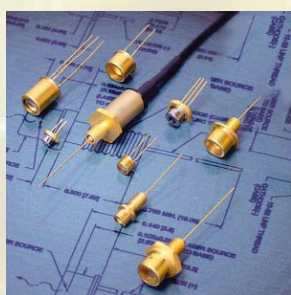


# Полупроводниковые лазеры и революция света

**Г.С. Соколовский**

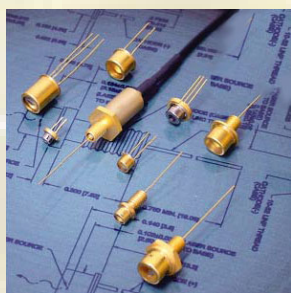


*ФТИ им. А.Ф. Иоффе*

[gs@mail.ioffe.ru](mailto:gs@mail.ioffe.ru)

## Laser diodes

**Grigori Sokolovskii<sup>1,2</sup>**



*<sup>1</sup>Ioffe Institute, St Petersburg, Russia*

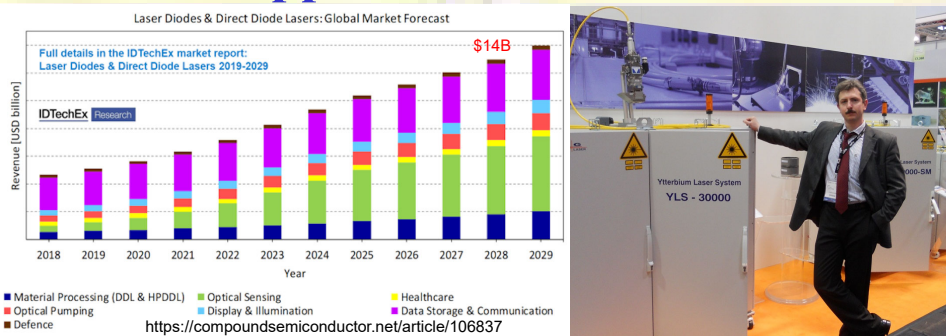
*<sup>2</sup>Aston University, Birmingham, UK*

[gs@mail.ioffe.ru](mailto:gs@mail.ioffe.ru)

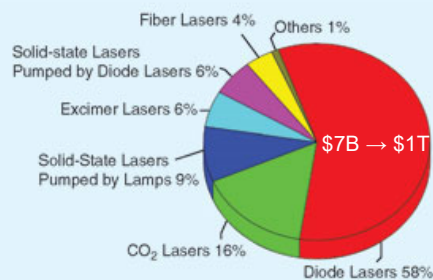
## Outline

- Applications of semiconductor lasers. ‘Revolution’ of light.
- Basic principles of laser operation (only to remind).
- Absorption and gain in semiconductors, inversion of population and conditions for it's achievement.
- Rate equations. Lasing threshold. Laser efficiency.
- Modulation of the laser signal. Gain clamping.
- Fiber-optical applications. DFB and DBR lasers. VCSELs and VECSELs
- Waveguide in LD structure. Modes of the waveguide.
- Beam-propagation (‘beam-quality’) parameter  $M^2$  and how to measure it. Achieving maximum power density with LDs.
- Interference focusing of LD beams.
- What’s next? New applications and problems to solve.

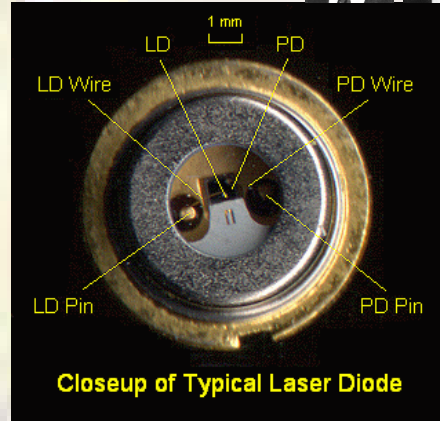
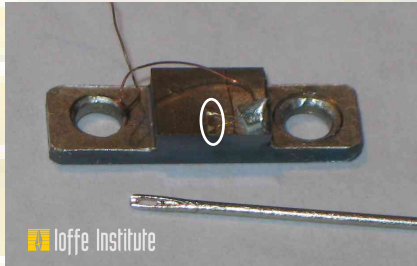
## Applications of LDs



- Telecoms
- Data reading/recording
- Laser printing
- Pumping of the solid-state
- ‘Direct’ applications: cutting
- Biology and medicine

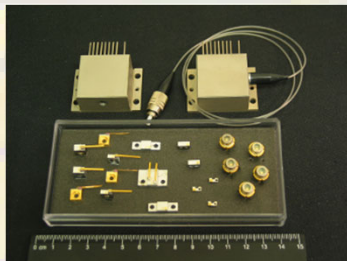


# Applications of LDs



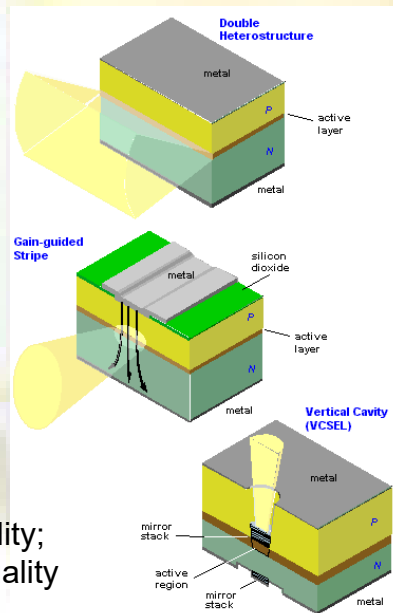
- Telecoms
- Data reading/recording
- Laser printing
- Pumping of the solid state and fiber lasers
- 'Direct' applications: drilling/cutting etc
- Biomedicine

# Types of LDs



1. Surface and edge-emitting (e.g. VCSELs, VECSELs, GCSELs, etc and 'striped' LDs)

2. Broad and narrow area  
 Broad: high power, poor beam quality;  
 Narrow: low power, better beam quality

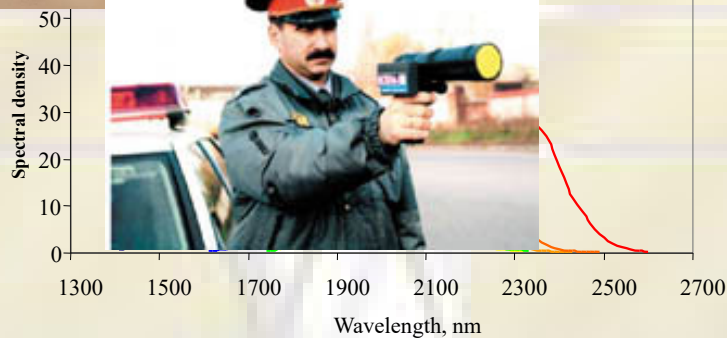


## LDs for Mid-IR (1600-5000 nm)

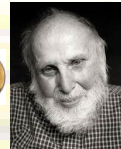


In Mid Infrared spectral range 1600-5000 nm lies strong absorption bands of such important gases and liquids as:

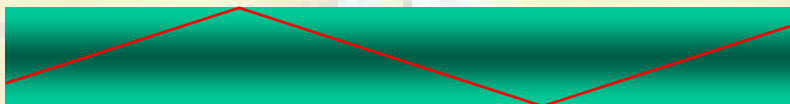
$\text{CH}_4$ ,  $\text{H}_2\text{O}$ ,  $\text{CO}_2$ ,  $\text{CO}$ ,  $\text{C}_2\text{H}_2$ ,  $\text{C}_2\text{H}_4$ ,  $\text{C}_2\text{H}_6$ ,  $\text{CH}_3\text{Cl}$ ,  $\text{OCS}$ ,  $\text{HCl}$ ,  $\text{HOCl}$ ,  $\text{HBr}$ ,  $\text{H}_2\text{S}$ ,  $\text{HCN}$ ,  $\text{NH}_3$ ,  $\text{NO}_2$ , glucose and many others.



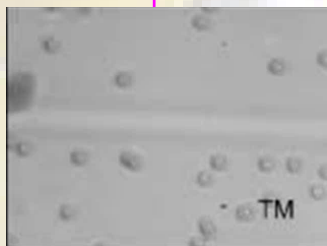
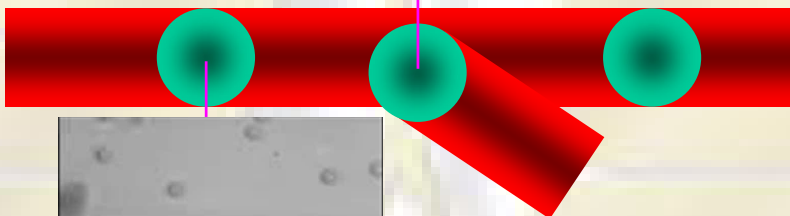
## Optical tweezers



Waveguide: light in matter



'Inverse' waveguide: matter in light – optical tweezers



## Laser projectors



Aiptek Pocket



no focus adjustment!



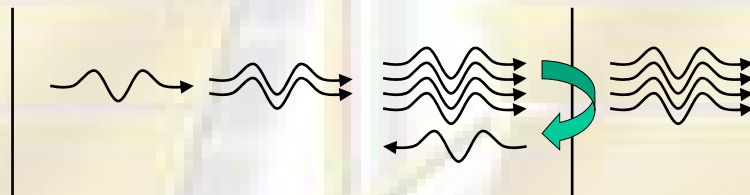
MacWorld 2010  
Best of Show award



Samsung H03  
Mini Projector

## Basic principles of laser operation

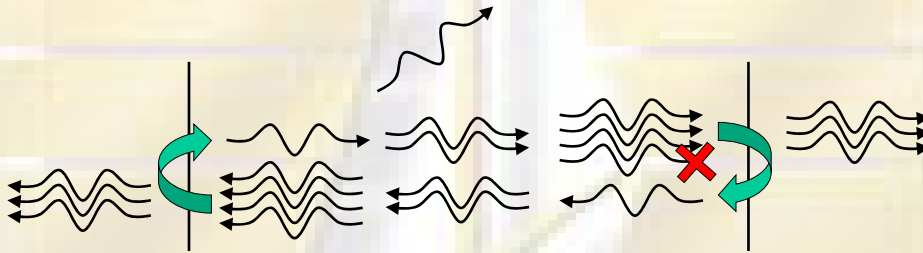
Gain + Feedback



$$S(x) = S_0 e^{\alpha x}$$

## Basic principles of laser operation

### Lasing threshold



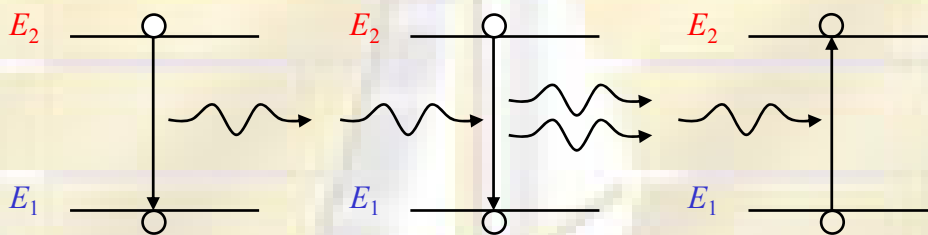
$$S(x) = S_0 e^{\alpha x} \quad S(2L) = R^2 S_0 e^{2\alpha L} = S_0$$

$$\alpha_{out} = \frac{1}{L} \ln R \quad \alpha_{threshold} = \alpha_{out} + \alpha_{in}$$

## Basic principles of laser operation



### Spontaneous and stimulated emission



Spontaneous

Stimulated

Absorption

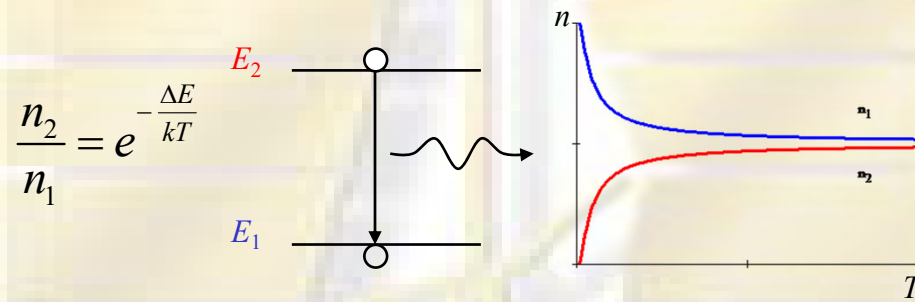
$$\frac{dS}{dt} = An$$

$$\frac{dS}{dt} = BSn$$

$$\frac{dS}{dt} = -BSn$$

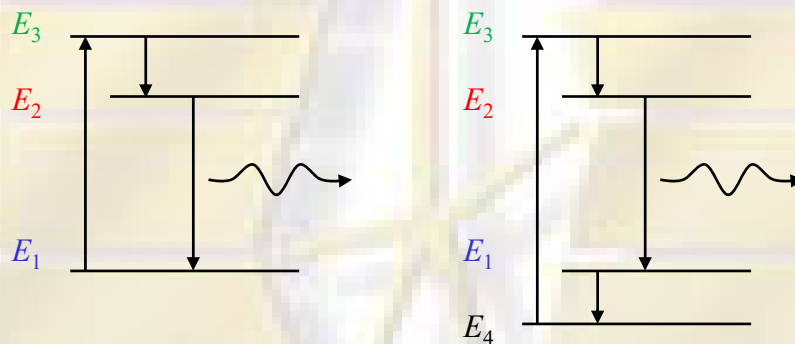
## Basic principles of laser operation

Gain: inversion of population

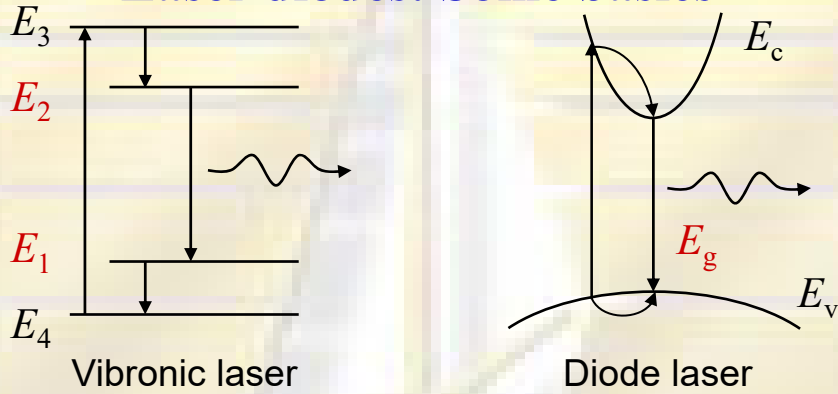


‘Negative’ temperature

## 3- & 4-level systems

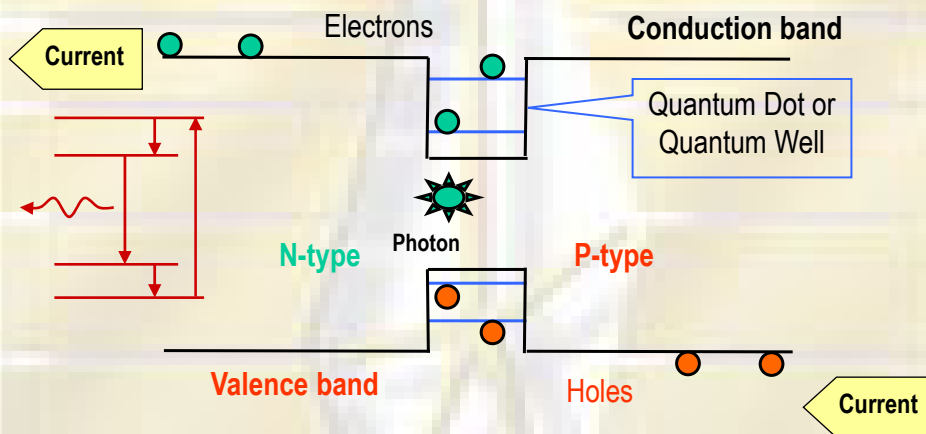


## Laser diodes: Some basics



$E_c$  – conduction band  
 $E_v$  – valence band  
 $E_g$  – energy gap

## Laser diodes: Some basics



QWs and QDs are the 'artificial atoms' :)



## Light generation and absorption in semiconductors

‘Golden’ rule of Quantum mechanics:  $W = \frac{2\pi}{\hbar} |\langle f|H|i\rangle|^2 \rho_f$

Absorption probability:  $W_{\text{absorption}} = B|M|^2 S\rho(E)f_v(1-f_c)$

Radiation probability:  $W_{\text{radiation}} = B|M|^2 (S+1)\rho(E)(1-f_v)f_c$

The total radiation probability:

$$W = W_{\text{radiation}} - W_{\text{absorption}} = B|M|^2 \rho(E)f_c(1-f_v) + SB|M|^2 \rho(E)(f_c - f_v)$$

$$W(E) = r_{sp}(E) + Sr_{st}(E) \quad \begin{aligned} r_{sp}(E) &= B|M|^2 \rho(E)f_c(1-f_v) \\ r_{st}(E) &= B|M|^2 \rho(E)(f_c - f_v) \end{aligned}$$

## Light generation and absorption in semiconductors

What is the condition for stimulated emission?  $r_{st}(E) > 0$

$$r_{st}(E) = B|M|^2 \rho(E)(f_c - f_v) > 0$$

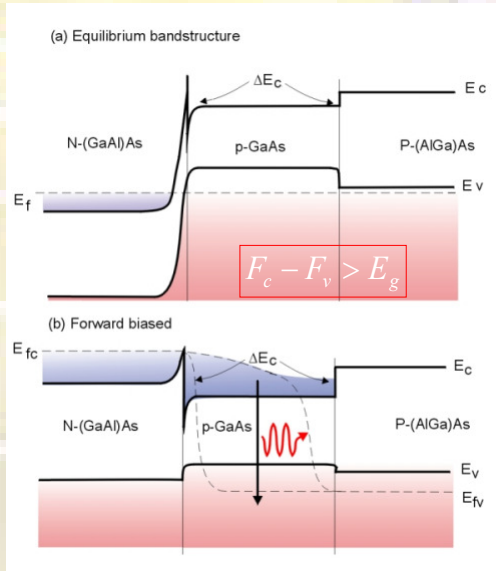
$$f_c - f_v > 0 \Leftrightarrow \frac{1}{1 + \exp\left(\frac{E_c - F_c}{kT}\right)} - \frac{1}{1 + \exp\left(\frac{E_v - F_v}{kT}\right)} > 0$$

$$F_c - F_v > E_c - E_v \geq E_g$$

Inversion of population:

$$\boxed{F_c - F_v > E_g}$$

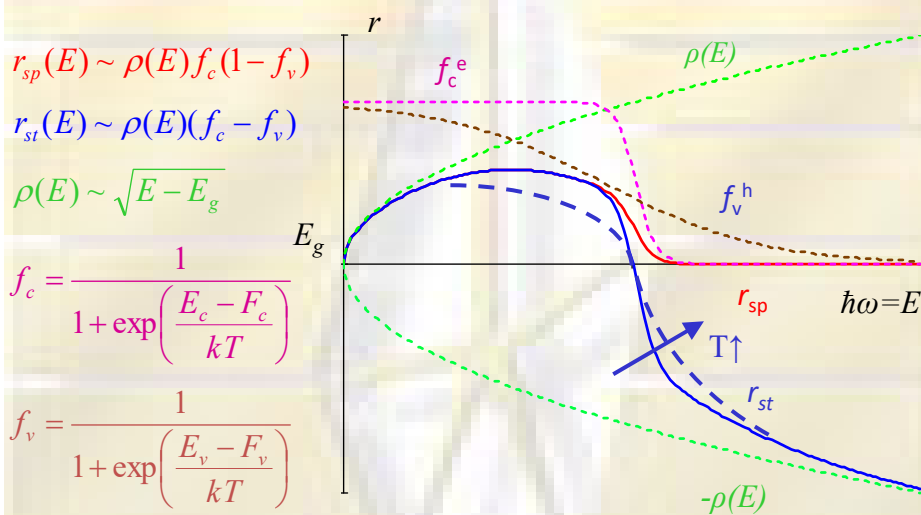
## Inversion of population



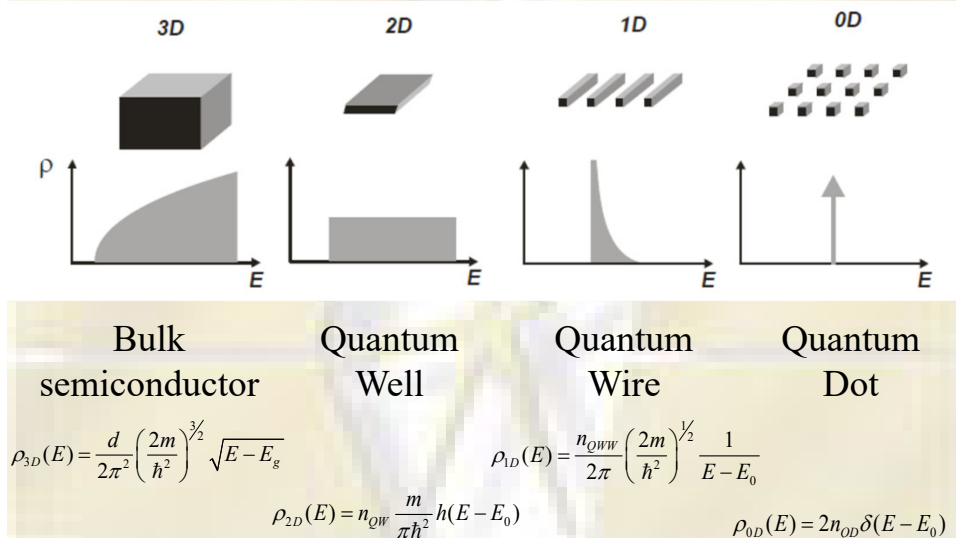
Forward-biased p-n-junction is both the source for the inversion of population and for the name of the “laser diodes”

<http://britneyspears.ac/physics/fplasers/fplasers.htm>

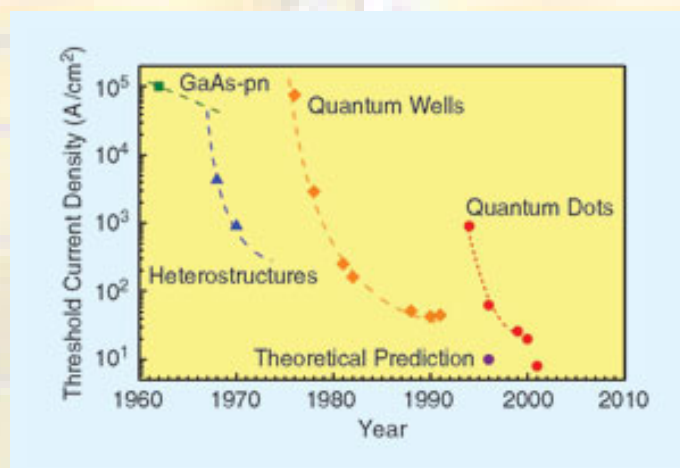
## Stimulated and spontaneous emission rate: 3D



## Density of states: Low-Dimensional vs 3D

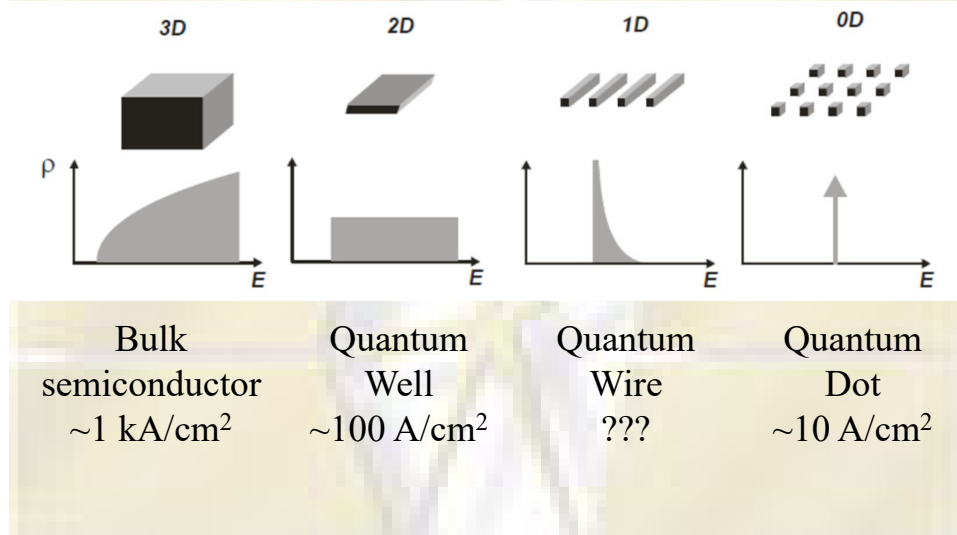


## Evolution of the threshold current density: from 3D to Low-Dimensional



[D.Bimberg, IEEE Phot. Soc. Newsletter 6, 3 (2013)]

## Density of states: Low-Dimensional vs 3D



## QDs vs 3D

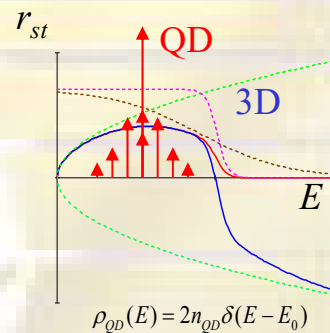
Lower threshold

But: at low QD density threshold may NOT be reached

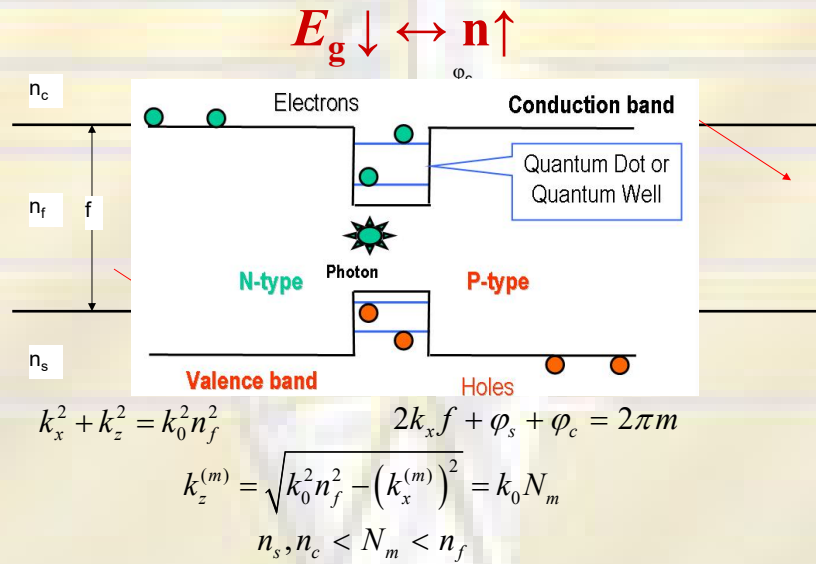
Lower temperature dependence of the laser parameters

Narrow spectrum for IDENTICAL QDs

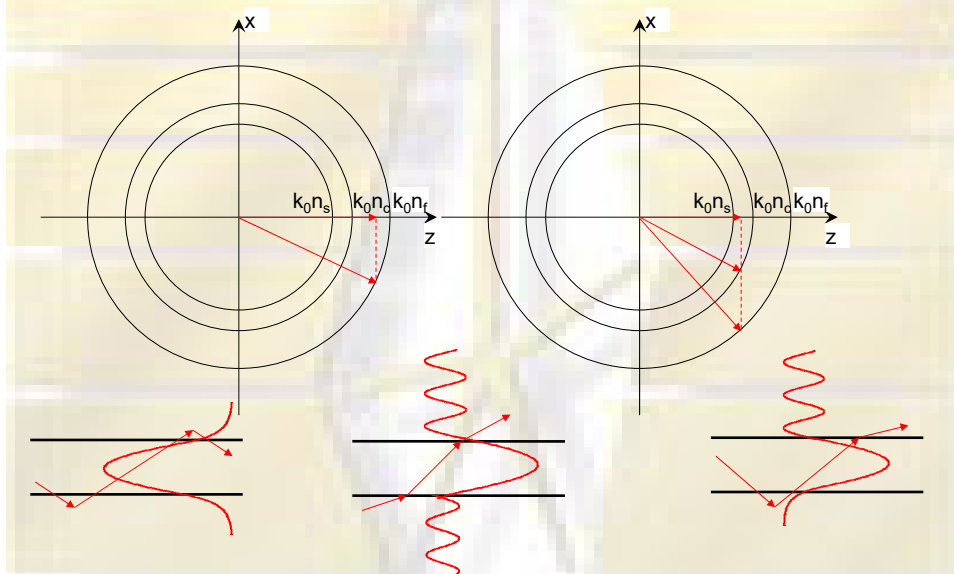
But: broad spectrum for inhomogeneously broadened QDs



## Laser diodes: The waveguide



## Modes of the waveguide



## Composing Rate Equations

### The simplest model

$J$  is the average pump current density

$n$  is the average carrier concentration in the active region

$S$  is the average photon concentration (average intensity)

Carriers, steady-state:

Pump rate = Recombination rate

$$\frac{J}{ed} = r$$

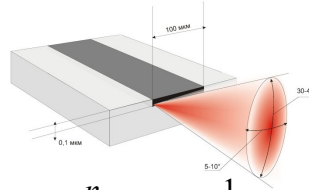
Carriers, time-dependent:

$$\frac{dn}{dt} = \frac{J}{ed} - r \quad \frac{dn}{dt} = \frac{J}{ed} - r_{sp} - Sr_{st}$$

$$\frac{dn}{dt} = \frac{J}{ed} - \frac{n}{\tau_s} - \frac{c}{N_{eff}} g(n) S$$

$$r_{sp} = Bn^2 = \frac{n}{\tau_s}, \quad \tau_s = \frac{1}{Bn}$$

$$r_{st} \sim \text{gain} \quad r_{st} = \frac{c}{N_{eff}} g(n)$$



## Composing Rate Equations

Photons, steady-state:

Total radiation probability = Recombination rate

$$\frac{S}{\tau_p} = r$$

Photons, time-dependent:

$$\frac{dS}{dt} = r - \frac{S}{\tau_p} \quad \frac{dS}{dt} = \Gamma(r_{sp} + Sr_{st}) - \frac{S}{\tau_p}$$

$$\frac{dS}{dt} = \Gamma \frac{c}{N_{eff}} g(n) S + \Gamma \beta \frac{n}{\tau_s} - \frac{S}{\tau_p}$$

$$\left\{ \begin{array}{l} \frac{dS}{dt} = \Gamma A(n - n_t) S + \Gamma \beta \frac{n}{\tau_s} - \frac{S}{\tau_p} \\ \frac{dn}{dt} = \frac{J}{ed} - \frac{n}{\tau_s} - A(n - n_t) S \end{array} \right.$$

$\Gamma$  - confinement factor

$\beta$  - spontaneous emission factor

$n_t$  - transparency concentration

$A$  - linear gain coefficient

$\tau_s$  - spontaneous recomb. time

$\tau_p$  - photon lifetime

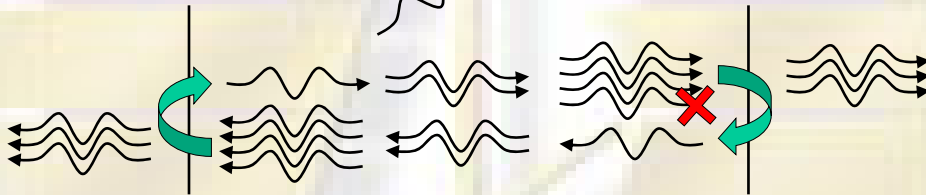
## Lifetimes

**Spontaneous:**

$$\tau_s = \frac{1}{Bn} \quad \tau_s \sim 1 \text{ ns}$$

**Photon lifetime:**

if  $n \sim n_p$ , then:  $\frac{dS}{dt} \approx -\frac{S}{\tau_p} \Rightarrow S = S_0 e^{-t/\tau_p}$



$$t_r = \frac{2LN_{eff}}{c} \quad S(t_r) = S_0 e^{-\frac{2LN_{eff}}{c\tau_p}} = S_0 e^{-2L\alpha_{in}R^2}$$

$$\frac{1}{\tau_p} = \frac{c}{N_{eff}} \left( \alpha_{in} + \frac{1}{L} \ln \frac{1}{R} \right) \quad \tau_p \sim 1 \text{ ps}$$

$L \uparrow \rightarrow \tau_p \uparrow$   
 $\alpha_{in} \uparrow \rightarrow \tau_p \downarrow$   
 $R \uparrow \rightarrow \tau_p \uparrow$

## Rate Equations for QD LD

$$\frac{\partial n_W}{\partial t} = \frac{J}{ed} + \gamma \frac{n_E}{\tau_E} f_W' - \frac{n_W}{\tau_0} f_E' - \frac{n_W}{\tau_s}$$

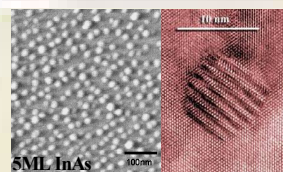
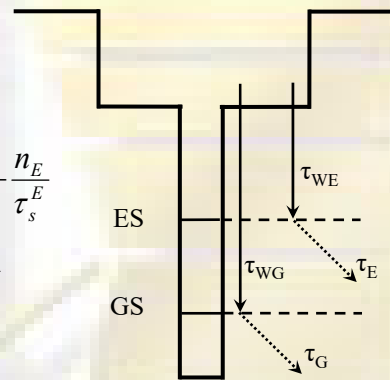
$$\frac{\partial n_E}{\partial t} = \gamma \frac{n_W}{\tau_0} f_E' - \frac{n_E}{\tau_E} f_W' - \frac{n_E}{\tau_0} f_G' + \frac{n_G}{\tau_G} f_E' - \frac{n_E}{\tau_s}$$

$$\frac{\partial n_G}{\partial t} = \frac{n_E}{\tau_0} f_G' - A(n_G - n_r)S - \frac{n_G}{\tau_G} f_E' - \frac{n_G}{\tau_s}$$

$$\frac{\partial S}{\partial t} = \Gamma A(n_G - n_r)S + \Gamma \beta \frac{n_G}{\tau_s} - \frac{S_p}{\tau_p}$$

...where  $f$  is the filling factor

Y. Wu, L.V. Asryan, JAP 115, 103105 (2014)  
<https://doi.org/10.1063/1.4868472>



## Steady-state solution of the Rate Eqs

$$\begin{cases} \frac{dS}{dt} = \Gamma A(n - n_t)S + \Gamma\beta \frac{n}{\tau_s} - \frac{S}{\tau_p} = 0 \\ \frac{dn}{dt} = \frac{J}{ed} - \frac{n}{\tau_s} - A(n - n_t)S = 0 \end{cases}$$

$$\begin{cases} \Gamma A(n - n_t)S = \frac{S}{\tau_p} - \Gamma\beta \frac{n}{\tau_s} \\ A(n - n_t)S = \frac{J}{ed} - \frac{n}{\tau_s} \end{cases}$$

$$\begin{cases} S = \Gamma\tau_p \left( \frac{J}{ed} - \frac{n}{\tau_s} \right) \\ \Gamma A(n - n_t)\tau_p \left( \frac{J}{ed} - \frac{n}{\tau_s} \right) = \frac{J}{ed} - \frac{n}{\tau_s} \end{cases}$$

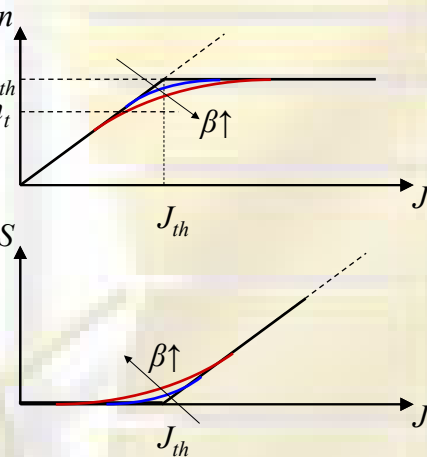
$$\begin{cases} n = \frac{J\tau_s}{ed} \\ n = n_t + \frac{1}{\Gamma A\tau_p} \\ S = 0 \\ S = \frac{\Gamma\tau_p}{ed} \left( J - \frac{ed}{\tau_s} \left( n_t + \frac{1}{\Gamma A\tau_p} \right) \right) \end{cases}$$

## Steady-state solution of the Rate Eqs

$$\begin{cases} n = \frac{J\tau_s}{ed} \\ n = n_t + \frac{1}{\Gamma A\tau_p} \end{cases}$$

$$\begin{cases} S = 0 \\ S = \frac{\Gamma\tau_p}{ed} \left( J - \frac{ed}{\tau_s} \left( n_t + \frac{1}{\Gamma A\tau_p} \right) \right) \end{cases}$$

$$J_{th} = \frac{ed}{\tau_s} \left( n_t + \frac{1}{\Gamma A\tau_p} \right) = \frac{ed}{\tau_s} n_{th}$$



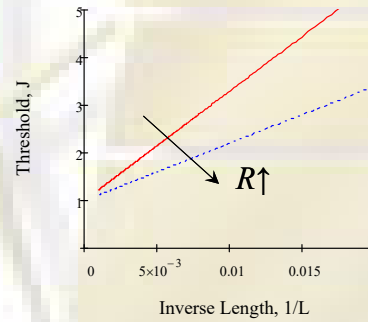
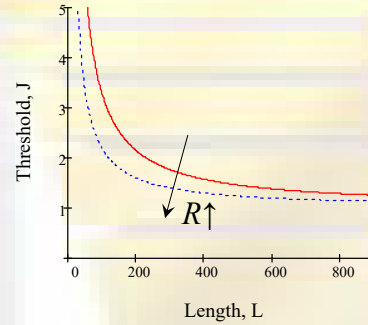
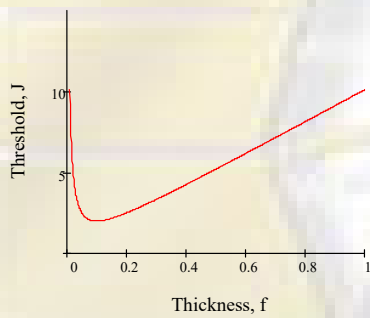
$$n_{th} = n_t + \frac{1}{\Gamma A\tau_p}$$



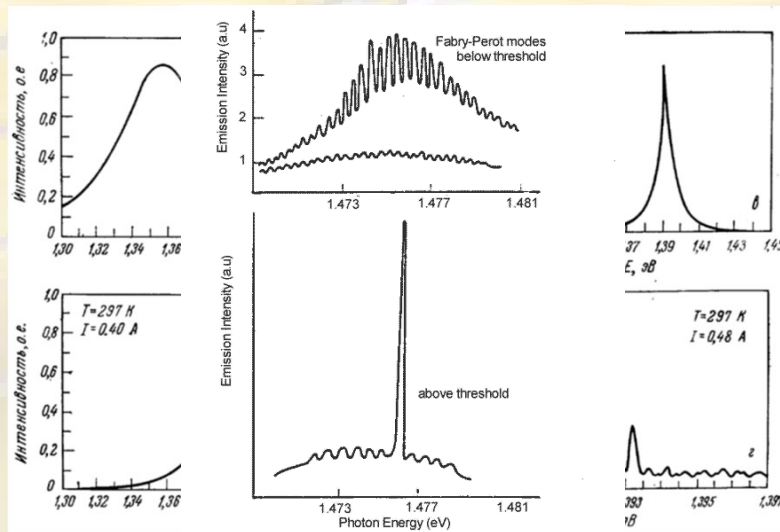
# Lasing Threshold

$$J_{th} = \frac{ed}{\tau_s} \left( n_i + \frac{1}{\Gamma A \tau_p} \right) = \frac{ed}{\tau_s} n_{th}$$

$$\frac{1}{\tau_p} = \frac{c}{N_{eff}} \left( \alpha_{in} + \frac{1}{L} \ln \frac{1}{R} \right)$$



# Lasing Threshold

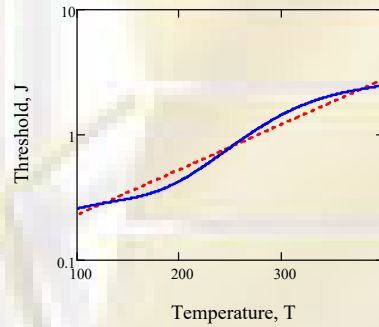
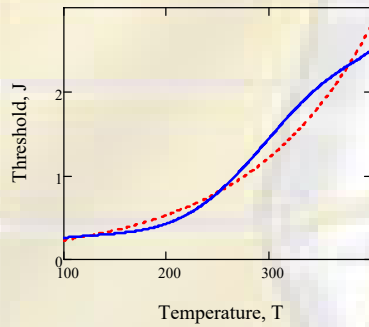


<http://britneyspears.ac/physics/fplasers/fplasers.htm>

## Threshold vs Temperature

$$J \sim \exp \frac{T}{T_0}$$

$$T_0 = f(T)$$



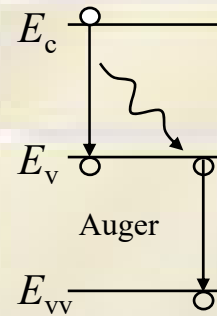
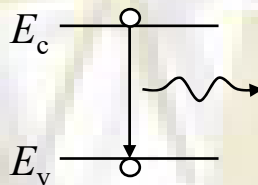
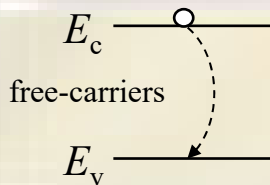
## Laser efficiency

$$J \rightarrow J\tilde{\eta}\eta_i \Rightarrow S = \tilde{\eta}\eta_i\Gamma \frac{\tau_p}{ed}(J - J_{th})$$

$$\tilde{\eta} = \frac{J_{active}}{J_{total}} \quad \text{Pumping efficiency}$$

$$\eta_i = \frac{\sum \text{photons}}{\sum \text{electrons}} \quad \text{Internal quantum efficiency}$$

$$\eta_i = \frac{Bn^2}{An + Bn^2 + Cn^3}$$



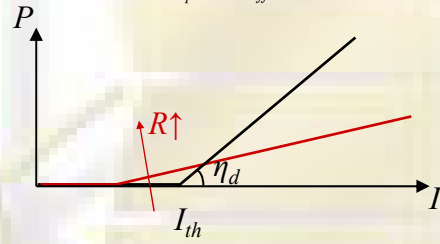
## Laser efficiency

$$S = \tilde{\eta} \eta_i \Gamma \frac{\tau_p}{ed} (J - J_{th}) \quad P^{(1/2)} = \frac{1}{2} S \hbar \omega \frac{c}{N_{eff}} \frac{V}{\Gamma} \frac{1}{L} \ln \frac{1}{R}$$

$$P^{(1/2)} = \frac{1}{2} \hbar \omega \frac{c}{N_{eff}} \frac{V}{\Gamma} \frac{1}{L} \ln \frac{1}{R} \tilde{\eta} \eta_i \Gamma \frac{\tau_p}{ed} (J - J_{th}) \quad \frac{1}{\tau_p} = \frac{c}{N_{eff}} \left( \alpha_{in} + \frac{1}{L} \ln \frac{1}{R} \right)$$

$$I = JWL = J \frac{V}{d}$$

$$P^{(1/2)} = \frac{1}{2} \tilde{\eta} \eta_i \frac{\hbar \omega}{e} \frac{\frac{1}{L} \ln \frac{1}{R}}{\alpha_{in} + \frac{1}{L} \ln \frac{1}{R}} (I - I_{th})$$

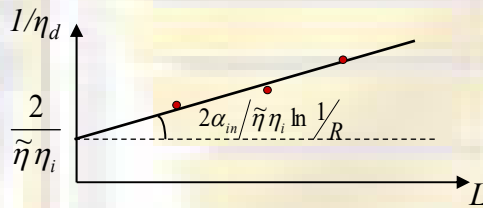


differential efficiency: 
$$\eta_d = \frac{P^{(1/2)}}{\frac{\hbar \omega}{e} (I - I_{th})} = \frac{1}{2} \tilde{\eta} \eta_i \frac{\frac{1}{L} \ln \frac{1}{R}}{\alpha_{in} + \frac{1}{L} \ln \frac{1}{R}} = \frac{1}{2} \frac{\tilde{\eta} \eta_i \alpha_{out}}{\alpha_{in} + \alpha_{out}}$$

## Laser efficiency

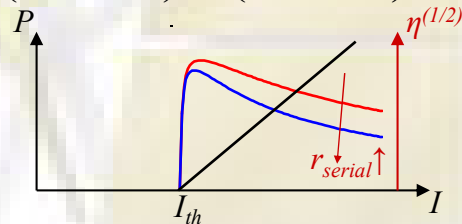
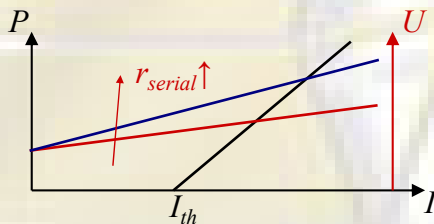
Measuring IQE:

$$\frac{1}{\eta_d} = \frac{2}{\tilde{\eta} \eta_i} \left( 1 + \frac{\alpha_{in} L}{\ln \frac{1}{R}} \right)$$



Lasing efficiency:

$$\eta^{(1/2)} = \frac{P^{(1/2)}}{I \left( \frac{\hbar \omega}{e} + I r_{serial} \right)} = \frac{\eta_d \frac{\hbar \omega}{e} (I - I_{th})}{I \left( \frac{\hbar \omega}{e} + I r_{serial} \right)}$$



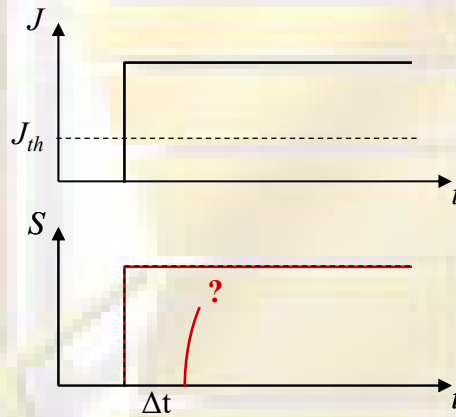
## Turn-on delay

$$\begin{cases} \frac{dS}{dt} = \Gamma A(n - n_i) S + \Gamma \beta \frac{n}{\tau_s} \frac{S}{\tau_p} \\ \frac{dn}{dt} = \frac{J}{ed} - \frac{n}{\tau_s} - A(n - n_i) S \end{cases}$$

$$n(t) = \frac{J\tau_s}{ed} \left( 1 - e^{-t/\tau_s} \right)$$

$$n(\Delta t) = n_{th}$$

$$\Delta t = \tau_s \ln \left( \frac{J}{J - J_{th}} \right) \approx \tau_s \frac{J_{th}}{J}$$

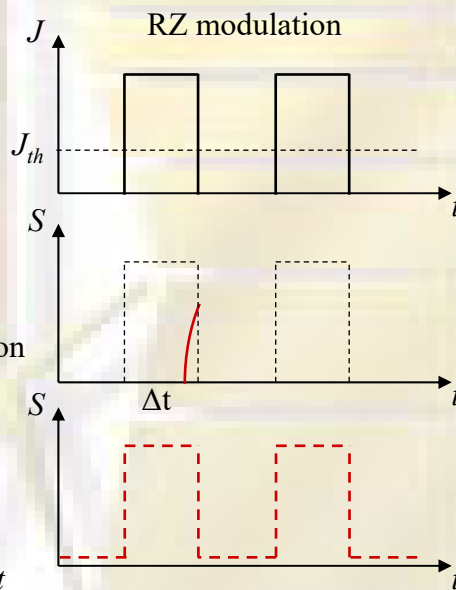
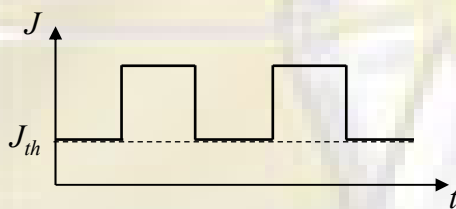


## Turn-on delay

$$\Delta t = \tau_s \ln \left( \frac{J}{J - J_{th}} \right) \approx \tau_s \frac{J_{th}}{J}$$

If  $J = 2J_{th}$ ,  $\tau_s = 1$  ns:  $\Delta t = 0.7$  ns

Non-return-to-zero (NRZ) modulation



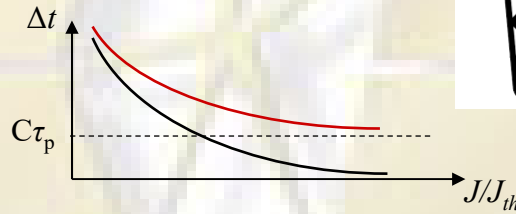
## Turn-on delay of QD LDs

Non-QD laser diode:

$$\Delta t = \tau_s \ln\left(\frac{J}{J - J_{th}}\right) \approx \tau_s \frac{J_{th}}{J}$$

QD LD:

$$\Delta t \approx \tau_s \frac{J_{th}}{J} + C\tau_p$$



## Small-signal modulation

$$\frac{dn}{dt} = \frac{J}{ed} - \frac{n}{\tau_s} - \frac{c}{N_{eff}} g(n) S$$

$$\frac{dS}{dt} = \Gamma \frac{c}{N_{eff}} g(n) S - \frac{S}{\tau_p} + \Gamma\beta \frac{n}{\tau_s}$$

$$J(t) = J_0 + \delta J(t), \quad J_0 \gg \delta J(t)$$

$$S(t) = S_0 + \delta S(t), \quad S_0 \gg \delta S(t)$$

$$n(t) = n_0 + \delta n(t), \quad n_0 \gg nJ(t)$$

$$\begin{cases} \Gamma \frac{c}{N_{eff}} g(n) S + \Gamma\beta \frac{n}{\tau_s} - \frac{S}{\tau_p} = 0 \\ \frac{J}{ed} - \frac{n}{\tau_s} - \frac{c}{N_{eff}} g(n) S = 0 \end{cases}$$

$$\delta \dot{n} = \frac{\delta J}{ed} - \frac{\delta n}{\tau_s} - \frac{c}{N_{eff}} (g_0 \delta S + g' S_0 \delta n)$$

$$\delta \dot{S} = \Gamma \frac{c}{N_{eff}} (g_0 \delta S + g' S_0 \delta n) - \frac{\delta S}{\tau_p} + \Gamma\beta \frac{\delta n}{\tau_s}$$

## Small-signal modulation

$$\begin{aligned} \delta \dot{n} &= \frac{\delta J}{ed} - \frac{\delta n}{\tau_s} - \frac{c}{N_{eff}} (g_0 \delta S + g' S_0 \delta n) & \delta J(t) &= j \exp(i\omega t) \\ \delta \dot{S} &= \Gamma \frac{c}{N_{eff}} (g_0 \delta S + g' S_0 \delta n) - \frac{\delta S}{\tau_p} + \Gamma \beta \frac{\delta n}{\tau_s} & \delta n(t) &= a(\omega) \exp(i\omega t) \\ & & \delta S(t) &= b(\omega) \exp(i\omega t) \end{aligned}$$

$$\begin{aligned} \left( i\omega + \frac{1}{\tau_s} + \frac{c}{N_{eff}} g' S_0 \right) a + \frac{c}{N_{eff}} g_0 b &= \frac{j}{ed} \\ - \left( \Gamma \frac{c}{N_{eff}} g' S_0 + \Gamma \frac{\beta}{\tau_s} \right) a + \left( i\omega + \frac{1}{\tau_p} - \Gamma \frac{c}{N_{eff}} g_0 \right) b &= 0 \end{aligned}$$

$$\gamma = \frac{1}{\tau_s} + AS_0, \quad A \equiv \frac{c}{N_{eff}} g', \quad \Delta = \frac{1}{\tau_p} - \Gamma \frac{c}{N_{eff}} g_0, \quad \omega_0^2 = \frac{AS_0}{\tau_p}$$

## Amplitude-freq. response of photons

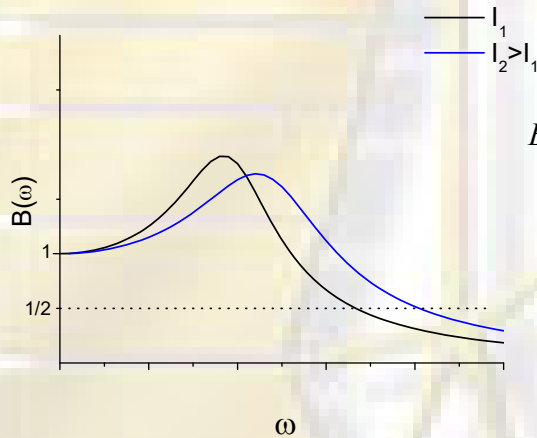
$$\begin{aligned} \left( i\omega + \frac{1}{\tau_s} + \frac{c}{N_{eff}} g' S_0 \right) a + \frac{c}{N_{eff}} g_0 b &= \frac{j}{ed} \\ - \left( \Gamma \frac{c}{N_{eff}} g' S_0 + \Gamma \frac{\beta}{\tau_s} \right) a + \left( i\omega + \frac{1}{\tau_p} - \Gamma \frac{c}{N_{eff}} g_0 \right) b &= 0 \end{aligned}$$

$$b(\omega) \approx \frac{\Gamma j \tau_p}{ed} \frac{\omega_0^2}{\omega_0^2 - \omega^2 + \gamma i \omega} = \frac{j}{ed} B(\omega) \exp(i\phi_B(\omega))$$

$$B(\omega) = \frac{\omega_0^2}{\sqrt{(\omega_0^2 - \omega^2)^2 + \gamma^2 \omega^2}}, \quad \phi_B(\omega) = \arctg\left(-\frac{\gamma \omega}{\omega_0^2 - \omega^2}\right)$$

$$\gamma = \frac{1}{\tau_s} + AS_0, \quad A \equiv \frac{c}{N_{eff}} g', \quad \Delta = \frac{1}{\tau_p} - \Gamma \frac{c}{N_{eff}} g_0, \quad \omega_0^2 = \frac{AS_0}{\tau_p}$$

## Amplitude-freq. response of photons

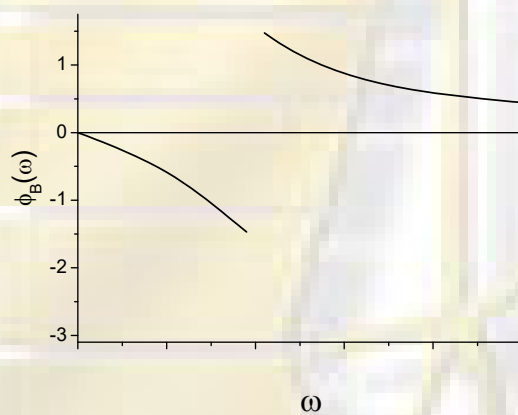


$$B(\omega) = \frac{\omega_0^2}{\sqrt{(\omega_0^2 - \omega^2)^2 + \gamma^2 \omega^2}}$$

$$\gamma = \frac{1}{\tau_s} + AS_0$$

$$\omega_0^2 = \frac{AS_0}{\tau_p}$$

## Phase-frequency response of photons



$$\phi_B(\omega) = \text{arctg}\left(-\frac{\gamma\omega}{\omega_0^2 - \omega^2}\right)$$

$$\gamma = \frac{1}{\tau_s} + AS_0$$

$$\omega_0^2 = \frac{AS_0}{\tau_p}$$

## Amplitude-freq. response of carriers

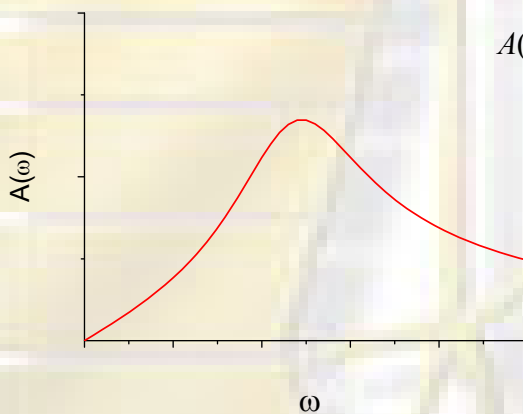
$$\left( i\omega + \frac{1}{\tau_s} + \frac{c}{N_{eff}} g' S_0 \right) a + \frac{c}{N_{eff}} g_0 b = \frac{j}{ed}$$

$$-\left( \Gamma \frac{c}{N_{eff}} g' S_0 + \Gamma \frac{\beta}{\tau_s} \right) a + \left( i\omega + \frac{1}{\tau_p} - \Gamma \frac{c}{N_{eff}} g_0 \right) b = 0$$

$$a(\omega) \approx \frac{j}{ed} \frac{-i\omega}{\omega_0^2 - \omega^2 + \gamma i\omega} = \frac{j}{ed} A(\omega) \exp(i\phi_A(\omega))$$

$$A(\omega) = \frac{\omega}{\sqrt{(\omega_0^2 - \omega^2)^2 + \gamma^2 \omega^2}} \quad \phi_A(\omega) = \text{arctg}\left(-\frac{\gamma\omega}{\omega_0^2 - \omega^2}\right) - \frac{\pi}{2}$$

## Amplitude-freq. response of carriers



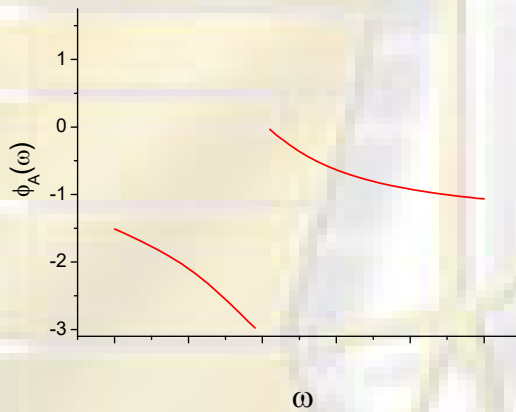
$$A(\omega) = \frac{\omega}{\sqrt{(\omega_0^2 - \omega^2)^2 + \gamma^2 \omega^2}}$$

$$\gamma = \frac{1}{\tau_s} + AS_0$$

$$\omega_0^2 = \frac{AS_0}{\tau_p}$$



## Phase-frequency response of carriers

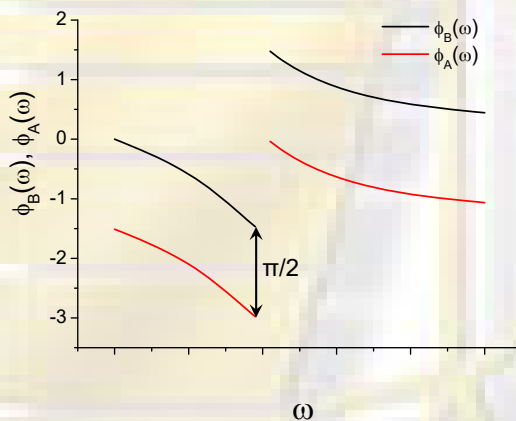


$$\phi_A(\omega) = \arctg\left(-\frac{\gamma\omega}{\omega_0^2 - \omega^2}\right) - \frac{\pi}{2}$$

$$\gamma = \frac{1}{\tau_s} + AS_0$$

$$\omega_0^2 = \frac{AS_0}{\tau_p}$$

## Phase-frequency response

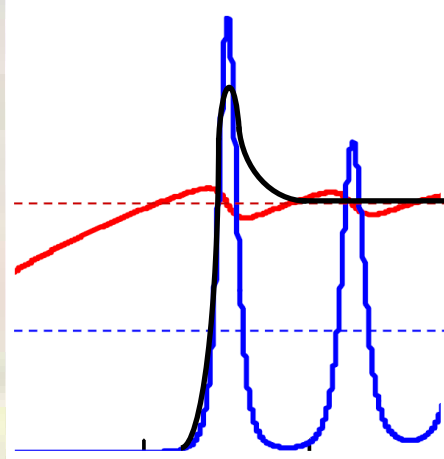
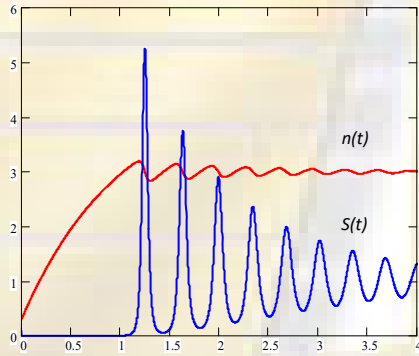


$$\phi_A(\omega) = \arctg\left(-\frac{\gamma\omega}{\omega_0^2 - \omega^2}\right) - \frac{\pi}{2}$$

$$\phi_B(\omega) = \arctg\left(-\frac{\gamma\omega}{\omega_0^2 - \omega^2}\right)$$

$\pi/2$  shift in the phase-frequency responses means the energy flow between the photons and carriers (similar to the kinetic and potential energy in pendulum) which is called 'relaxation oscillations'.

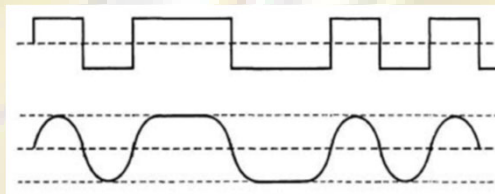
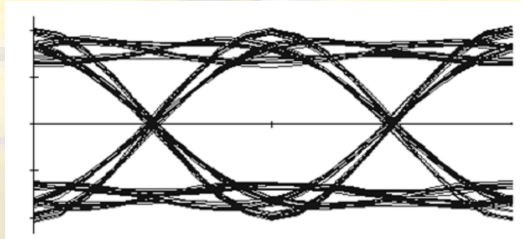
## Relaxation oscillations



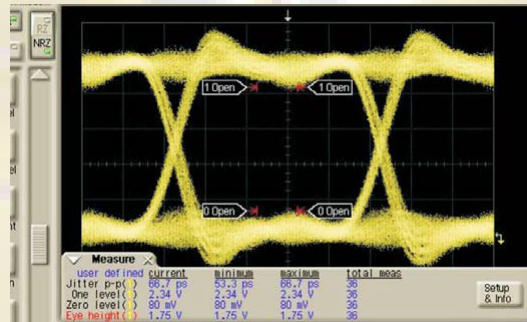
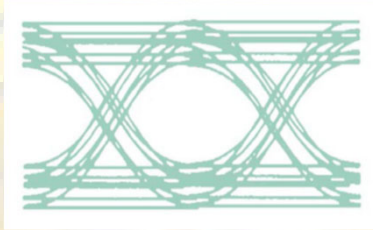
NB: no relaxation oscillations in QD lasers!

Typical explanations: 1) QD LDs are too fast; 2) QD LDs are too slow

## Eye-diagram

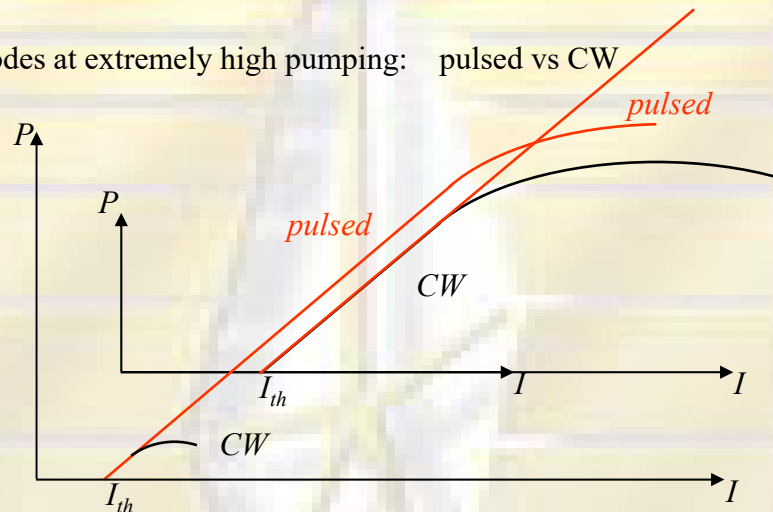


## Eye-diagram



## Gain clamping (gain saturation)

Laser diodes at extremely high pumping: pulsed vs CW

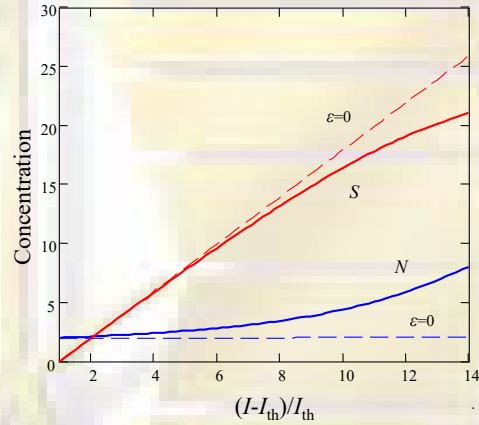


## Gain clamping

$$\begin{cases} \frac{dN}{dt} = \frac{J}{ed} - A(N - N_r)S(1 - \varepsilon S) - \frac{N}{\tau} \\ \frac{dS}{dt} = \Gamma A(N - N_r)S(1 - \varepsilon S) + \frac{\Gamma \beta N}{\tau} - \frac{S}{\tau_p} \end{cases}$$

Quasi-analytical:

$$\begin{cases} N = N_{th} + \frac{1}{\Gamma A \tau_p} \frac{\varepsilon S}{1 - \varepsilon S}, & N_{th} = N_r + \frac{1}{\Gamma A \tau_p} \\ S = \frac{\Gamma \tau_p}{ed} \left( J - J_{th} - \frac{ed}{\Gamma A \tau_p} \frac{\varepsilon S}{1 - \varepsilon S} \right), & J_{th} = \frac{N_{th} ed}{\tau} \end{cases}$$



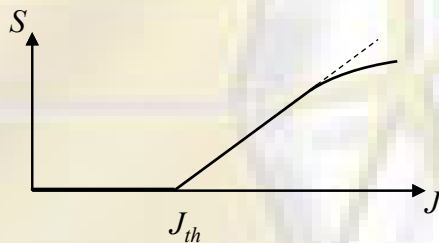
Asymptotical:

$$N = N_{th} + \frac{\tau}{ed} \frac{\varepsilon}{A\tau + \varepsilon} (J - J_{th}) + \frac{\Gamma A^2 \tau \tau_p}{A\tau + \varepsilon} \left[ \frac{\tau}{ed} \frac{\varepsilon}{A\tau + \varepsilon} (J - J_{th}) \right]^2$$

## Gain clamping

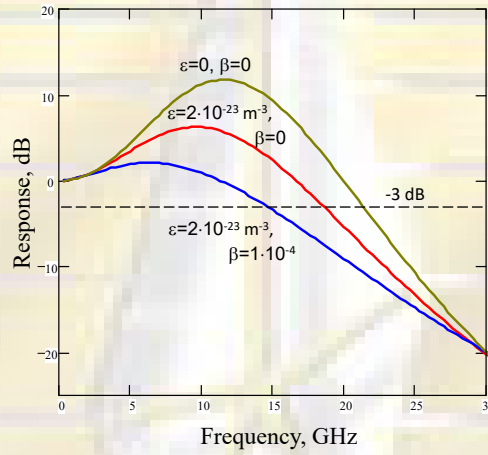
$$\frac{dn}{dt} = \frac{J}{ed} - \frac{n}{\tau_s} - \frac{c}{N_{eff}} g(n) S(1 - \varepsilon S)$$

$$\frac{dS}{dt} = \Gamma \frac{c}{N_{eff}} g(n) S(1 - \varepsilon S) - \frac{S}{\tau_p} + \Gamma \beta \frac{n}{\tau_s}$$



$$\omega_p \approx \sqrt{\frac{AS_0}{\tau_p} (1 - \varepsilon S_0)}$$

## Gain clamping



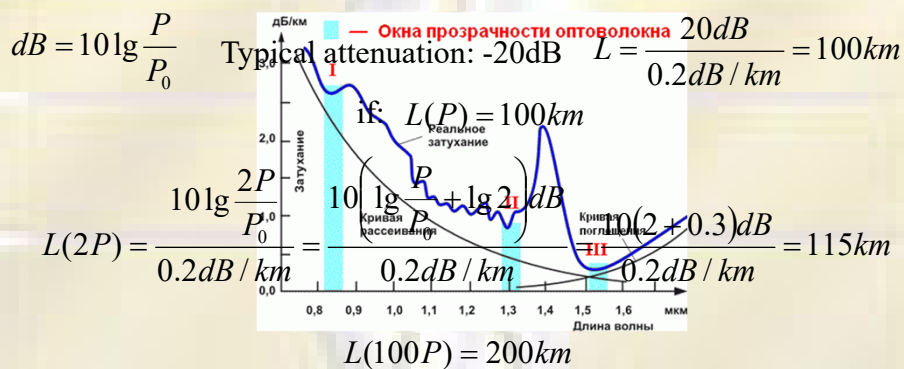
$$\omega_p \approx \sqrt{\frac{AS_0}{\tau_p} (1 - \epsilon S_0)}$$

## Application to the fiber optical communications

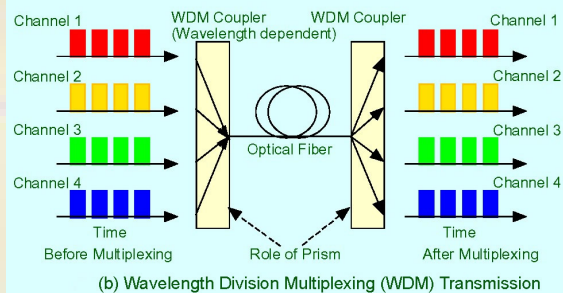
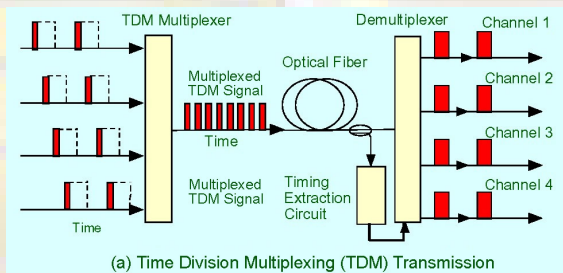


# Application to the fiber optical communications

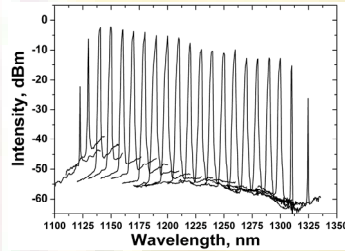
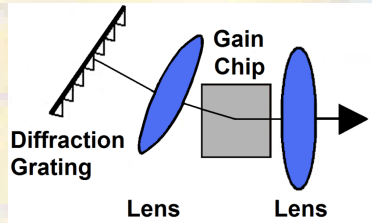
Double power: Double distance?



# Wavelength / Time Division Multiplexing

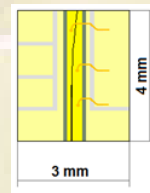


## Wavelength Selection

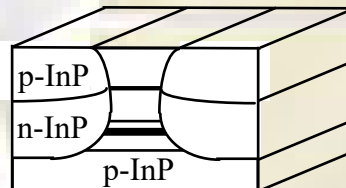
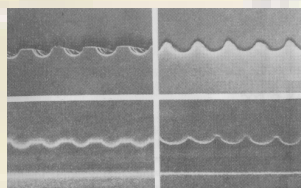
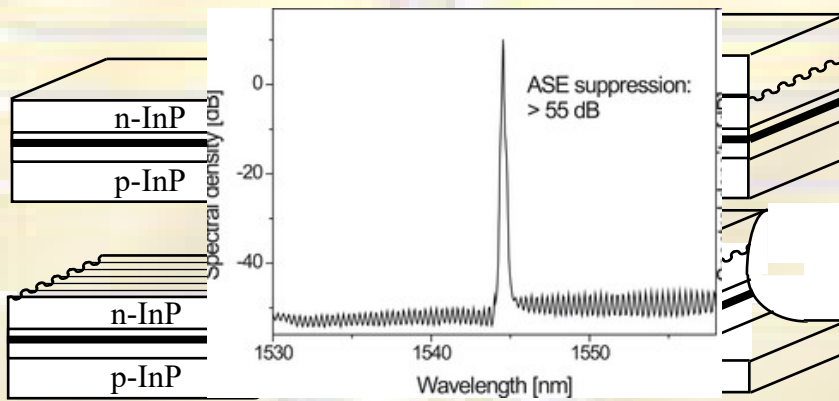


### Gain Chip:

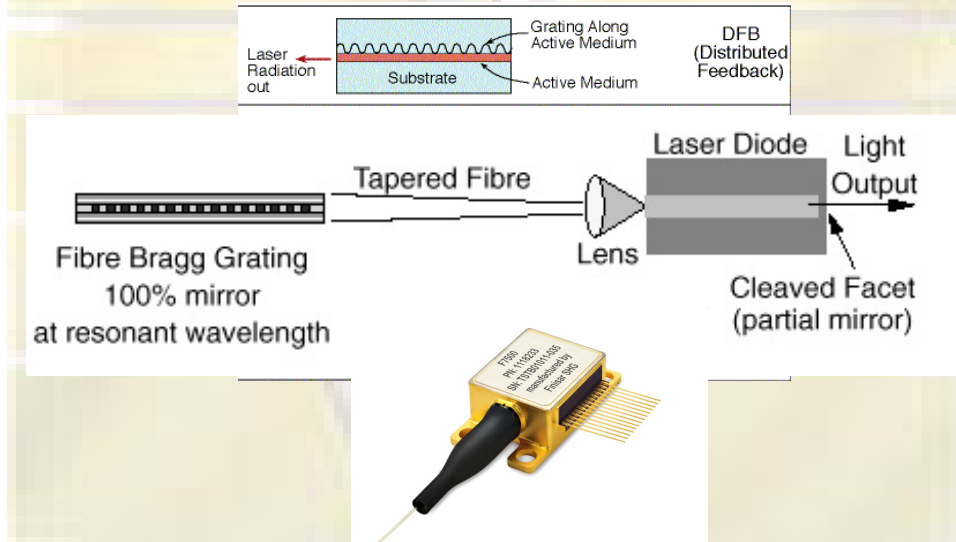
- 4mm length, 5 $\mu$ m wide waveguide
- 10 layers InAs QDs, grown on GaAs substrate
- waveguide angled at 5 $^\circ$
- facets AR coated:  $R_{\text{angled}} < 10^{-5}$   
 $R_{\text{front}} \sim 2 \cdot 10^{-3}$



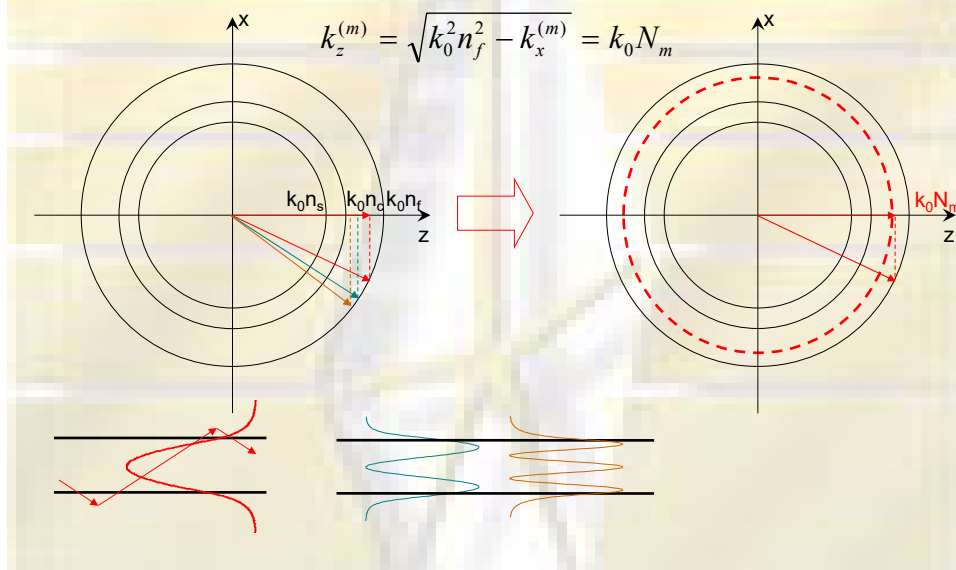
## Distributed feedback (DFB) laser diodes



## Distributed Bragg reflector (DBR) LDs

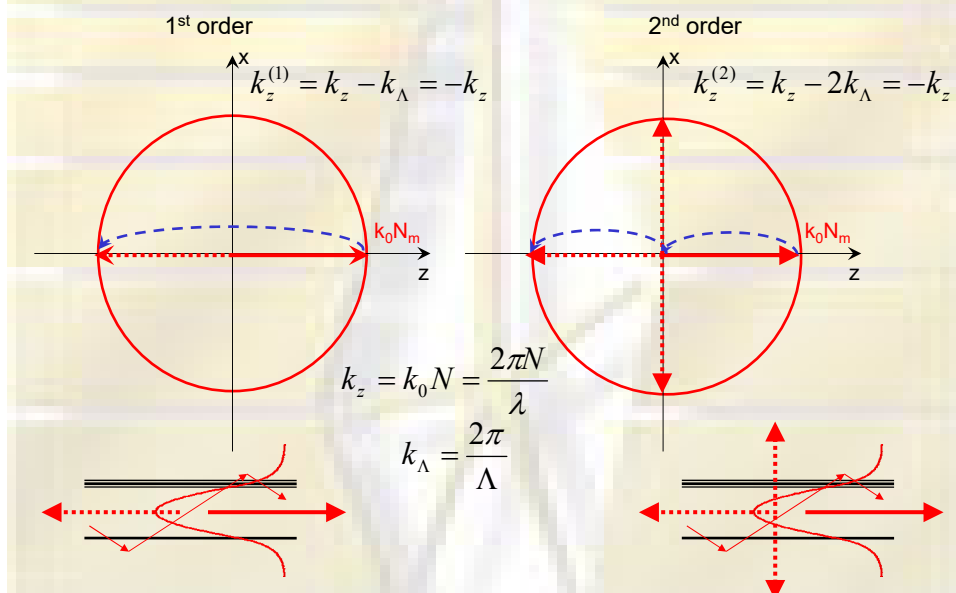


## Diffraction grating in a waveguide

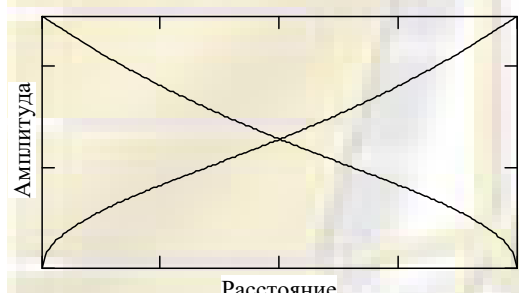
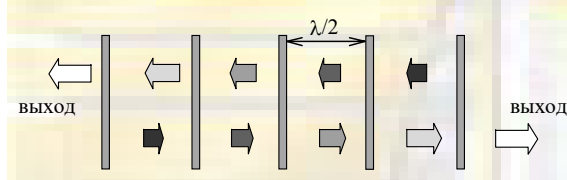




## Diffraction grating in a waveguide



## Distributed feedback



$$\delta = \frac{(\beta^2 - \beta_0^2)}{2\beta} \quad \beta - \beta_0 = \frac{n(\omega - \omega_0)}{c}$$

$$\frac{\partial^2 E}{\partial z^2} + \kappa^2 E = 0$$

$$\alpha(z) = \alpha + \alpha_1 \cos(2\beta_0 z)$$

$$n(z) = n + n_1 \cos(2\beta_0 z)$$

$$\beta_0 = n\omega_0 / c \approx n\omega / c$$

$$\kappa^2 = \beta^2 + 2i\alpha\beta + 4k\beta \cos(2\beta_0 z)$$

$$\beta = n\omega/c \quad k = \pi n_1 / \lambda_0 + i\alpha_1 / 2$$

$$E(z) = R(z)e^{-i\beta_0 z} + S(z)e^{i\beta_0 z}$$

$$-\frac{dR}{dz} + (\alpha - i\delta)R = ikS$$

$$\frac{dS}{dz} + (\alpha - i\delta)S = ik^* R$$

Р.Ф.Казаринов, Р.А.Сулис, ФТП, 1972, т.6(7), с. 1359-1365.  
 Н.Кogelnik, С. V.Shank, Journal of Appl. Phys, 1972, v.43(5), pp.2327-2335.

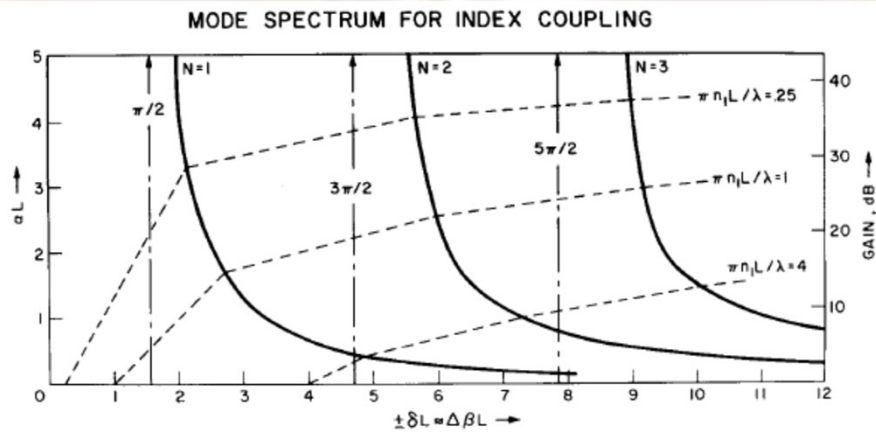
## Distributed feedback

$$-\frac{dR}{dz} + (\alpha - i\delta)R = ikS$$

$$\frac{dS}{dz} + (\alpha - i\delta)S = ik^*R$$

$$\delta = \frac{(\beta^2 - \beta_0^2)}{2\beta} \quad \beta - \beta_0 = \frac{n(\omega - \omega_0)}{c}$$

$$k = \pi n_1 / \lambda_0 + i\alpha_1 / 2$$

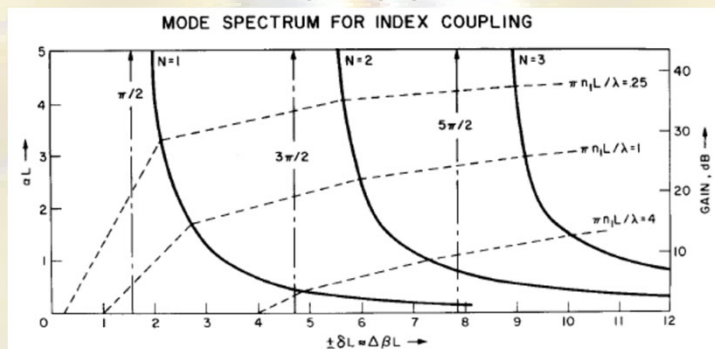


## Distributed feedback

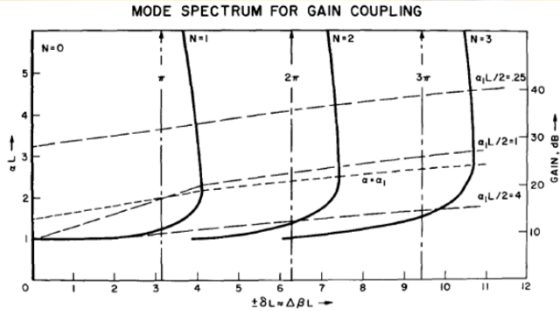


$$\delta = \frac{(\beta^2 - \beta_0^2)}{2\beta}$$

$$k = \pi n_1 / \lambda_0 + i\alpha_1 / 2$$



# Distributed feedback

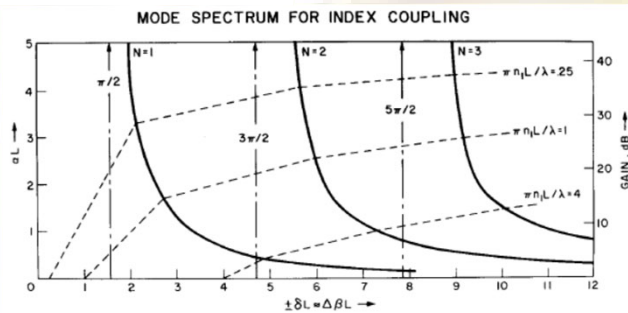


Связь по усилению

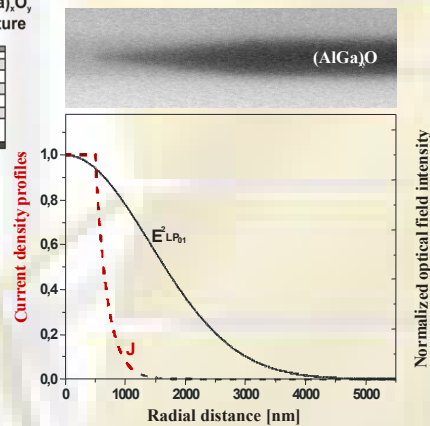
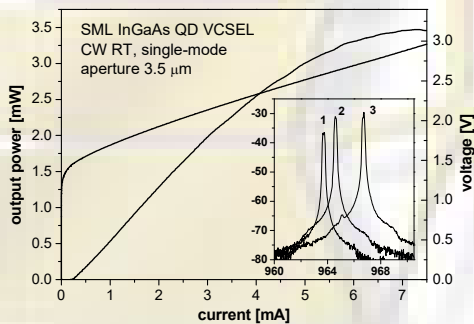
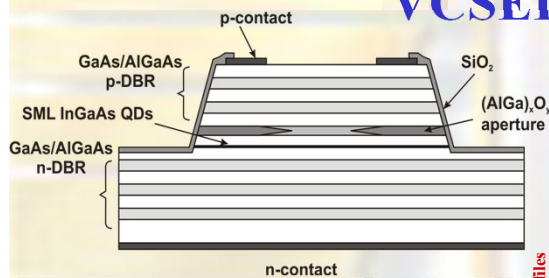
$$k = \pi n_1 / \lambda_0 + i \alpha_1 / 2$$

Связь по коэф. преломления

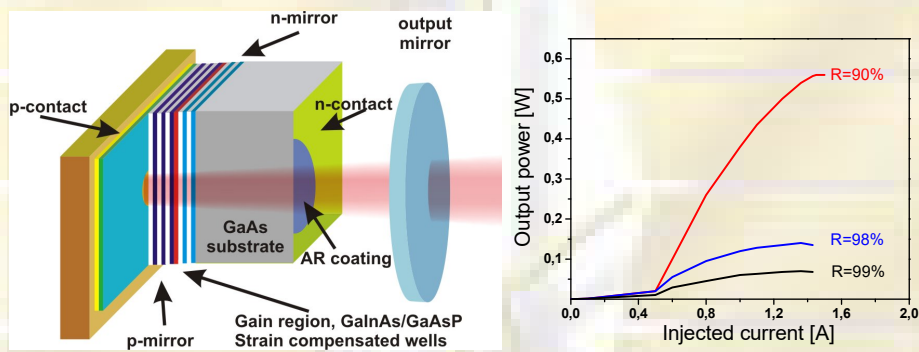
$$k = \pi n_1 / \lambda_0 + i \alpha_1 / 2$$



# Vertical-cavity surface-emitting lasers VCSELs



## Vertical-extended-cavity surface-emitting lasers (VECSELs)

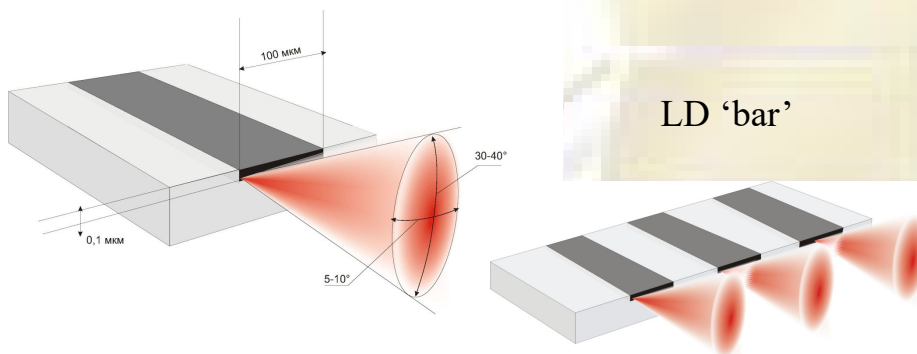


A.Mooradian, "High brightness cavity-controlled surface emitting GaInAs lasers operating at 980 nm", Proceedings of the Optical Fiber Communications Conference, 17-22 March 2001

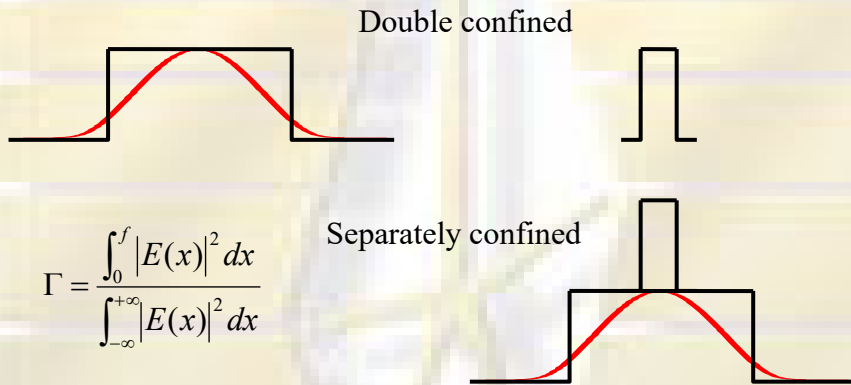
## Problems of LDs

- Power
  - Beam quality
  - ~~• Spectral quality~~
- } Power density & intensity gradient

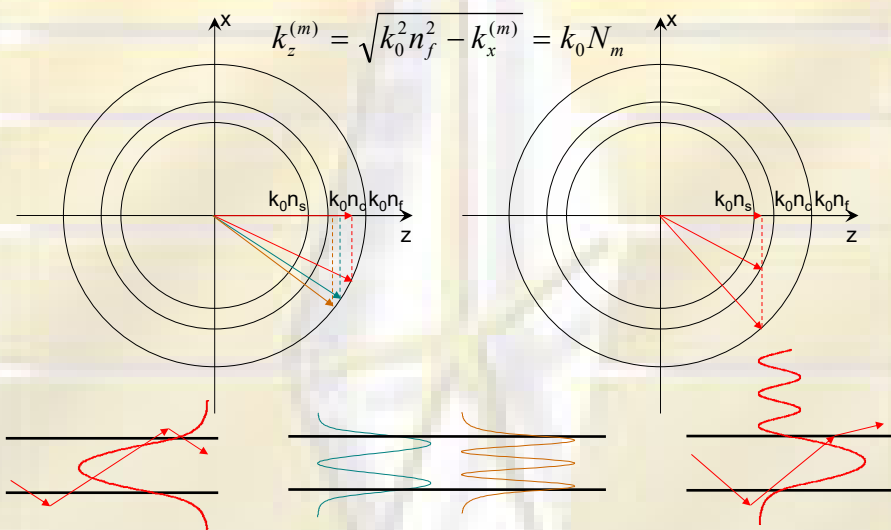
Broad-stripe LD



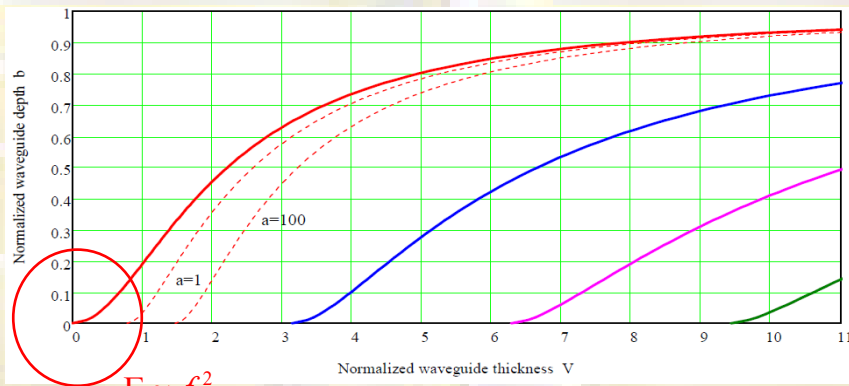
## Optical confinement



## Modes of the waveguide



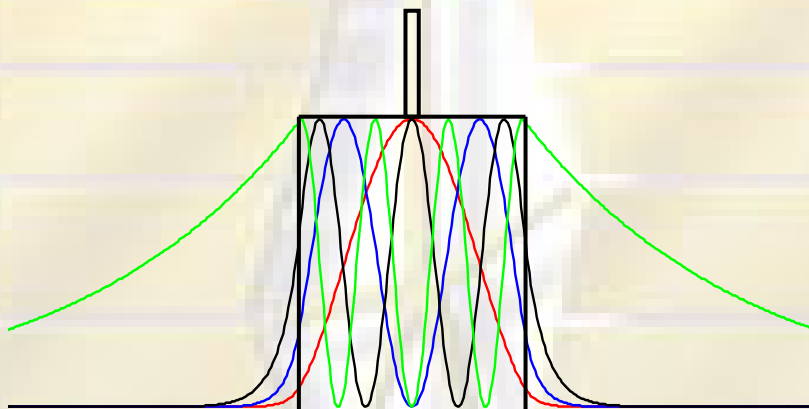
## Optical confinement



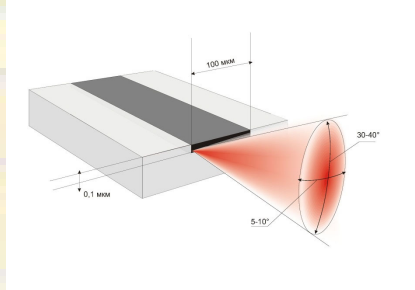
$$\Gamma \approx f^2$$

$$\Gamma = \frac{\int_0^f |E(x)|^2 dx}{\int_{-\infty}^{+\infty} |E(x)|^2 dx}$$

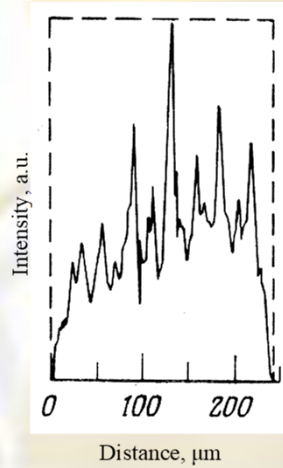
## Managing mode structure with optical confinement



## Power density and LD power



'Not impossible' near-field distribution of the broad-stripe LD

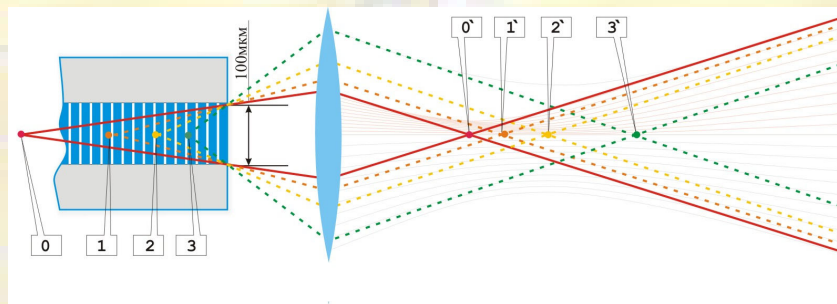


Higher power:

- Higher modes
- Spots (filaments)
- Astigmatism

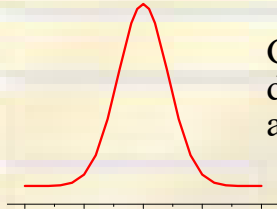
Higher power does not mean higher power density... :(

## Multimode LDs



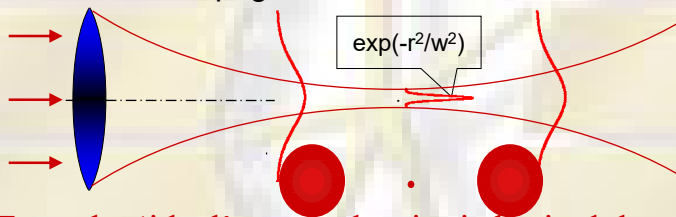
Multi-moded and 'spotted' (filamented) radiation is difficult (and sometimes impossible) to focus!

## Gaussian beams



Gaussian  $\exp(-r^2/w^2)$  is the most dense distribution. Therefore, Gaussian beams are the beams of the highest 'quality'.

### Propagation of the Gaussian beam



Even the 'ideal' power density is limited due to the quantum mechanical uncertainty principle  $\Delta p \Delta x = h$

## Gaussian beams

$$w_0 = \frac{\lambda}{\pi NA}$$

Beam waist size on the level of  $1/e^2$  (13.5%)

$$w(z) = w_0 \sqrt{1 + \left(\frac{\lambda z}{\pi w_0^2}\right)^2}$$

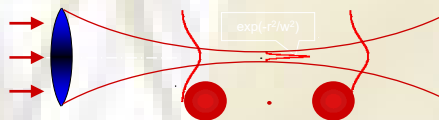
Propagating beam size (beam diameter)

$$R(z) = z \left[ 1 + \left(\frac{\pi w_0^2}{\lambda z}\right)^2 \right]$$

Wavefront curvature

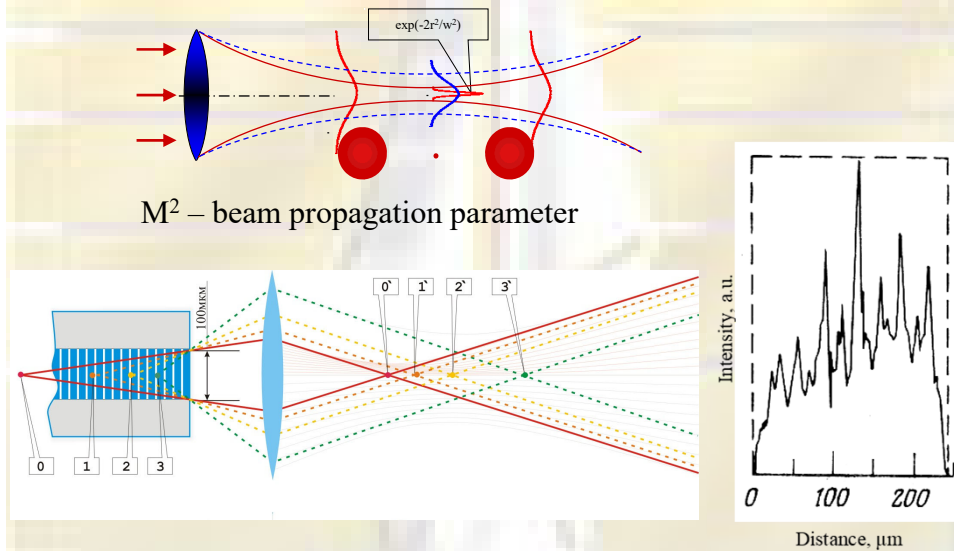
$$z_R = \frac{\pi w_0^2}{\lambda}$$

Rayleigh range (distance of  $\sqrt{2}$ -fold increase of diameter of the beam)





## LD beams: From multi-mode to quasi-Gaussian



## Second moment properties

Definition of the second moment (or variance):

$$\sigma_x^2 = \frac{\int_{-\infty}^{\infty} (x - x_0)^2 I(x, y) dx dy}{\int_{-\infty}^{\infty} I(x, y) dx dy}$$

Applied to a zero-order Gaussian beam yields:

$$\sigma_x^2 = \frac{\int_{-\infty}^{\infty} x^2 \exp(-x^2/w_x^2) dx}{\int_{-\infty}^{\infty} \exp(-x^2/w_x^2) dx} = \frac{w_x^2}{2}$$

This leads to a parabolic propagation rule:

$$\sigma_x^2(z) = \sigma_{x0}^2 + \sigma_{\theta}^2 (z - z_0)^2$$

$$w^2(z) = w_0^2 + \left( \frac{\lambda}{\pi w_0} \right)^2 (z - z_0)^2 = w_0^2 \left[ 1 + \left( \frac{z - z_0}{z_R} \right)^2 \right]$$

*A.E. Siegman / Proc. SPIE 1868, 2 (1993)*

## Second-moment-based beam width definition

Therefore for arbitrary real beams similar beam width definitions can be adopted:

$$w_x = \sqrt{2}\sigma_x$$

These second-moment-based beam widths will propagate exactly quadratically with distance in free space. For any arbitrary beam (coherent or incoherent), one can then write using the second-moment width definition

$$w_{x,y}^2(z) = w_{0x,y}^2 + M_{x,y}^4 \left( \frac{\lambda}{\pi w_{0x,y}} \right)^2 (z - z_0)^2$$

With  $M^2 \geq 1$  being ‘times diffraction-limited’ factor for an arbitrary real beam compared to the zeroth-order Gaussian beam

$$M^2 \equiv \frac{\pi w_0 w(z)}{z \lambda}$$

*A.E.Siegman / Proc. SPIE 1868, 2 (1993)*

## Basic properties of the $M^2$ parameter

$M^2$  is a ‘times-diffraction-limited’ parameter based on measured near and far field second-moment beam widths

$$M^2 = \frac{w_{measured}}{w_0} \quad M^2=1 \text{ for a zero-mode Gaussian beams}$$

Arbitrary real beam width can then be fully described by 6 parameters:

$$w_{0x}, w_{0y}, z_{0x}, z_{0y}, M_x^2, M_y^2$$

Requires time-averaged intensity measurements only.

NO phase or wavefront measurements!

- Can develop such a universal propagation rule only for the second-moment beam width definition
- Actually holds true for propagation through arbitrary paraxial optical systems as well

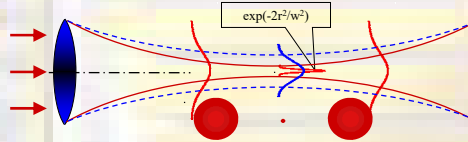
*A.E.Siegman / Proc. SPIE 1868, 2 (1993)*

## Focusing of the quasi-Gaussian beams

$$\lambda \rightarrow M^2 \lambda$$

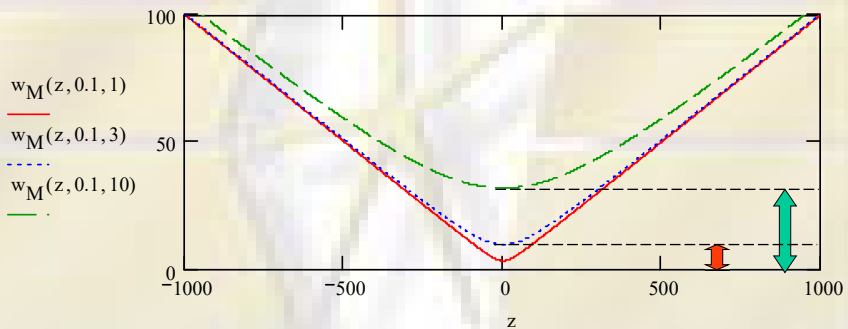
$$w_{M^2} = \frac{M^2 \lambda}{\pi NA}$$

$$w(z) = w_{M^2} \sqrt{1 + \left( \frac{M^2 \lambda z}{\pi w_{M^2}^2} \right)^2}$$



Beam waist size on the level of  $1/e^2$  (13.5%)

Propagating beam size



$w_M(z, 0.1, 1)$   
 $w_M(z, 0.1, 3)$   
 $w_M(z, 0.1, 10)$

## How to measure $M^2$ parameter?

Measurable: beam diameter as a function of propagation length:

$$w^2(z) = w_{M^2}^2 + NA^2(z - z_0)^2 = w_{M^2}^2 + z_0^2 NA^2 - 2zz_0 NA^2 + z^2 NA^2 = A + B_1 z + B_2 z^2$$

$$A = w_{M^2}^2 + z_0^2 NA^2$$

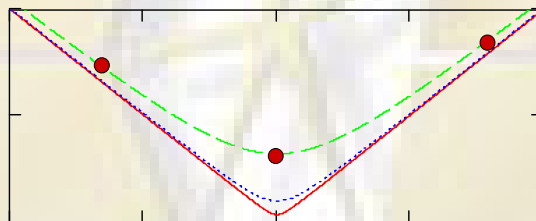
$$B_1 = -2z_0 NA^2 \quad \text{with} \quad w_{M^2} = \frac{M^2 \lambda}{\pi NA}$$

$$B_2 = NA^2$$

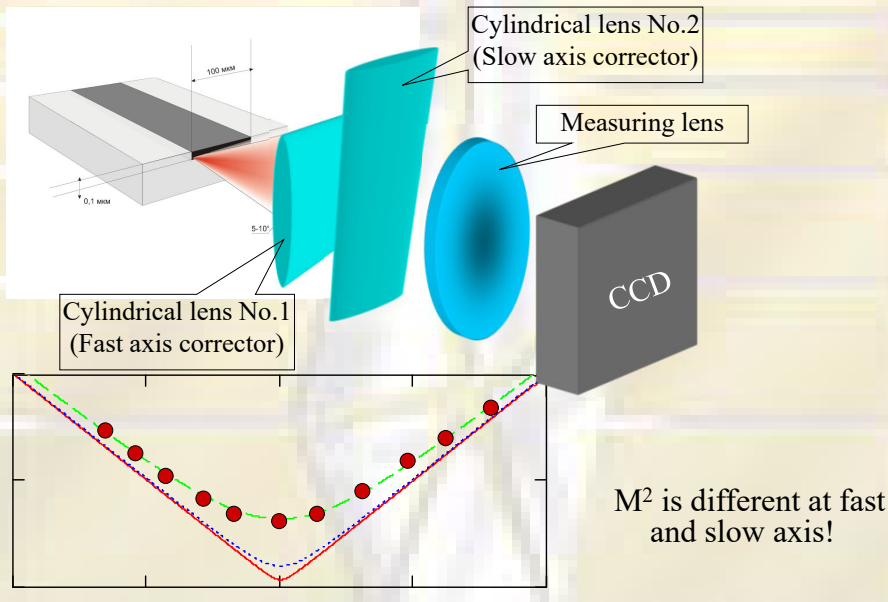
$$NA = \sqrt{B_2}$$

follows:  $z_0 = -\frac{B_1}{2B_2}$

$$M^2 = \frac{\pi}{\lambda} \sqrt{AB_2 - \frac{B_1^2}{4}}$$

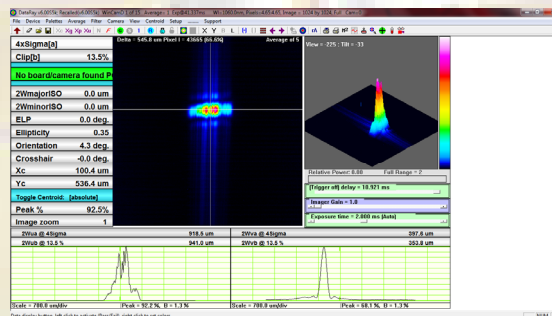


## How to measure $M^2$ parameter?



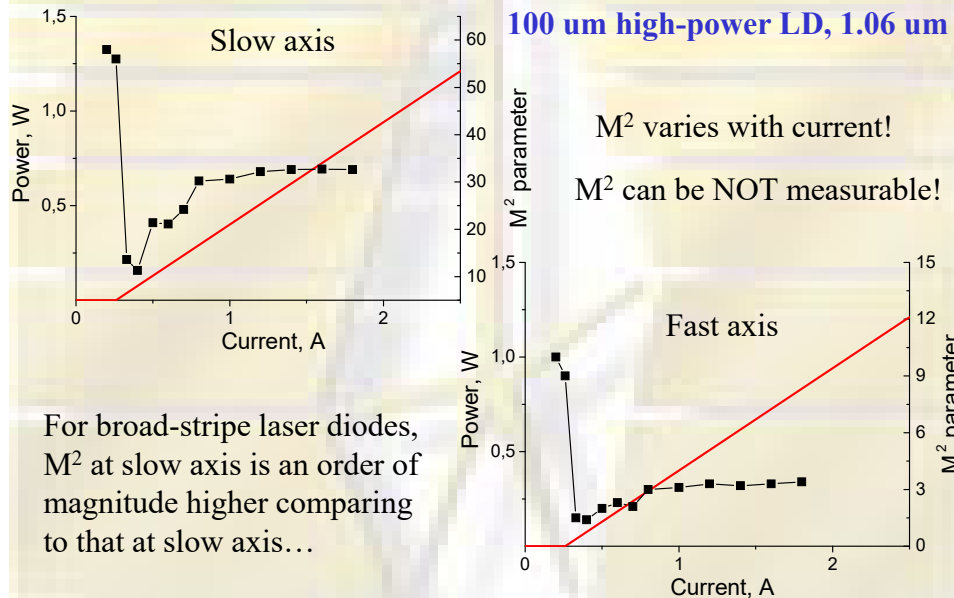
## How to measure $M^2$ in your lab?

1. Use house-built setup with any CAL or CAD software (e.g. MicroCal Origin)  
Approx. time for one laser: 3 hrs (when you've got some experience)
2. Use specialist hard- and soft-ware (e.g. DataRay)  
Approx. time for one laser: 15 mins

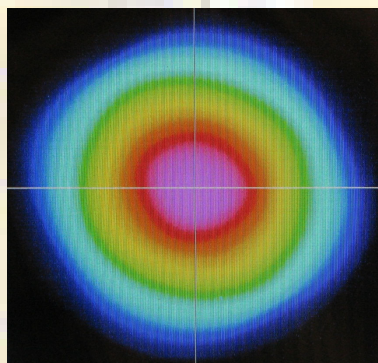


Standard:  
ISO 11146

## Measuring $M^2$ parameter

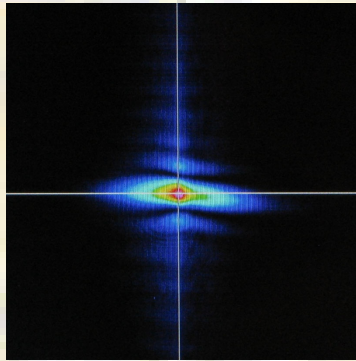


Don't call it 'Gaussian' before you are sure...



Light emitting diode Lumiled, 0.63  $\mu\text{m}$  350 mA,  $M^2=500$

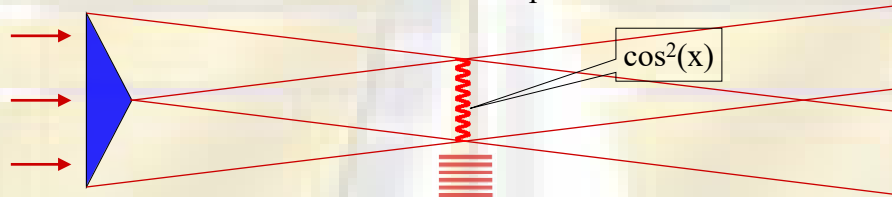
## Don't overestimate the $M^2$ parameter of the 'nasty' LD beams...



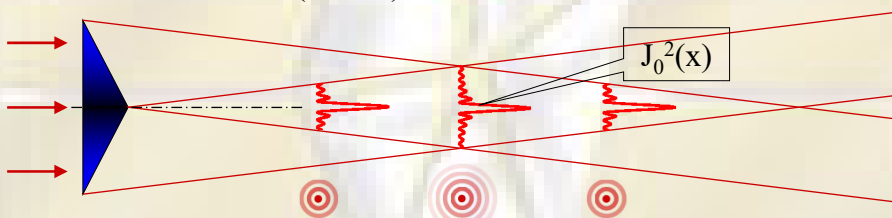
Narrow stripe LD, 1.06  $\mu\text{m}$ , 100 mA,  $M^2=4$

## Interference focusing (Generation of Bessel beams)

Prism: interference of plane waves



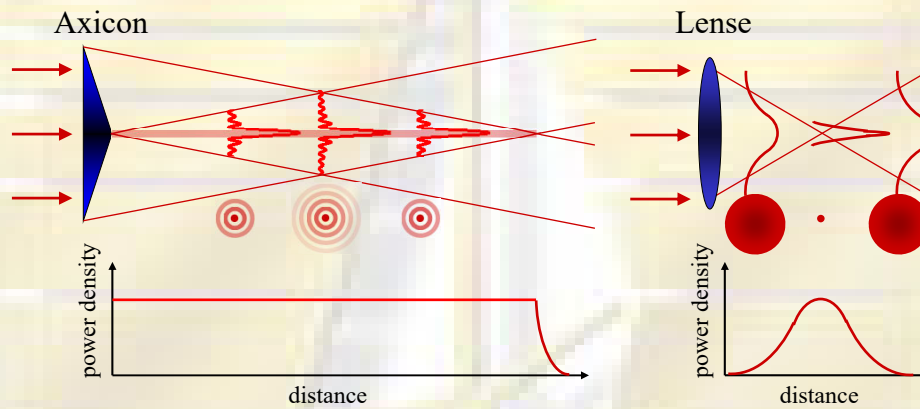
Conical lens (axicon): interference of conical waves



*J. Durnin // J. Opt. Soc. Am. 1987, A 4, P. 651-654*

*B.Ya.Zel'dovich, N.F.Pilipetskii // Izvestia VUZov, Radiophysics, 1966, 9(1), P.95-101*

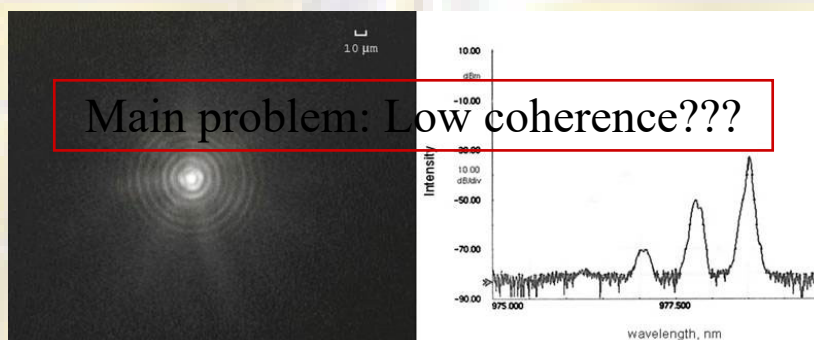
## Bessel beams vs Gaussian beams



Bessel beams are ideal for optical tweezers, micromanipulation and lab-on-a-chip applications

**Bessel beams are typically generated with vibronic lasers**

## Interference focusing with Laser Diodes



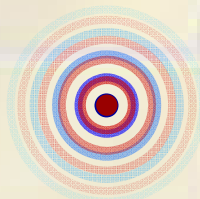
VCSEL DO-701d, Innolume GmbH, axicon 170<sup>0</sup>, I=1 mA

*G.S.Sokolovskii et al. // Tech Phys Lett, 2008, 34(24) 75-82*

## Interference focusing with High-power Laser Diodes

Main problem: Poor beam quality

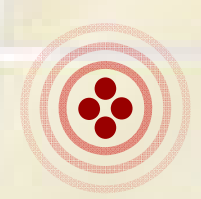
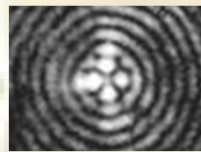
Multimode



Filamented

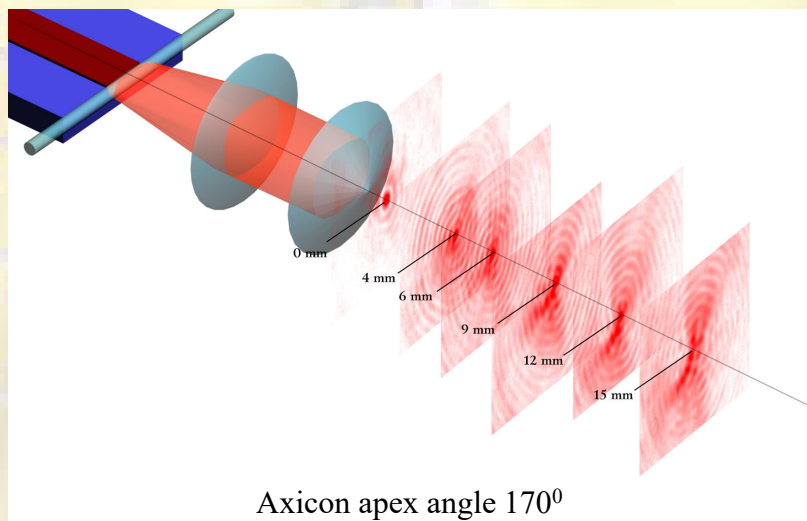


Oblique



Radiation wavelength  $\lambda = 1.06 \mu\text{m}$ . Axicon apex angle  $170^\circ$ . Central lobe size  $d_0 = 10 \mu\text{m}$ .

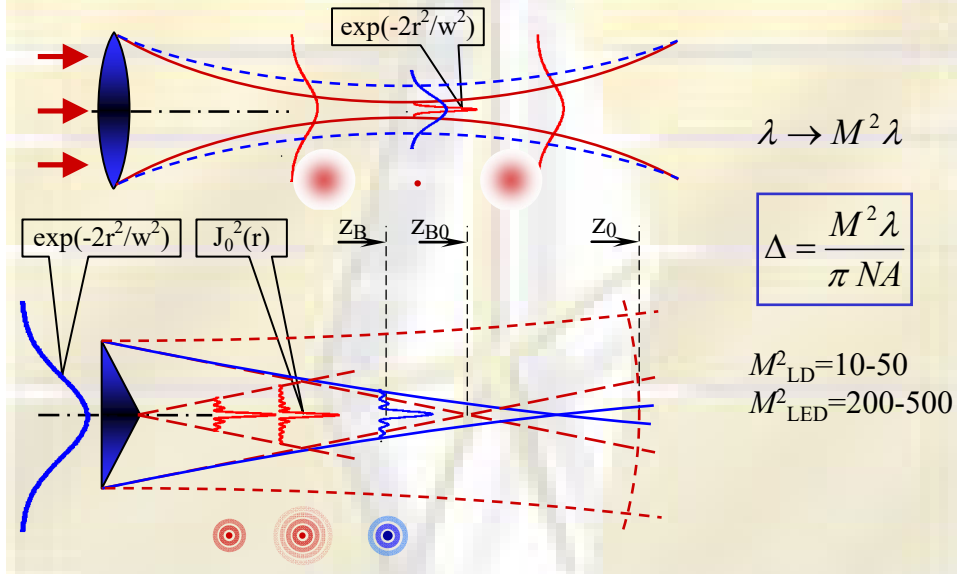
## Bessel beams from the broad-stripe LD



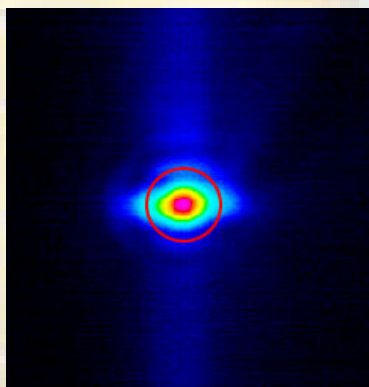
*G.S.Sokolovskii et al. // Tech Phys Lett, 2010, 36(1) 22-30*



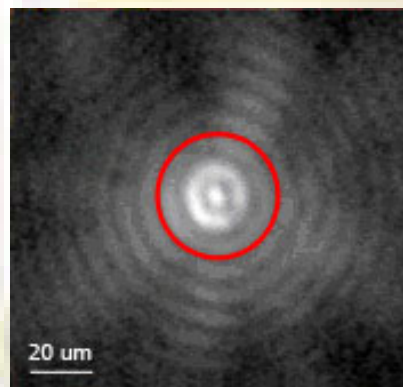
## Propagation length of Bessel beams generated from Laser Diodes



## Achieving 'non-achievable' intensity gradients with broad-stripe LDs



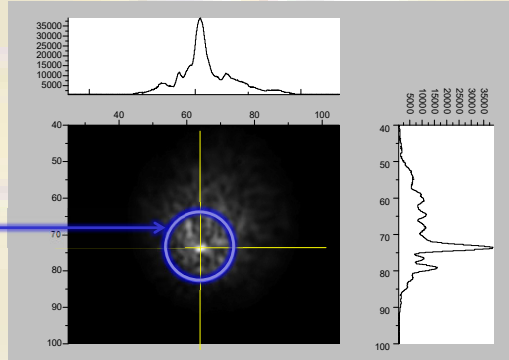
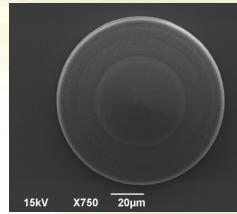
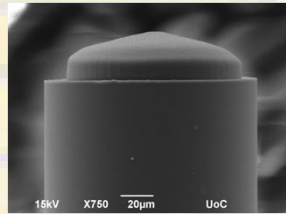
Broad-stripe LD 1.06  $\mu\text{m}$ ,  $M^2=22$ ,  
interference focal spot dia = 4  $\mu\text{m}$



LED 0.63  $\mu\text{m}$ ,  $M^2=200$ ,  
interference focal spot dia = 6  $\mu\text{m}$

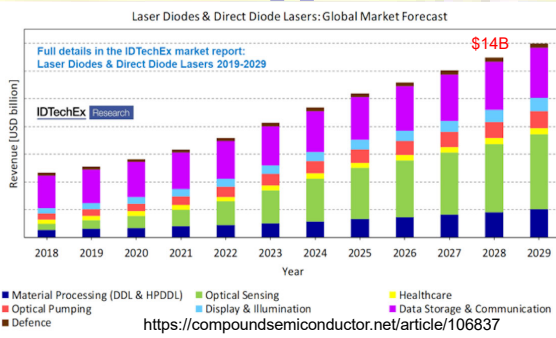
# Fiber axicons

(Fiber axicons manufactured by Maria Farsari and colleagues, FORTH)

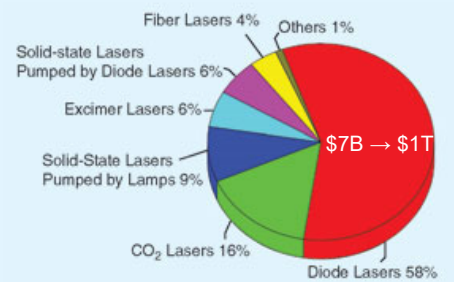


$$\Delta = \frac{M^2 \lambda}{\pi NA} \quad 1$$

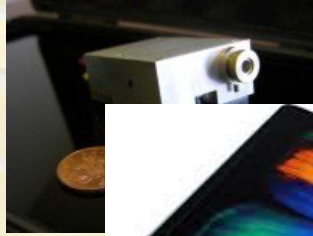
# Applications of LDs



- Telecoms
- Data reading/recording
- Laser printing
- Pumping of the solid-state
- 'Direct' applications: cutting
- Biology and medicine



## Laser projectors: Green Light?



Aiptek Pocket



no focus adjustment!

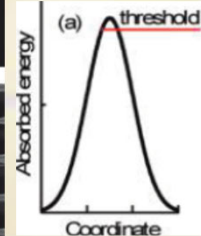
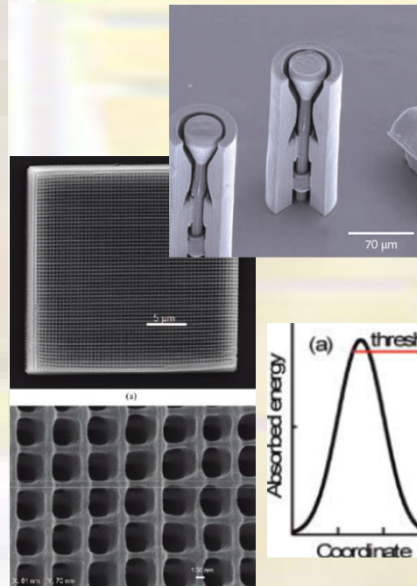
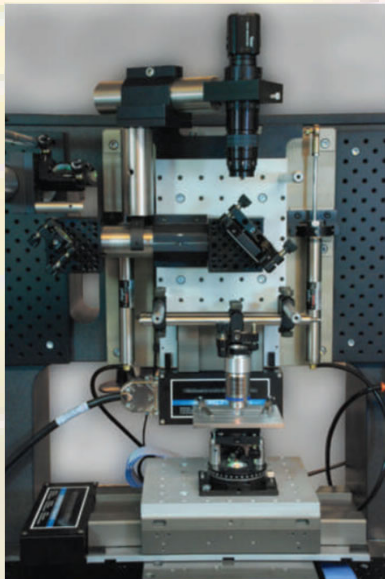


MacWorld 2010  
Best of Show award

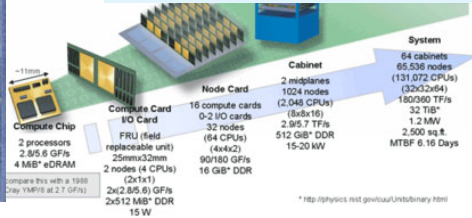
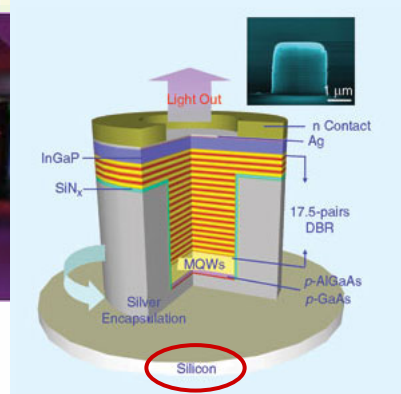
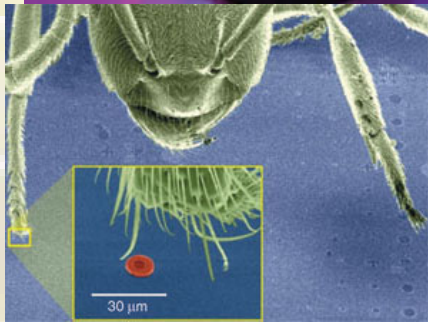


Samsung H03  
Mini-sized LED Projector

## From laser printers to 3D printing



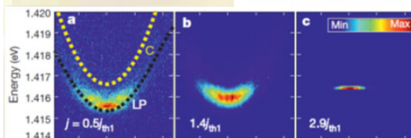
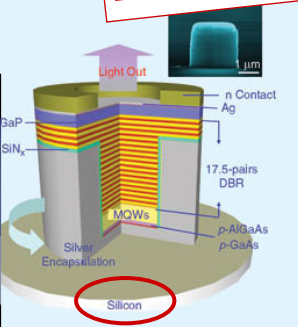
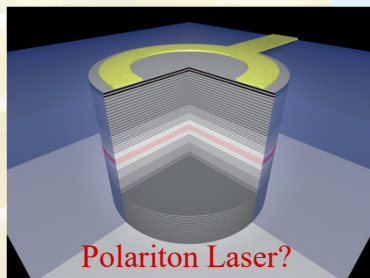
# Telecoms/Datacoms: Silicon Photonics?



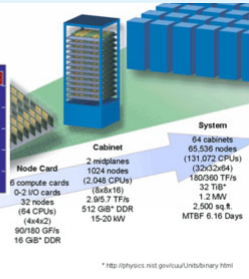
# On-chip Datacoms

**Energy matters!**

What counts? fJ/bit!



C.Schneider et al. // Nature 497, 348 (2013)



## Automotive applications: laser ignition?



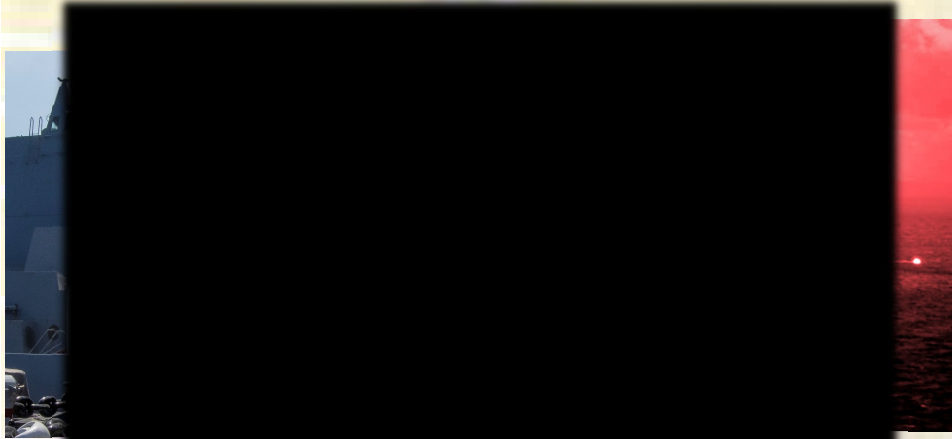
## US Army first laser weapon

Offers lethality against unmanned aircraft systems (drones) and rockets, artillery and mortars



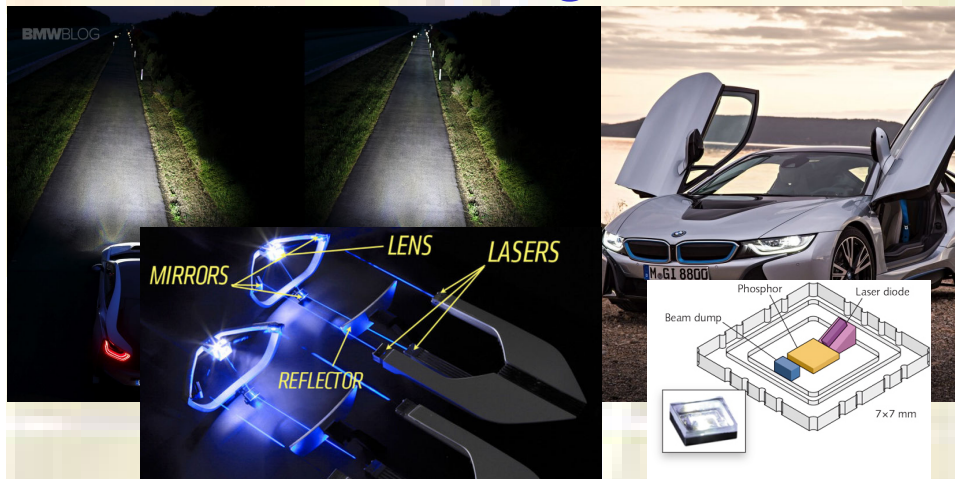
Citadel Defense has been awarded a sole source contract for \$6M from US DoD to build and deploy an AI-powered counter drone solution. The system will be deployed at “sensitive government locations” and operated by non-specialist military personnel and first responders.

## USS Portland tests high-energy laser



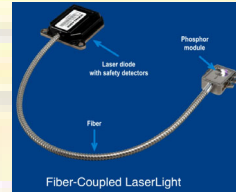
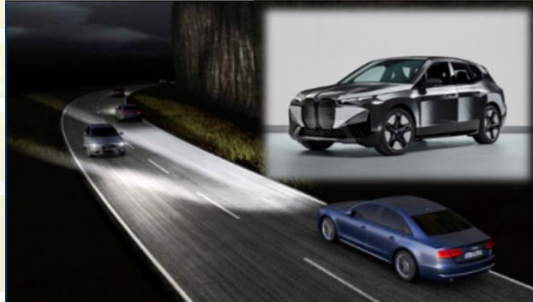
Solid State Laser aboard USS Portland successfully engages a static surface training target. Portland previously tested the LWSD in May 2020 when it successfully disabled a small unmanned aerial system while operating in the Pacific Ocean.

## Automotive applications: laser headlights?

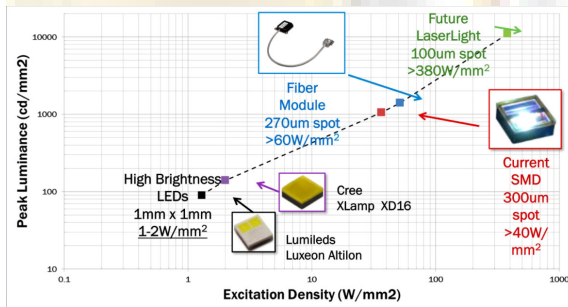
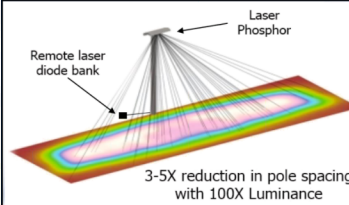


[www.bmw.com](http://www.bmw.com) 'In the laser headlight, the beams of light are bundled together to attain a luminous intensity that is ten times greater than conventional light sources such as halogen, xenon or LED. BMW Laserlight has a visual range of up to 600 metres, twice that of a headlight with conventional light technology.'

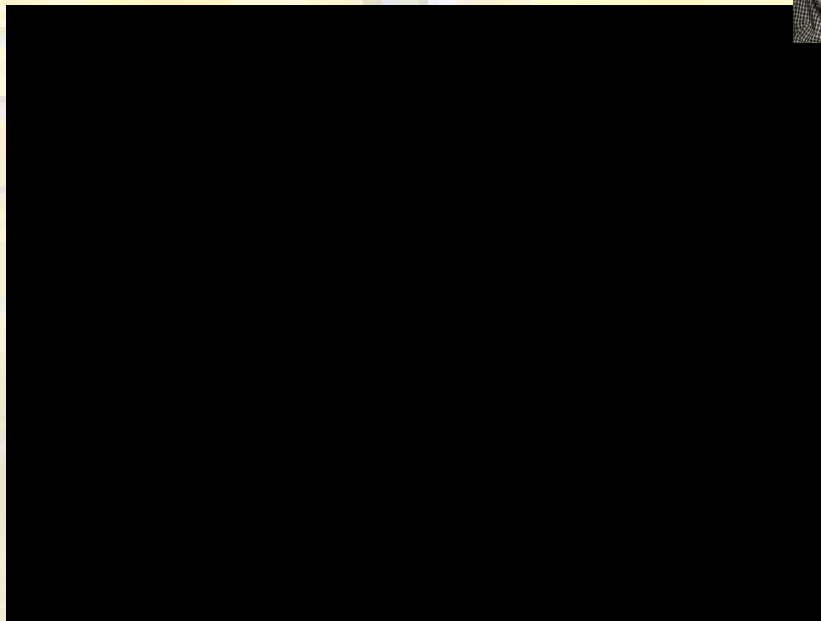
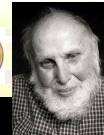
## Laser lighting?



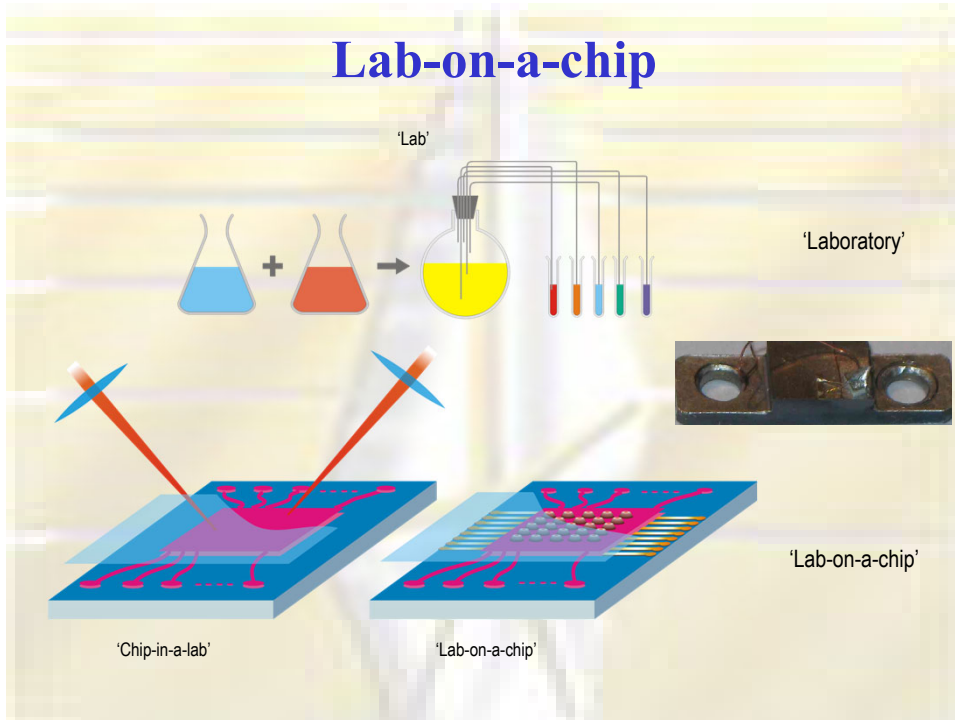
LED Street Lighting with < 30m pole spacing



## Optical tweezers: lab-on-a-chip?



# Lab-on-a-chip



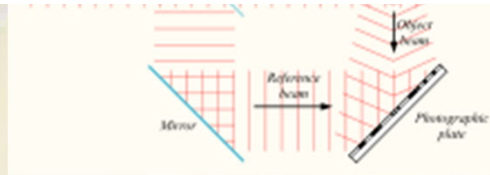
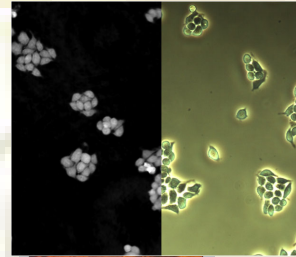
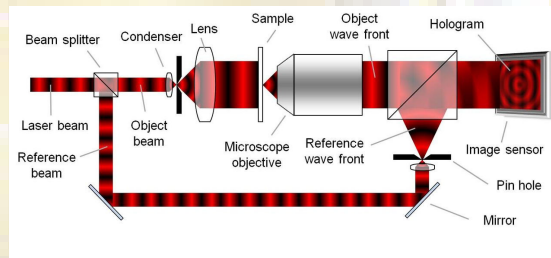
# Fluorescent microscopy

The slide features a schematic of an optical setup for fluorescent microscopy. The components include a Gain Chip 1, a Beam Splitter, a  $\lambda/2$  waveplate, a PPKTP crystal, a Filter, and two Lenses. A graph shows Pump Power in mW on a logarithmic scale, ranging from 1 to 100. Three portraits of researchers are shown at the top right. Several microscopy images are included, including a Confocal image of a cell and a 1  $\mu\text{m}$  scale bar.

Despite the small footprint of only 25 mm x 25 mm, FBH modules achieve optical output powers of  $> 8$  W cw at 1064 nm as well as 1156 nm with very good beam quality of  $M^2 < 2$  and a spectral linewidth of  $< 5$  MHz. <https://optics.org/news/13/11/3>

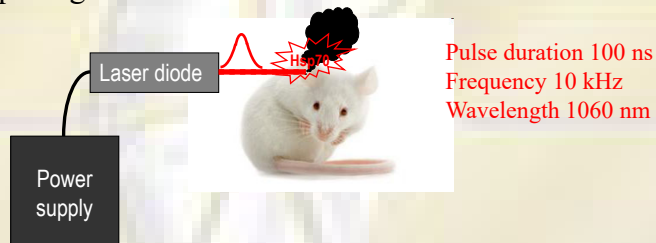


## Lensless microscopy



## Generation of Heat-Shock Proteins and Ceramides with Laser Diodes

Extracellular heat shock proteins (Hsp70) and ceramides are known to possess high adjuvant activity for cancer vaccines and low-immunogenic vaccines against dangerous infections. Hsp70 and ceramide can activate receptors of the innate immunity (TLR4) and boost protective immune response reactions against abiotic stress factors and biopathogens.



# Quantum time standards and gyroscopes

СТАНДАРТ ЧАСТОТЫ

ГИРОСКОП

[P. Porsandeh Khial et al, Nature Photonics, 12 (11), 671, 2018]

Alkali atom cell with Xenon

# LDs for Mid-IR (1600 nm - 16 μm)



In Mid Infrared spectral range 1600-5000 nm lies strong absorption bands of such important gases and liquids as:

CH<sub>4</sub>, H<sub>2</sub>O, CO<sub>2</sub>, CO, C<sub>2</sub>H<sub>2</sub>, C<sub>2</sub>H<sub>4</sub>, C<sub>2</sub>H<sub>6</sub>, CH<sub>3</sub>Cl, OCS, HCl, HOCl, HBr, H<sub>2</sub>S, HCN, NH<sub>3</sub>, NO<sub>2</sub>, glucose and many others.



# Quantum-cascade lasers

Вып. 4 УДК 621.382.8

**О ВОЗМОЖНОСТИ УСИЛЕНИЯ ЭЛЕКТРОМАГНИТНЫХ ВОЛН В ПОЛУПРОВОДНИКЕ СО СВЕРХРЕШЕТКОЙ**

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Рядом авторов отмечалась возможность возникновения отрицательного дифференциального сопротивления в полупроводниках, в которых зона проводимости (или валентная зона) расщеплена на ряд подзон (минус с периодом, превосходящим период кристаллической решетки [1-3]). Это отрицательное сопротивление обусловлено брегговскими отражениями, которые испытывает электрон, ускоренный внешним электрическим полем. Для того чтобы такой механизм мог реализовываться, необходимо выполнение условия

$$\frac{\hbar}{\tau} < e\mathcal{E} \ll I_0$$

Здесь  $I_0$  — ширина нижней мини-зоны,  $e\mathcal{E}$  — энергия, приобретаемая электроном в электрическом поле  $\mathcal{E}$  на периоде сверхрешетки  $a$ ,  $\tau$  — время его свободного пробега.

Интерес к явлениям, протекающим в таких системах, сейчас особенно возрос в связи с тем, что уже получены первые экспериментальные образцы этих структур на основе монокристаллов полупроводников АIIIВV с периодически меняющимися на расстояниях порядка  $10^8$  Å составом и соответственно шириной запрещенной зоны [4, 5].

$$J = e n v_y (1 - e^{-\Delta/\hbar}) \times \frac{2 |U|^2 \hbar^2 \tau}{\hbar^2 \tau + 4 |U|^2 \hbar^2 \tau^2} \quad (1)$$

$$\hbar \xi = e\mathcal{E} - \Delta$$

Здесь  $\Omega = e\mathcal{E}a$ , где  $x$  — матричный элемент координаты между волновой функцией основного состояния  $n$ -й и первого возбужденного состояния  $(n+1)$ -й зоны. Эта величина порядка постоянной сверхрешетки, умноженной на прозрачность разделяющего яма барьера, которая в случае сильной связи много меньше единицы;  $\tau_1^{-1}$  — частота перехода электрона из возбужденного состояния в основное;  $\tau$  — время свободного пробега по

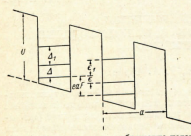
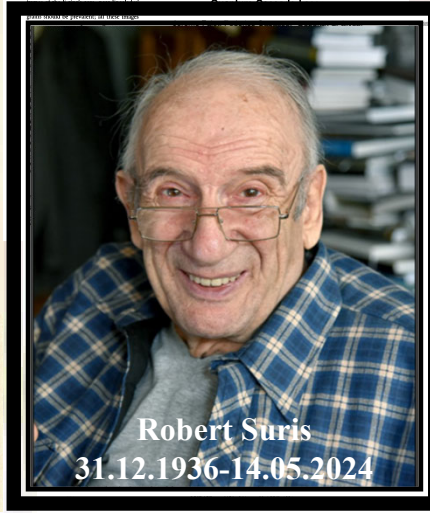
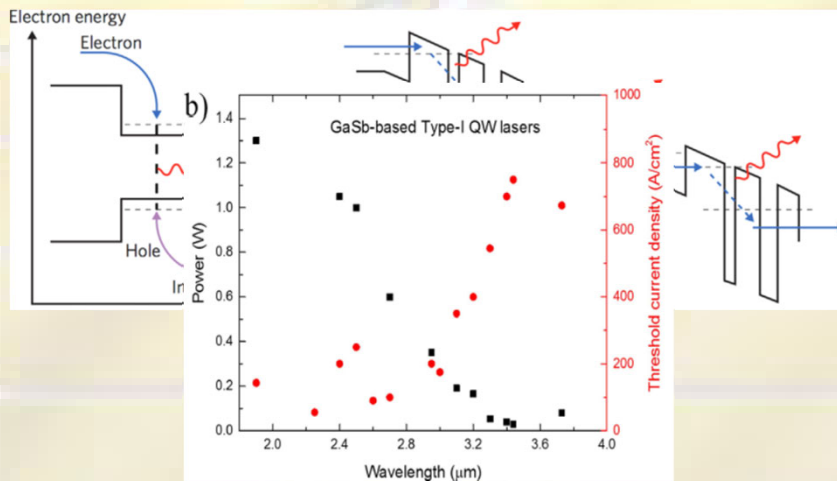


Fig. 1. Schematic representation of the potential of a superlattice and the energy levels of the conduction band.



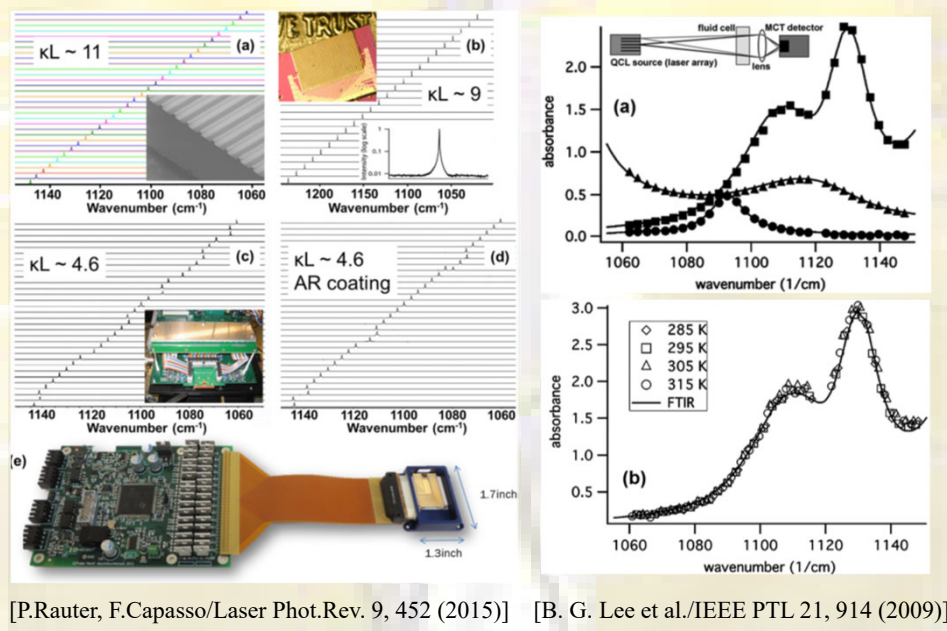
[R.F.Kazarinov, R.A.Suris, Semiconductors 5, 797 (1971)]  
[J.Faist, F.Capasso et al., Science 264, 553 (1994)]

# Quantum cascade lasers: End of $E_g$ slavery



[G. Taubes, 'A new laser promises to put an end to band gap slavery' Science 264, 508 (1994)]  
[D. Jung et al. / J of Optics 19, 123001 (2017)]

## QCL arrays for spectroscopy



## Thank you for your attention!

- Applications of semiconductor lasers. ‘Revolution’ of light.
- Basic principles of laser operation (only to remind).
- Absorption and gain in semiconductors, inversion of population and conditions for it's achievement.
- Rate equations. Lasing threshold. Laser efficiency.
- Modulation of the laser signal. Gain clamping.
- Fiber-optical applications. DFB and DBR lasers. VCSELs and VECSELs
- Waveguide in LD structure. Modes of the waveguide.
- Beam-propagation (‘beam-quality’) parameter  $M^2$  and how to measure it. Achieving maximum power density with LDs.
- Interference focusing of LD beams.
- What’s next? New applications and problems to solve.



**Thank you!**  
**Questions?**  
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