ever!

- Universe has to reheat after! T_{reh}

$$\rho_e \gtrsim (3\,\text{MeV})^4\,,\ \text{and better}\ \rho_e \gtrsim (100\,\text{GeV})^4\,,$$

- How long? Horizon problem:
  present size of the horizon at the end of inflation

$$l_{H,e}(t_0) = a_0 \int_{t_{Pl}}^{t_e} \frac{dt}{a(t)} = a_0 \int_{t_{Pl}}^{t_e} \frac{da}{a^2}\frac{1}{H} \sim \frac{a_0}{a(t_{Pl})}\cdot\frac{1}{H(t_{Pl})}$$

Solution to the horizon problem:

$$1 \lesssim \frac{l_{H,e}(t_0)}{l_{H,0}} \sim \frac{a_0}{a(t_{Pl})}\frac{H_0}{H(t_{Pl})} = \frac{a_0}{a(t_{reh})}\frac{a_{reh}}{a(t_e)}\frac{a(t_e)}{a(t_{Pl})}\cdot\frac{H_0}{H(t_{Pl})}$$

Introducing the number of e-foldings

$$N_e^{tot} = \ln\frac{a(t_e)}{a(t_{Pl})}\,,\quad N_e^{tot} = \int_{t_{Pl}}^{t_e} dt\, H(t) \sim H_e\cdot\Delta t_{infl}$$

For relativistic particles $\rho \propto T^4 \propto 1/a^4 \ \Rightarrow\ a_0/a(t_{reh}) \sim T_{reh}/T_0$

Inflation: general remarks

- How long? Solution to the horizon problem:

$$1 \lesssim \frac{l_{H,e}(t_0)}{l_{H,0}} \Rightarrow N_e^{tot} \gtrsim \log\frac{T_0}{H_0} + \ln\frac{a(t_e)}{a_{reh}} + \ln\frac{H(t_{Pl})}{T_{reh}} \simeq 50-60$$

  Inflation lasts not less than (accepting $H^2 \sim \rho/M_{Pl}^2$)

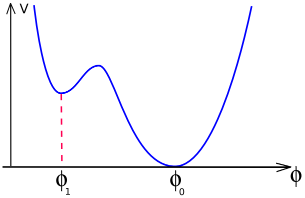
$$\Delta t_{infl} \sim N_e^{tot}/H_e \sim 10^{-11}\,\mathrm{c}\cdot\left(\frac{1\ \mathrm{TeV}}{T_{reh}}\right)^2$$

  we must reheat the Universe then!

- In realistic models $N_e^{tot} \ggg 100$!!!
  Inflatinary stage may be short, but expansion is enormous!

Inflatinary stage: simplest models

"Old inflation" by Guth



does not work in fact!

starts from a hot stage
and ends up in a false vacuum

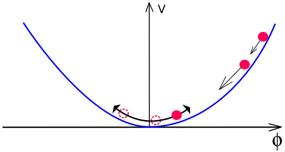
reheating due to percollations

However: for sufficiently long inflationary stage requires

$\Gamma < H^4_{infl}$

hence the bubbles never collide!

"Chaotic inflation"



needs superplanckian field values!

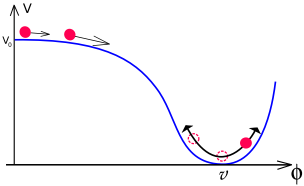
$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$

$$p = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

$$\varepsilon = \frac{M_{Pl}^2}{16\pi}\left(\frac{V'}{V}\right)^2 , \ \eta = \frac{M_{Pl}^2}{8\pi}\frac{V''}{V} ,$$

"New inflation"



Initial condition is very specific!

$$H^2 = \frac{8\pi}{3M_P^2}V(\phi) , \quad a(t) \propto e^{Ht}$$

and we require

$$V(\phi) < M_{Pl}^4$$

$V(\phi) \propto \phi^n \Rightarrow \varepsilon, \eta \sim M_{Pl}^2/\phi^2 \ll 1$ ← slow-roll conditions

Unexpected bonus: generation of perturbations

- Quantum fluctuations of wavelength λ of a free massless field φ have an amplitude of $\delta\varphi_\lambda \simeq 1/\lambda$
- In the expanding Universe: $\lambda \propto a$

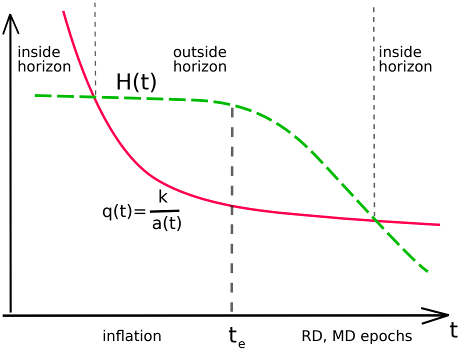
  inflation: $l_H \sim 1/H = \text{const}$, so modes "exit horizon"
  Ordinary stage: $l_H \sim 1/H \propto t$, $l_H/\lambda \nearrow$, modes "enter horizon"

Evolution at inflation

- inside horizon: $\lambda < l_H$
  $\lambda \propto a \Rightarrow$ $\delta\varphi_\lambda \propto 1/\lambda \propto 1/a$

- outside horison: $\lambda > l_H$
  $\lambda \propto a \Rightarrow$ $\delta\varphi_\lambda = \text{const} = H_{infl}$!!!

- got "classical" fluctuations:
  $\delta\varphi_\lambda = \delta\varphi_\lambda^{quantum} \times e^{N_e}$



Power spectrum of perturbations

In the Minkowski space-time:

- fluctuations of a free quantum field φ are gaussian its power spectrum is defined as

$$\int_0^\infty \frac{dq}{q}\,\mathscr{P}_\varphi(q) \equiv \langle\varphi^2(x)\rangle = \int_0^\infty \frac{dq}{q}\,\frac{q^2}{(2\pi)^2}$$

We define amplitude as $\delta\varphi(q) \equiv \sqrt{\mathscr{P}_\varphi} = q/(2\pi)$

- In the expanding Universe momenta $q = k/a$ gets redshifted
- Cast the solution in terms $\phi(\mathbf{x},t) = \phi_c(t) + \varphi(\mathbf{x},t)\,, \quad \varphi(\mathbf{x},t) \propto \mathrm{e}^{\pm i\mathbf{kx}}\,\varphi(\mathbf{k},t)$
  φ solves the equation

$$\ddot{\varphi} + 3H\dot{\varphi} + \frac{k^2}{a^2}\varphi + V''(\phi_c)\varphi = 0$$

- $q = k/a \gg H \;\Rightarrow$ as in Minkowski space-time
- $q = k/a \ll H \;\Rightarrow$ for inflaton $\varphi = \text{const}$
- Matching at t_k: $q(t_k) = k/a(t_k) = H(t_k) \equiv H_k$ gives

$$\delta\varphi(q) = \frac{H_k}{2\pi} \;\Rightarrow\; \mathscr{P}_\varphi(q) = \frac{H_k^2}{(2\pi)^2}$$

amplification $H_k/q = \mathrm{e}^{N_e(k)}$!!!

$H_k \approx \text{const} = H_{infl}$ hence (almost) flat spectrum

Transfer to matter perturbations: simple models

Illustration: Local delay(advance)δt in evolution due to impact of $\delta\phi$ of all modes with $\lambda > H$:

$$\delta\phi = \dot{\phi}_c \, \delta t \,, \quad \delta\rho \sim \dot{\rho} \, \delta t$$

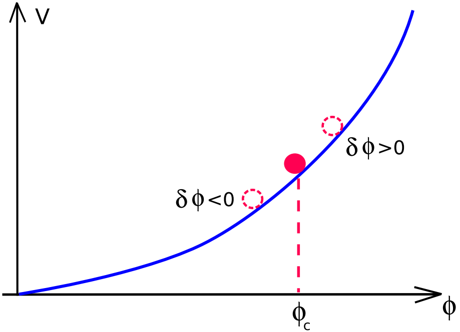
at the end of inflation $\dot{\rho} \sim -H\rho$, then

$$\frac{\delta\rho}{\rho} \sim \frac{H}{\dot{\phi}_c} \delta\phi$$

Hence, $\delta\rho/\rho$ is also gaussian.
Power spectrum of scalar perturbations

$$\mathscr{P}_{\mathscr{R}}(k) = \left(\frac{H^2}{2\pi \, \dot{\phi}_c} \right)^2 \,,$$

everything is calculated at $t = t_k : H = k/a$



Analogously for the tensor perturbations: each of the two polarizations of the gravity waves solves the free scalar field equation!

$$\mathscr{P}_T(k) = \frac{16}{\pi} \frac{H_k^2}{M_{Pl}^2}$$

To the leading order no k-dependence: both spectra are “flat” (scale-invariant)!

Inflaton parameters and spectral parameters

- Observation of CMB anisotropy gives $\delta T/T$

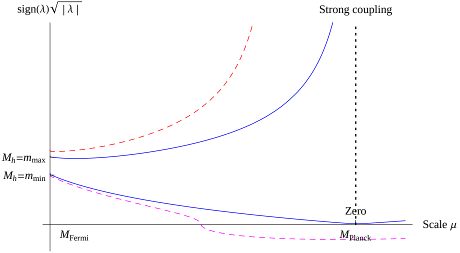
$$\frac{\delta T}{T} \sim \frac{\delta\rho}{\rho} \;\Rightarrow \Delta_{\mathscr{R}} \equiv \sqrt{\mathscr{P}_{\mathscr{R}}} = 5\times 10^{-5}$$

- These are so-called adiabatic perturbations!
  Other possibles (isocurvature) modes (e.g. $\delta T = 0$, but $\delta n_B/n_B \neq 0$) are not found.
- $\Delta_{\mathscr{R}} = 5\times 10^{-5} \Rightarrow$ fixes model paramaters, e.g.:

$$V(\phi) = \frac{\beta}{4}\phi^4 \to \lambda \sim 10^{-13}$$

  With such a tiny coupling perturbations are obviously gaussian
  So far confirmed by observations
- That's why Higgs boson in the SM does not help!
  However, it can be exploited as inflaton if non-minimally coupled to gravity $\xi RH^\dagger H$

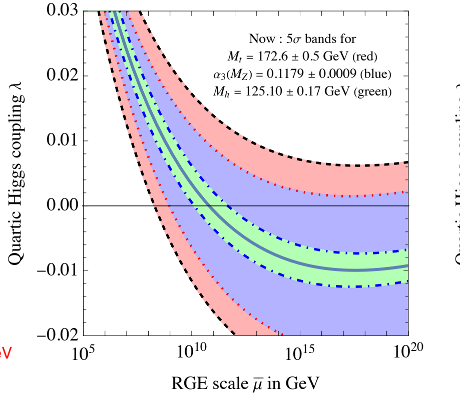
Critical point: where EW-vacuum becomes unstable



F.Bezrukov, M.Shaposhnikov (2009)
F.Bezrukov, D.G. (2011)
F.Bezrukov, M.Kalmykov, B.Kniehl, M.Shaposhnikov (2012)

G. Degrassi et al (2012)

$$m_h^{cr} > \left[129.0 + \frac{m_t - 172.9\,\text{GeV}}{1.1\,\text{GeV}} \times 2.2 - \frac{\alpha_s(M_Z) - 0.1181}{0.0007} \times 0.56\right]\,\text{GeV}$$



2203.17197

present theoretical uncertainties 0.5 GeV

Important for inflation, when usually $h \sim H$

Inflation & Reheating: simple realization with Higgs

$$\ddot{X} + 3H\dot{X} + V'(X) = 0$$

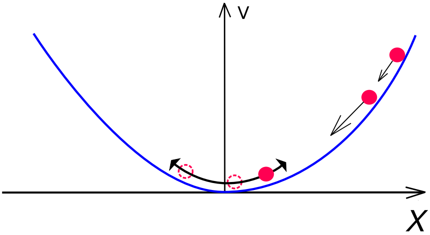
$X_e > M_{Pl}$

generation of scale-invariant scalar (and tensor) perturbations from exponentially stretched quantum fluctuations of X

$\delta\rho/\rho \sim 10^{-5}$ requires
$V = \beta X^4 : \beta \sim 10^{-13}$

reheating ? renormalizable?

the only choice: $\alpha H^\dagger H X^2$
"Higgs portal"



Chaotic inflation, A.Linde (1983)

larger α larger T_{reh}

quantum corrections $\propto \alpha^2 \lesssim \beta$

No scale, no problem

Inflaton parameters and spectral parameters

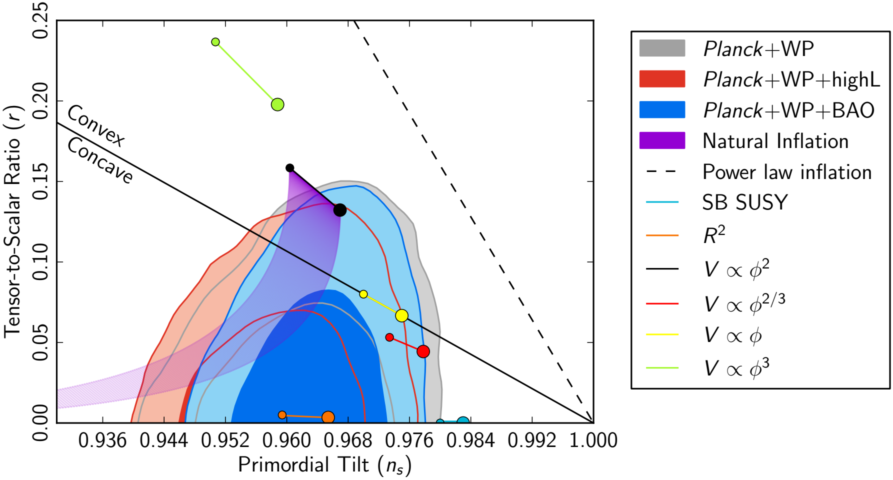
- In fact, spectra are a bit tilted, as H_{infl} slightly evolves

$$\mathscr{P}_{\mathscr{R}}(k) = A_{\mathscr{R}} \left(\frac{k}{k_*} \right)^{n_s - 1} , \qquad \mathscr{P}_T(k) = A_T \left(\frac{k}{k_*} \right)^{n_T} .$$

- Measure $\Delta_{\mathscr{R}}$ at present scales $q \simeq 0.002/\mathrm{Mpc}$, it fixes the number of e-foldings left N_e
- For tensor perturbations one introduces:

$$r \equiv \frac{\mathscr{P}_T}{\mathscr{P}_{\mathscr{R}}} = \frac{1}{\pi} \frac{M_{Pl}^2 V'^2}{V} = 16\,\varepsilon \;\; \rightarrow \;\; \frac{16}{N_e} \text{ for } \beta\phi^4$$

Recent analysis (Planck) of cosmlogical data



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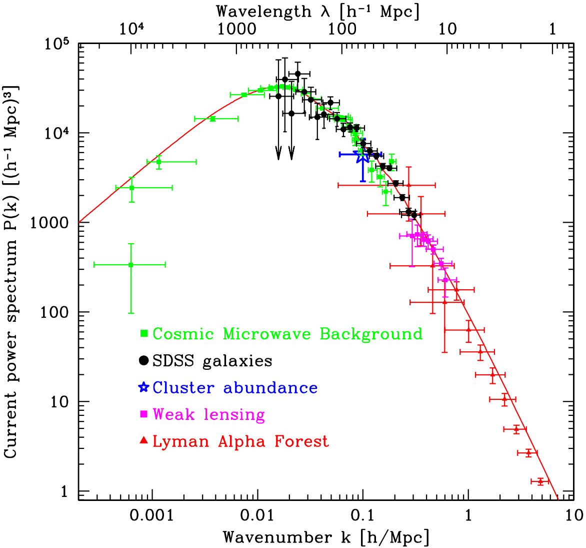
$N_e = 50 - 60$

Actually we observe rather narrow range



Observable range:

$$\frac{k_{max}}{k_{min}} \sim 10^5$$

$$\Delta N_e \simeq 10$$

Small scales cannot describe:
for a long time in nonlinear regime

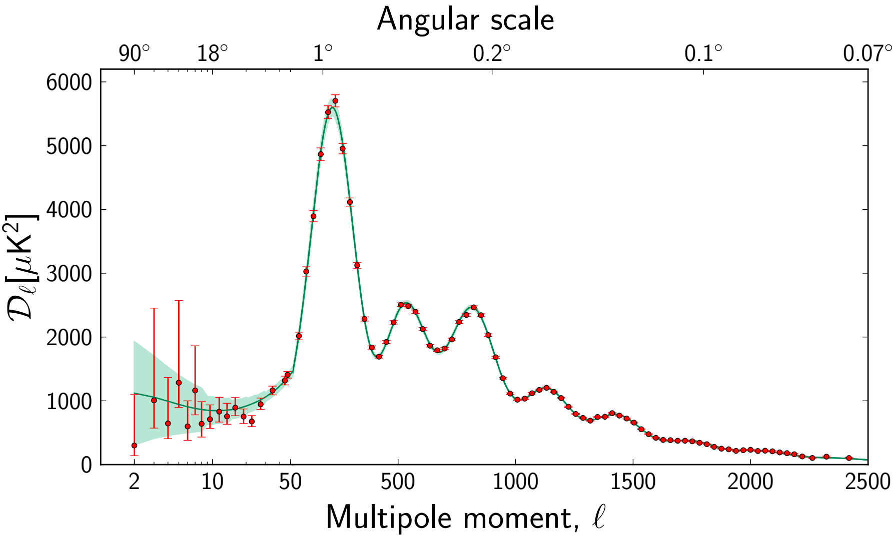
Mode evolution

- Amplitude remains constant, while superhorizon, e.g. $k/a < H$
- Subhorizon Inhomogeneities of DM start to grow at MD-stage, $\delta\rho_{CDM}/\rho_{CDM} \propto a$ from $T \approx 0.8\,\text{eV}$
  Smaller objects (first stars, dwarf galaxies) are first to form
- Subhorizon Inhomogeneities of baryons join those of DM only after recombination, $\delta\rho_{CDM}/\rho_{CDM} \propto a$ from $T_{rec} \approx 0.25\,\text{eV}$
- at recombination $\delta\rho_B/\rho_B \sim \delta T/T \sim 10^{-4}$ and would grow only by a factor $T_{rec}/T_0 \sim 10^3$ without DM
- Subhorizon Inhomogeneities of photons $\delta\rho_\gamma/\rho_\gamma$ oscillate with constant amplitude at RD and with decreasing amplitude at MD, thus we can measure $T_{RD/MD}/T_{rec}$
- Phase of oscillations decoupled after recombination depends on the wave-length, recombination time and sound speed

$$\delta\rho_\gamma/\rho_\gamma \propto \cos\left(k\int_0^{t_r} \frac{v_s\, dt}{a(t)}\right) = cos(kl_{sound})$$

- $$\delta T(\theta,\varphi) = \sum a_{lm} Y_{lm}(\theta,\varphi)\,, \qquad \langle a^*_{lm} a_{lm}\rangle = C_l \equiv 2\pi\mathscr{D}_l/(l(l+1))$$

CMB measurements (Planck) $H_0, \Omega_{DM}, \Omega_B, \Omega_\Lambda, \Delta_{\mathscr{R}}, n_s$



Other ways of testing inflation

- Curvature: the World is flat
  not convincing for many
- Relic tensor modes (gravitational waves)
  low-l *B*-mode: well below Galactic foreground
- preheating: $T_{reh} \to N_e$, GW ?
  tiny effects, $n_s, r = f(log(N_e))$, GW from clumps
- Direct tests: inflaton potential
  only in specific models with light inflaton
- Generic for many-field inflation are
  isocurvature modes, non-Gaussianity
- Exotic signatures
  primordial black holes, GW from oscillons, etc