Экситоны в двумерных материалах

М.М. Глазов

ФТИ им. А.Ф. Иоффе, Санкт-Петербург

Лекция 2. Экситонный транспорт: классические и квантовые эффекты

1. Режимы экситонного транспорта
   - Прыжковый транспорт
   - Полуклассическое распространение
   - Слабая локализация экситонов
2. Нелинейный транспорт экситонов
   - Экситон-экситонные столкновения
   - Экситонные жидкости
3. Диффузия экситонов в море Ферми электронов

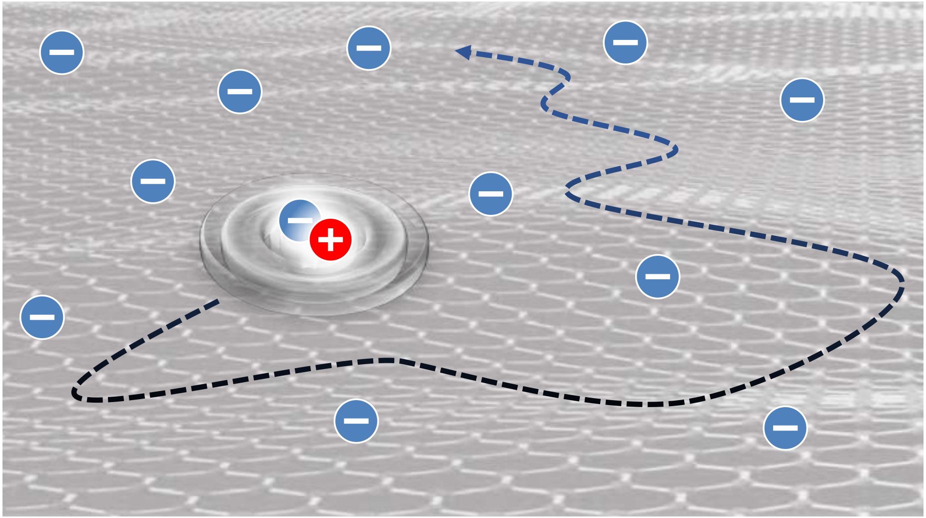
ЛЕТНЯЯ ШКОЛА ФОНДА «БАЗИС»

БАЗИС

Физический факультет МГУ имени М.В. Ломоносова

Exciton transport: what are we looking for?

Periodic crystal $\Rightarrow$ Bloch theorem $\psi_{nk} = e^{ikr} u_{nk}(\boldsymbol{r})$ $\Rightarrow$ free propagation



static disorder (defects) + phonons + charge carriers + other excitons

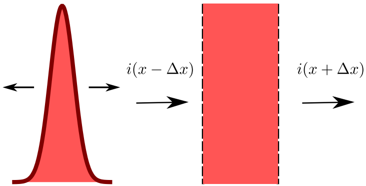
How can we solve this very complex problem?

Phenomenological approach: macroscopic (coarse grain) description

R. Brown (1827); A. Einstein (1905); M. Smoluchowski (1906); N. Wiener (1918)

- Continuity equation for exciton density $n(\boldsymbol{r}, t)$

$$\frac{\partial n(\boldsymbol{r}, t)}{\partial t} + \operatorname{div} \boldsymbol{i}(\boldsymbol{r}, t) = 0$$



- Ficks law for exciton flux $\boldsymbol{i}(\boldsymbol{r}, t)$

$$\boldsymbol{i}(\boldsymbol{r}, t) = -D \boldsymbol{\nabla} n(\boldsymbol{r}, t)$$

$D \geqslant 0$ is the diffusion coefficient

$\sim$ expansion in small gradient of density

Density should be smooth on the microscopic length and timescales

Diffusion equation allowing for the generation, G, and recombination, R

$$\frac{\partial n}{\partial t} - D \Delta n + R\{n\} = G$$

Linear diffusion equation

$$\frac{\partial n}{\partial t} - D\Delta n + \frac{n}{\tau} = G(\boldsymbol{r}, t)$$

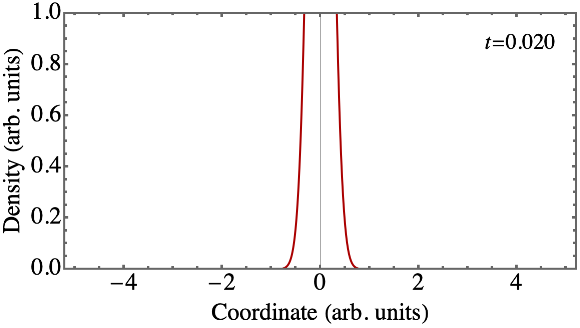
General solution in 2D:

$$n(\boldsymbol{r}, t) = \int_{-\infty}^{t} dt' \int d\boldsymbol{r}' g(\boldsymbol{r} - \boldsymbol{r}', t - t') e^{-\frac{t-t'}{\tau}} G(\boldsymbol{r}', t'), \quad g(\boldsymbol{r}, t) = \frac{1}{4\pi D t} \exp\left(-\frac{r^2}{4Dt}\right)$$

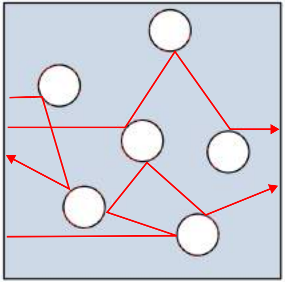
The mean squared displacement along x or y

$$\sigma^2(t) = \int d\boldsymbol{r} x^2 g(\boldsymbol{r}, t) = \int d\boldsymbol{r} y^2 g(\boldsymbol{r}, t) = 2Dt$$

displacement $\sigma \propto \sqrt{t}$



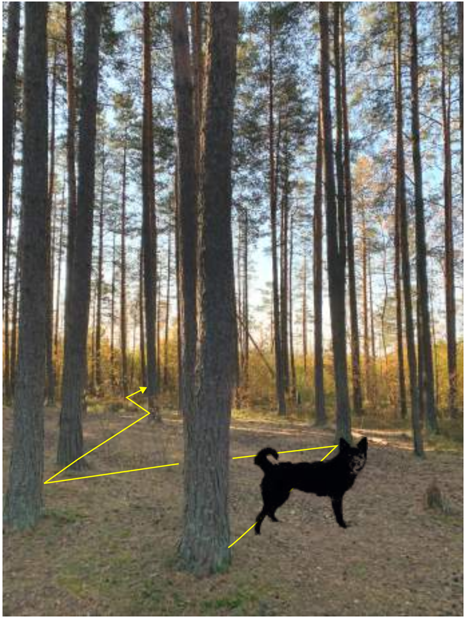
Diffusion and random walk: Free propagation interrupted by scattering



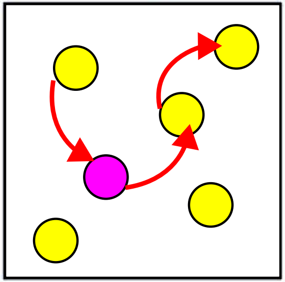
$$x^2(t) = \left(\sum_i \Delta x_i\right)^2 = \underbrace{\sum_i (\Delta x_i)^2}_{\sim \ell^2 n_{\rm steps}} + \underbrace{\sum_{i\neq j} \Delta x_i \Delta x_j}_{=0} = \ell^2 \frac{t}{\tau_p}$$

mean free path $\ell = v\tau_p$, scattering time τ_p, $n_{\rm steps} \approx t/\tau_p$

diffusion coefficient $D = \frac{v^2 \tau_p}{2}$



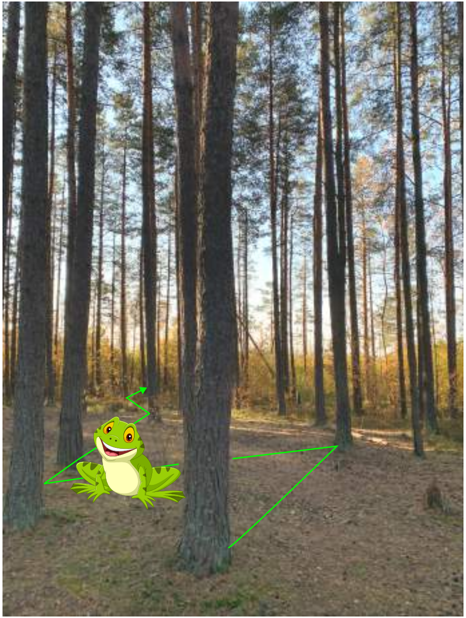
Diffusion and random walk: Hopping between localization sites



$$x^2(t) = \left(\sum_i \Delta x_i\right)^2 = \underbrace{\sum_i (\Delta x_i)^2}_{\sim d^2 n_{\text{steps}}} + \underbrace{\sum_{i\neq j} \Delta x_i \Delta x_j}_{=0} = d^2 \frac{t}{\tau_h}$$

hopping distance d, hopping time τ_h, $n_{\text{steps}} \approx t/\tau_h$

diffusion coefficient $D = \frac{d^2}{2\tau_h}$



Diffusion coefficient is determined by velocity correlations

General approach (valid in any transport regime):

$$x(t) = \int_0^t v_x(t_1)dt_1$$

We calculate the mean squared displacement

$$\langle x^2(t)\rangle = \left\langle \int_0^t v_x(t_1)dt_1 \int_0^t v_x(t_2)dt_2 \right\rangle = \int_0^t dt_1 \int_0^t dt_2 \langle v_x(t_1)v_x(t_2)\rangle$$

Velocity autocorrelation function

$$\langle v_x(t)v_x(t+\tau)\rangle = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} v_x(t)v_x(t+\tau)$$

depends on the absolute difference of times $\tau = |t_2 - t_1|$

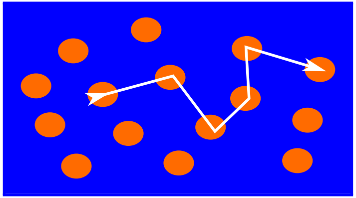
$$\langle x^2(t)\rangle = 2t\int_0^t \langle v_x(0)v_x(\tau)\rangle d\tau \xrightarrow[t\to\infty]{} 2Dt$$

diffusion cofficient $D = \int_0^\infty \langle v_x(0)v_x(\tau)\rangle d\tau$

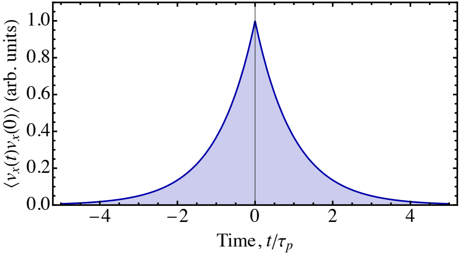
J. Johnson (1928); H. Nyquist (1928); M. Green (1954); R. Kubo (1957)

Velocity autocorrelation function: example

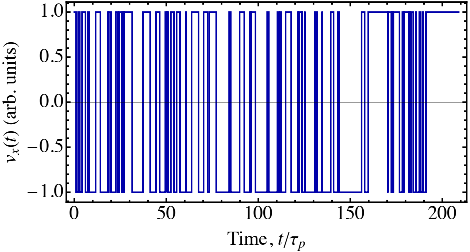
diffusion cofficient $D = \int_0^\infty \langle v_x(t) v_x(0) \rangle dt$



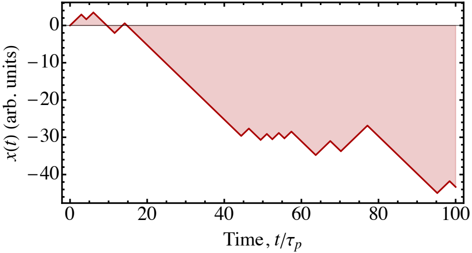
$$\langle v_x(t) v_x(0) \rangle \propto \exp(-|t|/\tau_p)$$



Velocity:



Coordinate:



Drift and diffusion: The Einstein relation

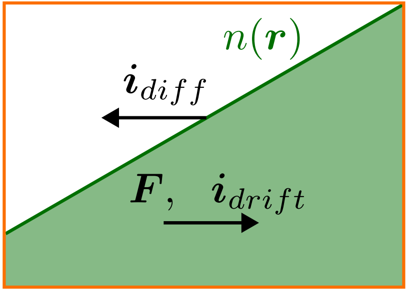
- Continuity equation for exciton density $n(\boldsymbol{r},t)$

$$\frac{\partial n(\boldsymbol{r},t)}{\partial t} + \operatorname{div} \boldsymbol{i}(\boldsymbol{r},t) = 0$$

- Ficks law for exciton flux $\boldsymbol{i}(\boldsymbol{r},t)$

$$\boldsymbol{i}(\boldsymbol{r},t) = -D\boldsymbol{\nabla} n(\boldsymbol{r},t)$$

$D \geqslant 0$ is the diffusion coefficient



Let an external force $\boldsymbol{F}$ create the drift exciton flux $\boldsymbol{i}$:

$$\boldsymbol{i}_{drift} = n\boldsymbol{v}_{dr} = n\mu\frac{\boldsymbol{F}}{|e|}, \quad \mu \text{ is the effective mobility}$$

In bounded system $\boldsymbol{i}_{tot} = \boldsymbol{i}_{drift} + \boldsymbol{i}_{diff} = 0$

the Einstein relation $\quad \mu k_B T = |e| D$

It is a simplest example of the fluctuation-dissipation theorem

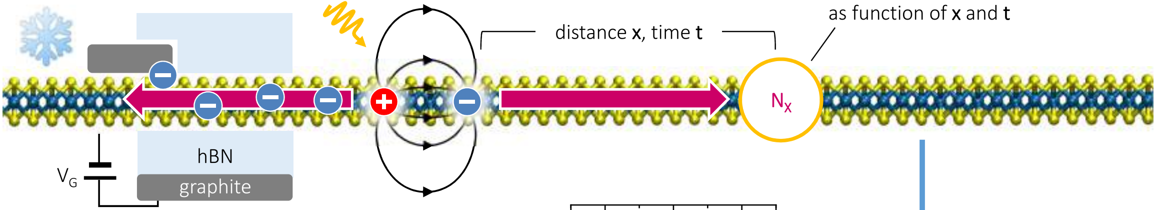
J. Johnson (1928); H. Nyquist (1928); H. Callen & T. Welton (1951); R. Kubo (1954)

How to measure exciton propagation?

5 K

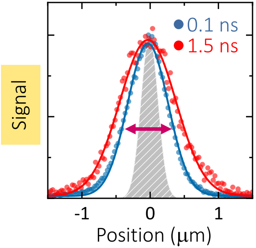
1. Create *local* exciton population

2. Monitor exciton density as function of **x** and **t**



❖ Changes of reflection, transmission, scattering

❖ Photoluminescence



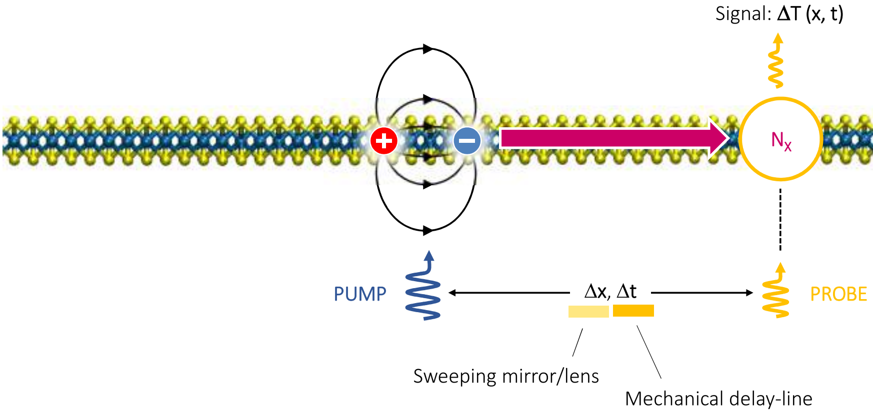
3. Analyze emission profile

4. Fit by a Gaussian

Review:

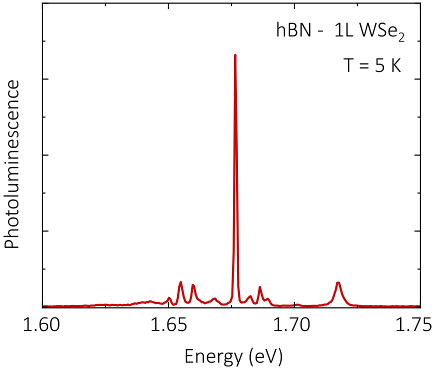
N. S. Ginsberg & W. A. Tisdale, *Annu. Rev. Phys. Chem.* 70 (2020)

(*differential*) absorption-based measurements

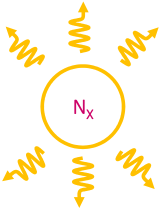


luminescence-based measurements

Detecting emitted light

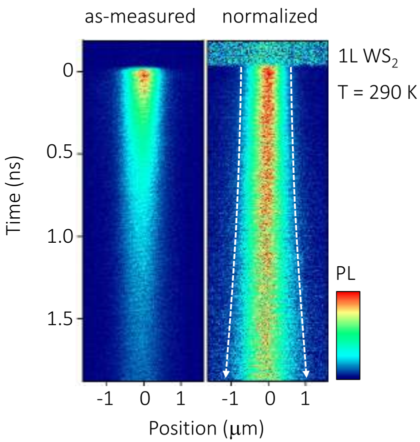


Signal: **PL (x, t)**

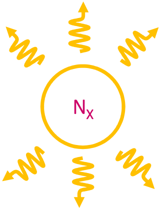


- Imaging of luminescence
- Time-resolved detection

luminescence-based measurements

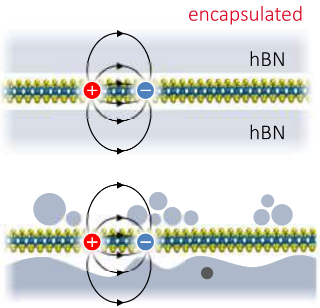


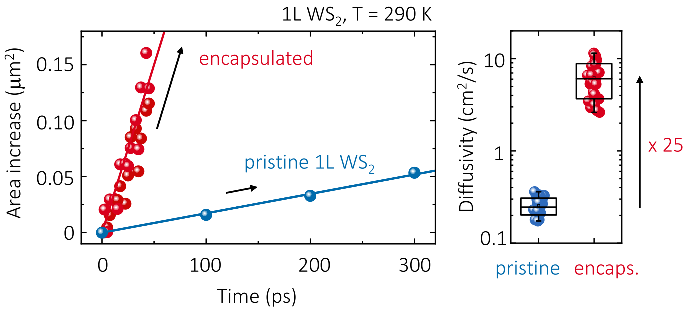
Signal: **PL (x, t)**



- Imaging of luminescence
- Time-resolved detection

Influence of environment / disorder





Dielectric disorder $\varepsilon(x) \neq const.$

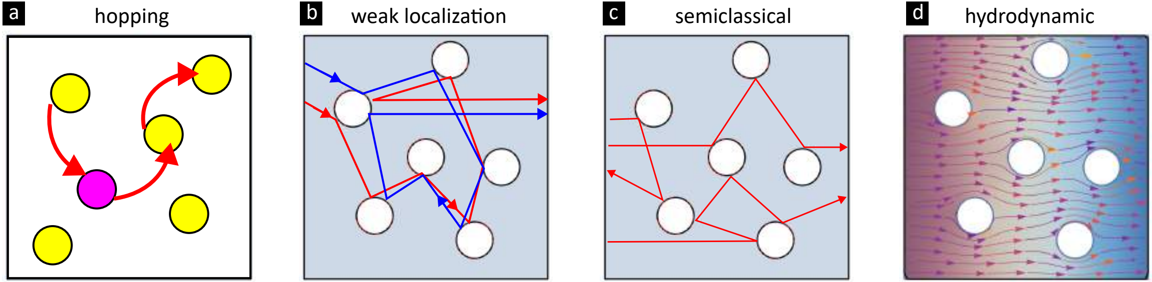
$\mu_{eff} = De/k_B T$

*assuming semiclassics $D = \dfrac{k_B T\, \tau_s}{M_X}$

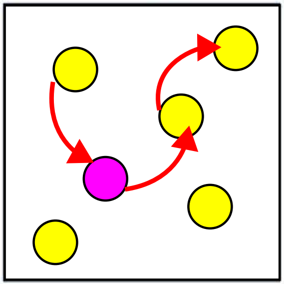
❖ Effective mobility up to 400 cm^2/Vs - 20 nm mean free path*, 100's nm diffusion length

A. Raja et al, *Nat. Nanotech.* 14, 832 (2019)

Regimes of exciton propagation



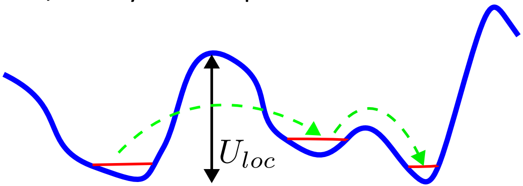
Exciton hopping between localization sites



Hopping is realized if excitons are localized in disorder potential

$$U_{loc} \gtrsim k_B T,$$

e.g., moire potentials in heterstructures and/or very low temperatures



hopping distance d, hopping time τ_h, $n_{\text{steps}} \approx t/\tau_h$:

diffusion coefficient $$D = \frac{d^2}{2\tau_h} \propto \exp\left[-\left(\frac{T_0}{T}\right)^s\right]$$

$s = 1/3$ for the variable range hopping in 2D

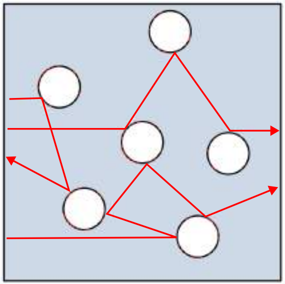
$$k_B T_0 \sim U_{loc}$$

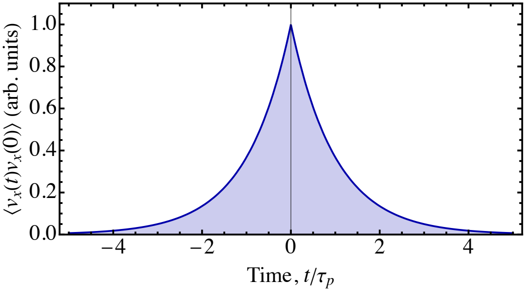
- Exponential temperature dependence $T \uparrow \Rightarrow D \uparrow$
- $\sigma(t)$ can be non-diffusive due to spread in d and τ_h

Hopping transport can be very complicated and very slow

Semiclassical propagation: basics

applicability: $k_BT \gg U_{loc} \quad \Leftrightarrow \quad \frac{k_BT\tau_p}{\hbar} \gg 1 \quad \Leftrightarrow \quad \ell_{\mathrm{mfp}} = v\tau_p \gg \lambda_{\text{de Broglie}} \sim \sqrt{\frac{\hbar^2}{Mk_BT}}$





$$D = \int_0^\infty \langle v_x(t)v_x(0)\rangle dt$$

$$\langle v_x(t)v_x(0)\rangle = \langle v_x^2(0)\rangle \exp(-|t|/\tau_p)$$

non-degenerate excitons $\left\langle \frac{Mv_x^2}{2} + \frac{Mv_y^2}{2} \right\rangle = k_BT \quad \Rightarrow \quad \langle v_x^2(0)\rangle = \frac{k_BT}{M}$

diffusion coefficient $D = \frac{k_BT}{M}\tau_p$

Kinetic equation: rigorous approach. Beyond coarse grain description

applicability: $k_B T \gg U_{loc} \quad \Leftrightarrow \quad \frac{k_B T \tau_p}{\hbar} \gg 1 \quad \Leftrightarrow \quad \ell_{\text{mfp}} = v\tau_p \gg \lambda_{\text{de Broglie}} \sim \sqrt{\frac{\hbar^2}{M k_B T}}$

Exciton distribution function $f_{m,\boldsymbol{k}}(t)$ obeys kinetic equation

$$\frac{\partial f_{m,\boldsymbol{k}}}{\partial t} + \underbrace{\boldsymbol{v}_{m,\boldsymbol{k}} \cdot \frac{\partial f_{m,\boldsymbol{k}}}{\partial \boldsymbol{r}}}_{\text{propagation}} + \underbrace{\frac{\boldsymbol{F}}{\hbar} \cdot \frac{\partial f_{m,\boldsymbol{k}}}{\partial \boldsymbol{k}}}_{\text{drift}} + \underbrace{\hat{Q}\{f_{m,\boldsymbol{k}}\}}_{\text{scattering}} + \underbrace{\hat{R}\{f_{m,\boldsymbol{k}}\}}_{\text{recombination}} = \underbrace{G_{m,\boldsymbol{k}}(\boldsymbol{r},t)}_{\text{generation}}$$

m enumerates bands, $\boldsymbol{k}$ is the wavevector.

Collision integral $\quad \hat{Q}\{f_{m,\boldsymbol{k}}\} = \sum_{m',\boldsymbol{k}'} W_{m,\boldsymbol{k};m',\boldsymbol{k}'} f_{m,\boldsymbol{k}} - W_{m',\boldsymbol{k}';m,\boldsymbol{k}} f_{m',\boldsymbol{k}'}$

Single band, effective mass model $E_k = \hbar^2 k^2/2M$, (quasi)elastic scattering:

diffusion coefficient $\quad D = \frac{k_B T}{M}\tau_p, \qquad \frac{1}{\tau_p} = \sum_{\boldsymbol{k}'} W_{\boldsymbol{k},\boldsymbol{k}'}(1 - \cos\vartheta_{\boldsymbol{k},\boldsymbol{k}'})$

Semiclassical regime works where excitons are well-defined quasiparticles/wavepackets

Landau & Lifshits, Kinetic theory; MMG, Phys. Rev. B **100***, 045426 (2019); Perea-Causin et al. Nano Lett.* **19***, 7317 (2019)*

Key scattering mechanisms

Single band, effective mass model $E_k = \hbar^2 k^2/2M$, (quasi)elastic scattering:

diffusion coefficient $$D = \frac{k_B T}{M}\tau_p, \quad \frac{1}{\tau_p} = \frac{2\pi}{\hbar}\sum_{\boldsymbol{k}'} |M_{\boldsymbol{k}'\boldsymbol{k}}|^2 \delta(E_k - E_{k'})(1-\cos\vartheta_{\boldsymbol{k},\boldsymbol{k}'})$$

Static defects

$$U(\boldsymbol{r}) = \sum_i V_0 \delta(\boldsymbol{r} - \boldsymbol{R}_i)$$

$$|M_{\boldsymbol{k}'\boldsymbol{k}}|^2 = V_0^2 n_d \quad \Rightarrow \quad \tau_p = const(T)$$

Diffusion coefficient increases with T:

$$D \propto T$$

Phonons (long wavelength acoustic)

$$U(\boldsymbol{r},t) = \sum_{\boldsymbol{q}} \sqrt{\frac{\hbar}{2\rho\Omega_q}} \mathrm{i}q\Xi \hat{b}_q e^{\mathrm{i}\boldsymbol{qr} - \mathrm{i}\Omega_q t} + \text{h.c.}$$

$$\tau_p^{-1} \propto T$$

Diffusion coefficient is T-independent:

$$D = const(T)$$

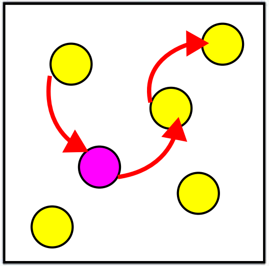
details

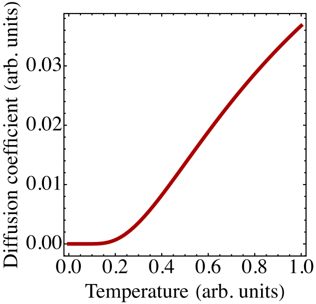
Selig et al., Nat. Communs **7***, 13279 (2016); S. Shree, et al., PRB* **98***, 035302 (2018); MMG, PRL* **124***, 166802 (2020); MMG et al., APL* **121***, 192106 (2022)*

These are simplest models, the reality is more complex

Summary of exciton propagation

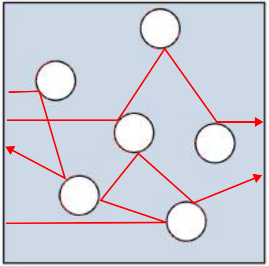
hopping

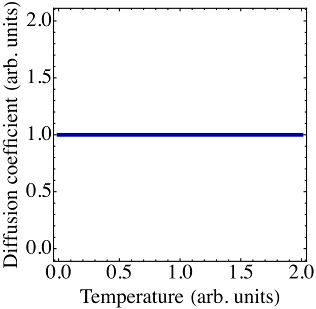




D strongly increases with increase in T

semiclassical

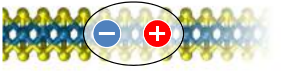


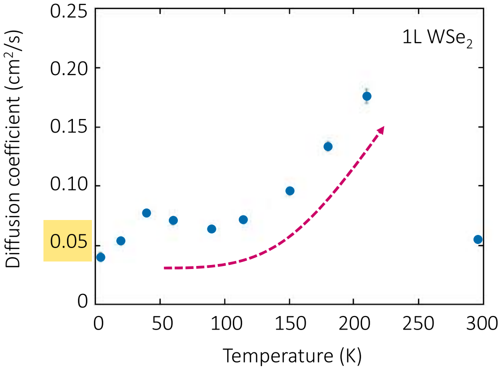


D weakly depends on T

Hopping

thermally activated diffusion



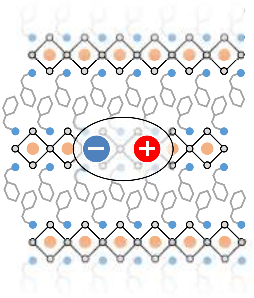


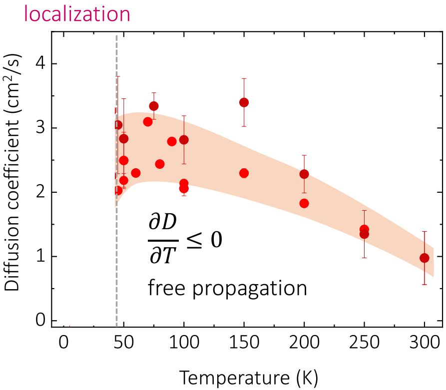
[P. Deotare lab]

Z. Li et al, *ACS Nano* 15, 1539 (2021)

Free propagation

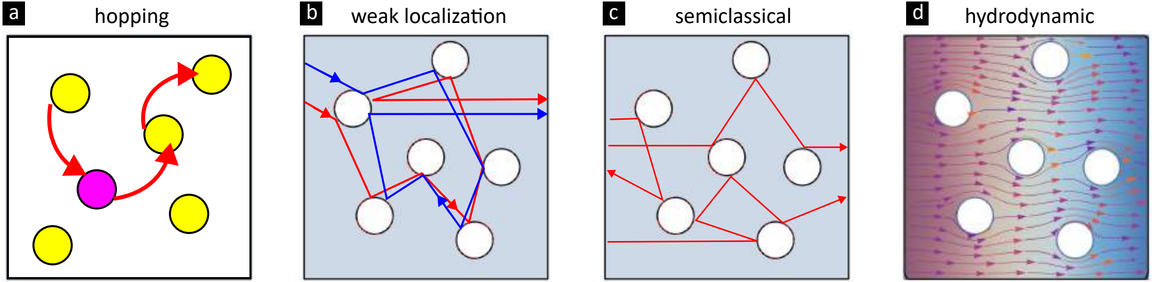
thermally "suppressed" diffusion





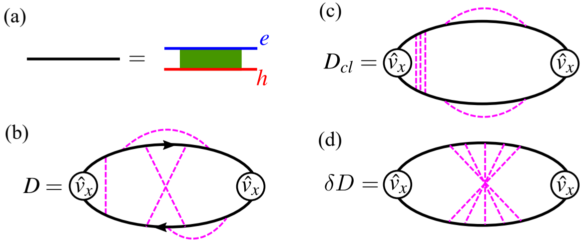
J. D. Ziegler et al, *Nano Lett.* 20, 6674 (2020)

Regimes of exciton propagation



General approach is to determine D from exciton propagator $D = \int_0^\infty \langle\langle \hat{v}_x(t)\hat{v}_x(0)\rangle\rangle\, dt$

$$D = \frac{\hbar \sum_{\boldsymbol{k}} \int_{-\infty}^{\infty} \frac{d\varepsilon}{2\pi}\, \mathrm{Tr}\{\hat{n}_{\boldsymbol{k}} \langle \hat{v}_x \hat{\mathcal{G}}^R_{\boldsymbol{k}}(\varepsilon)\hat{v}_x \hat{\mathcal{G}}^A_{\boldsymbol{k}}(\varepsilon)\rangle\}}{\sum_{\boldsymbol{k}} \mathrm{Tr}\{\hat{n}_{\boldsymbol{k}}\}}$$



It is impossible to calculate and sum up all diagrams. In some cases, only specific ones contribute.

- Ladder (c) at the semiclassical regime

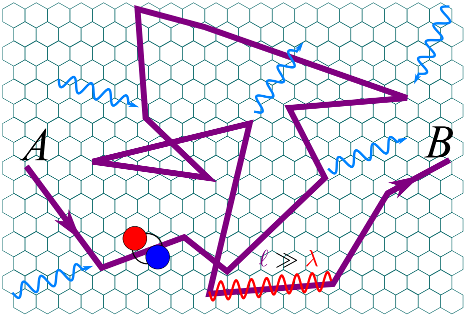
Do we need to go beyond semiclassics?

- Check diagram (d)!

Quantum interference of pathways

$$D = \int_0^\infty \langle\langle \hat{v}_x(t)\hat{v}_x(0)\rangle\rangle \, dt$$

Non-degenerate excitons: $\ell \gg \lambda \quad \Rightarrow \quad \frac{k_B T \tau}{\hbar} \gg 1$



semiclassical diffusion coefficient from

$$\langle v_x(t)v_x(0)\rangle = v_x^2(0)e^{-t/\tau}, \quad \frac{1}{\tau} = \sum_{\boldsymbol{k}'} W_{\boldsymbol{k}\boldsymbol{k}'}(1 - \cos\vartheta)$$

$$D = \left\langle \frac{v^2\tau}{2} \right\rangle = \frac{k_B T \tau}{M} \gg \frac{\hbar}{M} \sim 1\ \mathrm{cm}^2/s,$$

LA-phonon scattering in MX_2 MLs

$$\tau = \frac{Ms^2}{k_B T}\tau_0, \quad \tau_0^{-1} = \frac{M^2(\Xi_c - \Xi_v)^2}{\rho\hbar^3} \quad \Rightarrow \quad D = s^2\tau_0 \sim 1\ldots3\ \mathrm{cm}^2/\mathrm{s} \sim \frac{\hbar}{M}$$

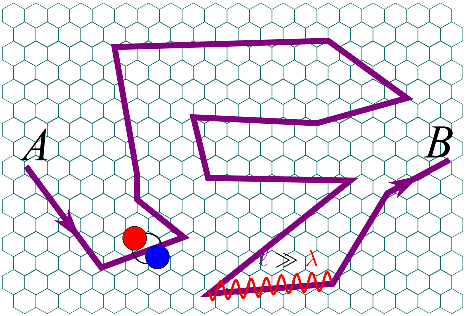
Here D is temperature independent; experiment at 4 K gives similar values (~ 2.5 cm^2/s).

Nonclassical effects should play a role

Quantum interference of pathways

$$D = \int_0^\infty \langle\langle \hat{v}_x(t)\hat{v}_x(0)\rangle\rangle \, dt$$

Non-degenerate excitons: $\ell \gg \lambda \quad \Rightarrow \quad \frac{k_B T\tau}{\hbar} \gg 1$



semiclassical diffusion coefficient from

$$\langle v_x(t)v_x(0)\rangle = v_x^2(0)e^{-t/\tau}, \quad \frac{1}{\tau} = \sum_{k'} W_{kk'}(1-\cos\vartheta)$$

$$D = \left\langle \frac{v^2\tau}{2} \right\rangle = \frac{k_B T\tau}{M} \gg \frac{\hbar}{M} \sim 1\,\mathrm{cm}^2/s,$$

Quantum mechanics: different trajectories should interfere

$$P(A\to B) = \left|\sum_i \mathcal{A}_i\right|^2, \quad \mathcal{A}_i = |\mathcal{A}_i| \exp(i\phi_i)$$

$$\phi_i = \int_i \boldsymbol{k}\cdot d\boldsymbol{l}, \quad |\phi_i - \phi_j| \gtrsim k\ell \sim \frac{\ell}{\lambda} \gg 1 \quad \Rightarrow \quad P(A\to B) = \sum_i P_i \;\; (?)$$

Quantum interference of pathways

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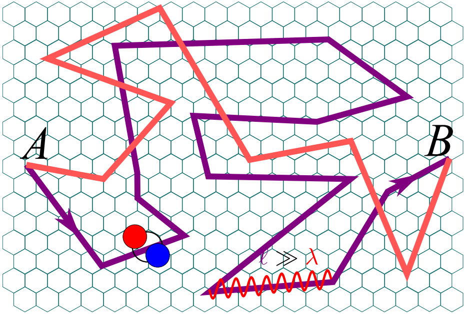

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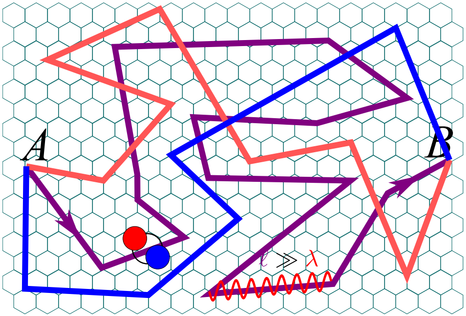

semiclassical diffusion coefficient from

$$\langle v_x(t) v_x(0)\rangle = v_x^2(0) e^{-t/\tau}, \quad \frac{1}{\tau} = \sum_{\mathbf{k}'} W_{\mathbf{k}\mathbf{k}'}(1 - \cos\vartheta)$$

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Quantum interference of pathways

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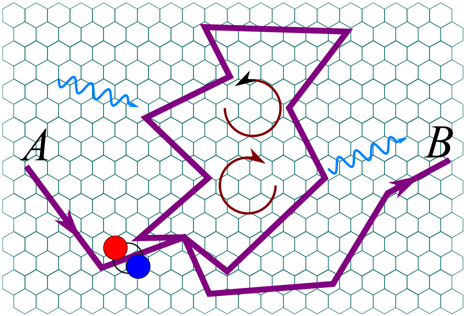

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$$D = \left\langle \frac{v^2 \tau}{2} \right\rangle = \frac{k_B T \tau}{M} \gg \frac{\hbar}{M} \sim 1\,\mathrm{cm}^2/s,$$

Self-intersecting trajectory: $\phi_{\circlearrowright} = \oint \boldsymbol{k} d\boldsymbol{l} = \oint (-\boldsymbol{k}) d(-\boldsymbol{l}) = \phi_{\circlearrowleft}$

Constructive interference at perfectly elastic scattering $\Rightarrow$ localization

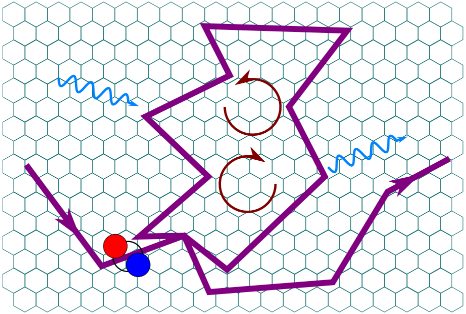
How well do the phases match in the case of the exciton-phonon scattering?

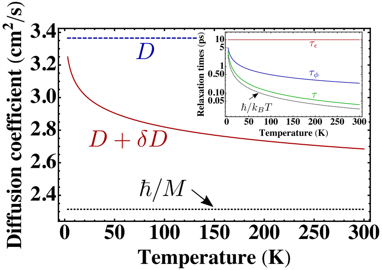
Non-trivial temperature dependence of the interference contribution

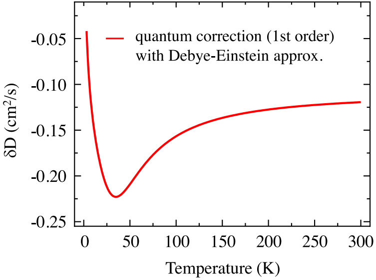
bulk crystals: *Ivchenko, Pikus, Razbirin, Starukhin (1977); Golubentsev (1984); Afonin, Galperin, Gurevich (1985)*

Acoustic phonon scattering is the main exciton scattering mechanism in MX_2 MLs at $1\text{ K} \lesssim T \lesssim 50\text{ K}$, $\tau \propto T^{-1}$:

$$D_{cl} = \frac{k_B T}{M}\tau = const(T)$$







$$\delta D = -\frac{\hbar}{2\pi M}\ln\left(\frac{\tau_\phi}{\tau}\right), \quad \frac{\hbar}{\tau_\phi} = \Delta\varepsilon(\tau_\phi), \quad \frac{\tau_\phi}{\tau} \propto T^{1/3}$$

details

Phys. Rev. Lett. **124**, *166802 (2020); Appl. Phys. Lett.* **121**, *192106 (2022)*

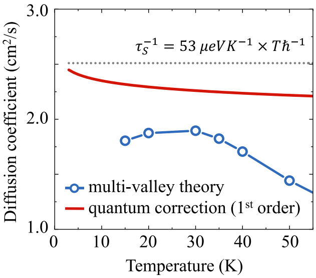
How to disentangle the $\tau(T)$ and weak localization? WSe_2

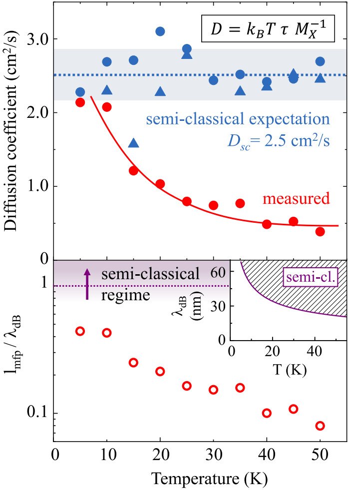
PRL **127**, 076801 (2021)

Semiclassical approach $D = \left\langle \frac{v^2 \tau}{2} \right\rangle, \quad \frac{1}{\tau} = \frac{2}{\hbar} \operatorname{Im} \Sigma(E^x_{\boldsymbol{k}}, \boldsymbol{k})$

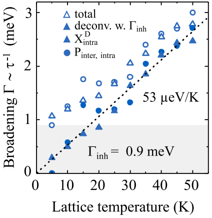
Scattering contribution to the linewidth $\Gamma = \frac{2}{\hbar} \operatorname{Im} \Sigma(0,0) \left(= \frac{1}{\tau} \right)$

Theoretical expectations





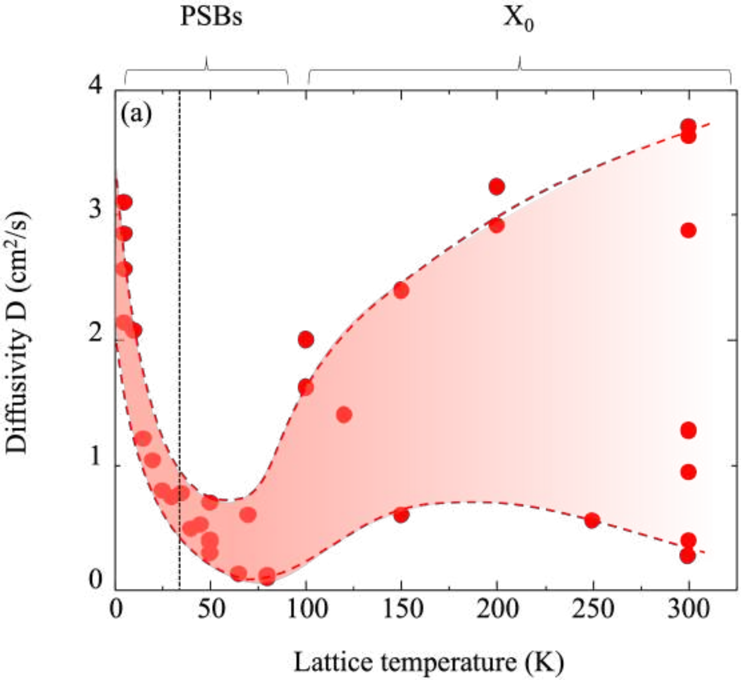
T-dependent linewidth



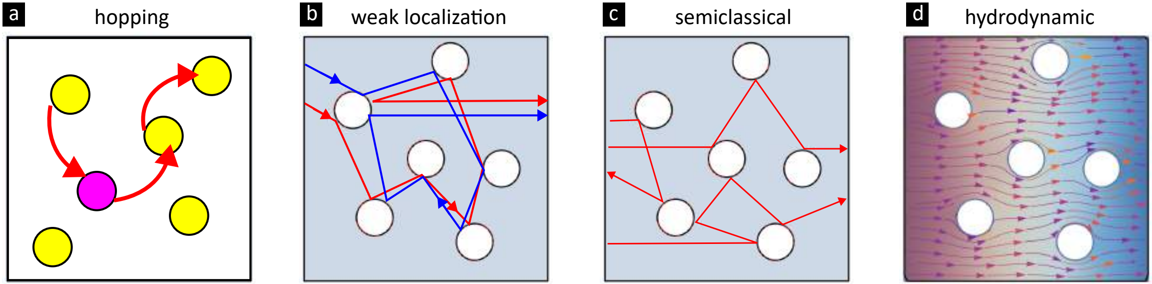
consistent
with acoustic phonon
scattering

Temperature dependence of D in WSe_2

by courtesy of Alexey Chernikov



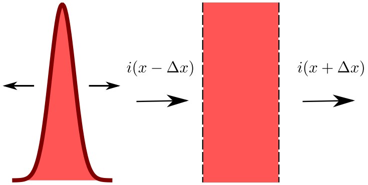
Regimes of exciton propagation



What happens at elevated exciton densities and more extreme conditions?

- Continuity equation for exciton density $n(\boldsymbol{r},t)$

$$\frac{\partial n(\boldsymbol{r},t)}{\partial t} + \operatorname{div} \boldsymbol{i}(\boldsymbol{r},t) + R\{n\} = 0$$



- Ficks law for exciton flux $\boldsymbol{i}(\boldsymbol{r},t)$

$$\boldsymbol{i}(\boldsymbol{r},t) = -D\boldsymbol{\nabla} n(\boldsymbol{r},t) + \frac{D}{k_B T} n\boldsymbol{F}, \quad \boldsymbol{F} = -\boldsymbol{\nabla} V$$

Nonlinearities

- Nonlinear recombination (Auger-like exciton-exciton annihilation): $R\{n\} = \frac{n}{\tau} + R_A n^2$
- Exciton-exciton repulsion: $\boldsymbol{F} = -U_0 \boldsymbol{\nabla} n$
- Temperature gradients (feeback from EEA): $\boldsymbol{F} \propto -\boldsymbol{\nabla} T$
- Screening, instabilities, etc., ...$D(n)$

XX interactions

Nonlinearities cause an increase of the **effective** diffusion coefficient

Nonlinear diffusion equation

$$\frac{\partial n}{\partial t} + \frac{n}{\tau} + R_A n^2 = D\Delta n + \frac{U_0 D}{k_B T}\boldsymbol{\nabla}\cdot(n\boldsymbol{\nabla} n) + \frac{D}{k_B T}\boldsymbol{\nabla}\cdot(n\boldsymbol{\nabla} V)$$

Gaussian initial condition

$$n(\boldsymbol{r},0) = \frac{N_0}{\pi r_0^2} e^{-r^2/r_0^2}$$

Effective diffusion coefficient

$$D_{\text{eff}} = -\frac{\pi r_0^4}{4N_0}\left[N_0 \frac{\partial}{\partial t}\frac{n}{N_0}\right]_{\boldsymbol{r}=0,t=0} = D + R_A \frac{N_0}{8\pi} + \frac{U_0 D}{k_B T}\frac{N_0}{\pi r_0^2} + \frac{D}{k_B T}\frac{r_0^2}{4}\Delta V\Big|_{\boldsymbol{r}=0}$$

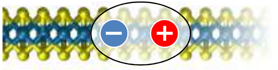
Note that D is only weakly renormalized by XX scattering due to momentum conservation

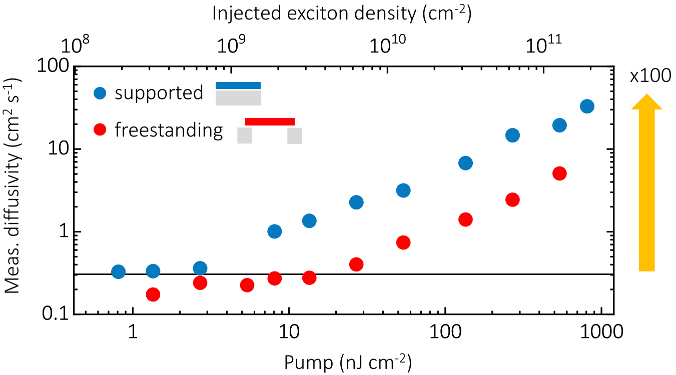
Kulig et al., Phys. Rev. Lett. ***120****, 207401 (2018); deQuilettes, et al. ACS Photonics* ***9****, 110 (2022); Wietek et al., Phys. Rev. Lett.* ***132****, 016202 (2024)*

Non-linear diffusion

1L WS_2

T = 290 K

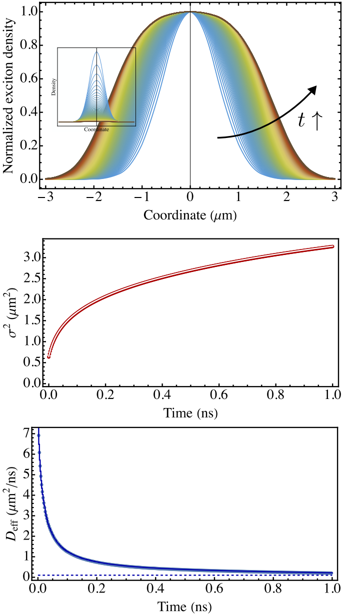




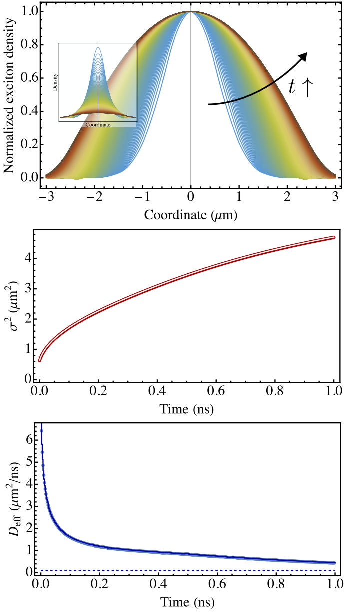
M. Kulig et al., *Phys. Rev. Lett.* 120, 207401 (2018)

Nonlinearities modify exciton density profile yielding $D_{\text{eff}} \neq D$

EEA enhanced diffusion



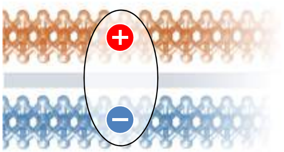
Repulsion enhanced diffusion

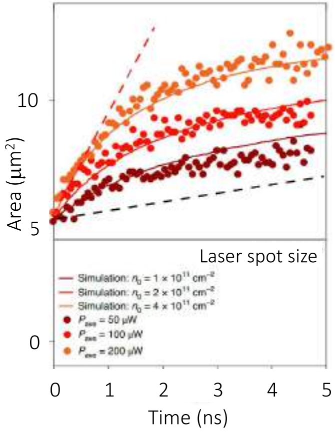


Non-linear diffusion in TMDC heterobilayers

$MoSe_2$/hBN/WSe_2

T = 5 K





Z. Sun et al., *Nat. Photon.* 16, 79 (2022)

[A. Kis lab]

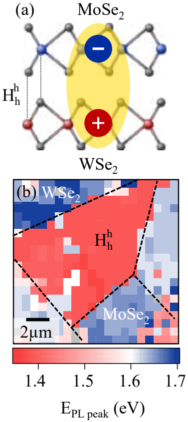
Density dependence

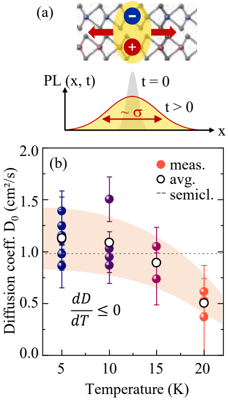
Subdiffusive behavior

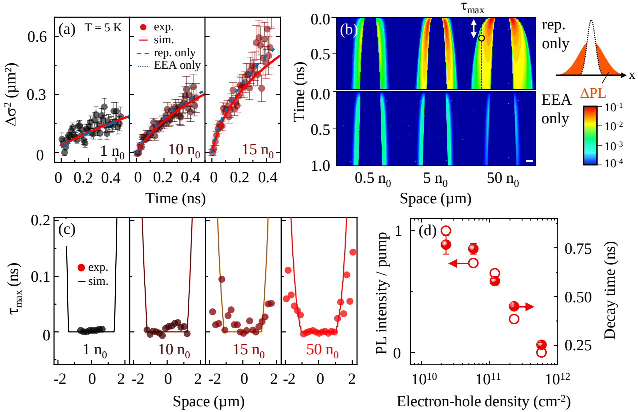
Enhancement of exciton diffusion by interactions: repulsion in BLs

Interactions and Auger recombination

$$\frac{\partial n}{\partial t} + \frac{n}{\tau} + R_A n^2 = D\Delta n + \frac{U_0 D}{k_B T}\nabla \cdot (n \nabla n)$$



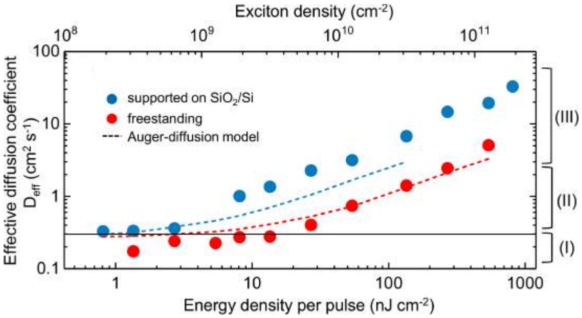


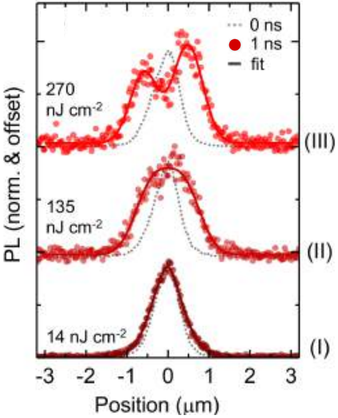


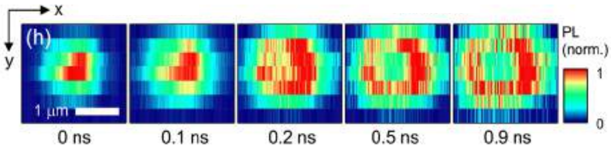
Two contributions to $D_{\mathrm{eff}} \propto N$ are disentangled: (i) Auger effect and (ii) exciton repulsion

Phys. Rev. Lett. **132**, *016202 (2024)*

Halo formation at high excitonic densities as a result of Auger effect

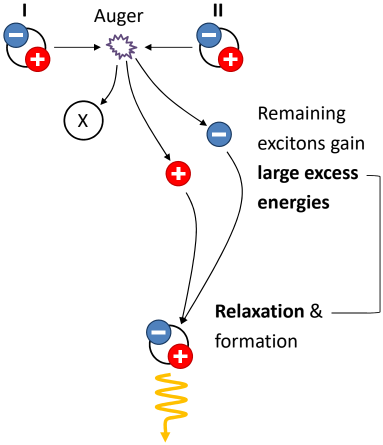






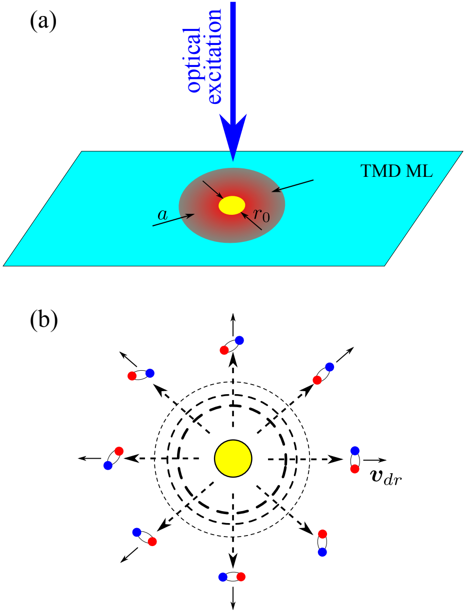
Phys. Rev. Lett. **120**, *207401 (2018); Phys. Rev. B* **101**, *115430 (2020)*

Heating via Auger scattering is the key nonlinear feedback effect



Hot spot: non-equilibrium phonons

bulk semiconductors: *Keldysh (1976); Zinov'ev, Ivanov, Kozub, Yaroshetskii (1983); Bulatov, Tikhodeev (1992)*



- Efficient Auger recombination
- Large energy release
- Excitation of non-equilibrium phonons

Phonons propagate out of the hot spot and drag excitons ⇒ halo-like pattern is formed

Drift-diffusion model

$$\frac{\partial n}{\partial t} + \boldsymbol{\nabla} \cdot \boldsymbol{j} + \frac{n}{\tau} + R_A n^2 = 0,$$

$$\boldsymbol{j} = -D\boldsymbol{\nabla} n + \frac{\tau_p}{m}\boldsymbol{F}(\boldsymbol{\rho})n$$

$$\boldsymbol{F} = \boldsymbol{F}_{\text{phonons}} + \boldsymbol{F}_{\text{Seebeck}}$$

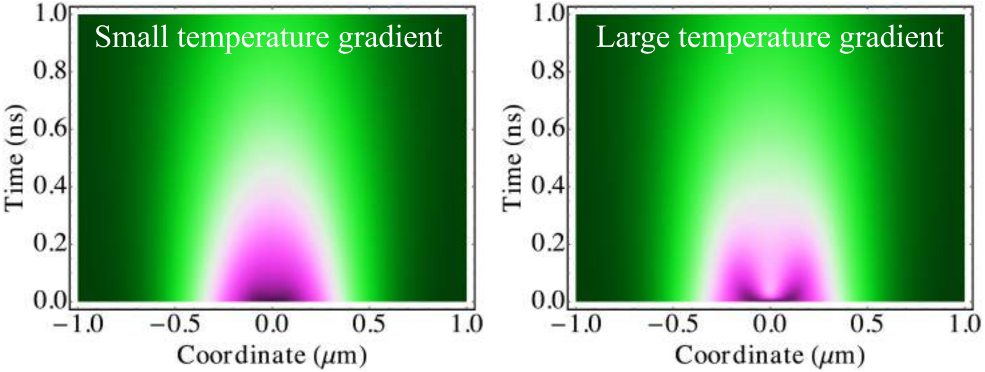
MMG, Phys. Rev. B **100**, *045426 (2019); Perea-Causin et al., Nano Lett.* **19**, *7317 (2019)*

High temperatures, diffusive phonons

Temperature gradients of lattice and of excitons are formed

$$\boldsymbol{F}_{\text{drag}} = -\frac{\tau_p}{\tau_x} k_B \boldsymbol{\nabla} T_{\text{latt}}, \quad \boldsymbol{F}_{\text{Seebeck}} = \frac{\mu}{k_B T} k_B \boldsymbol{\nabla} T_{\text{exc}}$$

Phonon drag scenario:



MMG, Phys. Rev. B **100**, *045426 (2019)*; **for Seebeck effect:** *Perea-Causin et al. Nano Lett.* **19**, *7317 (2019)*

Phonon wind vs. phonon drag

Phonon wind

Phonons propagate ballistically

$$F_{\text{wind}}(\boldsymbol{\rho}) = \frac{U}{\rho}\frac{\boldsymbol{\rho}}{\rho}$$

These bluestripe snapper (кашмирский луциан) are schooling (стая).

They are all swimming in the same direction in a coordinated way.

details

Phonon drag

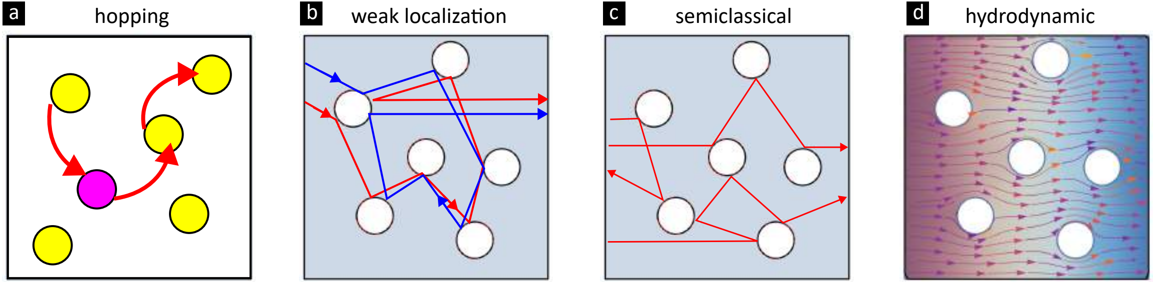
Phonons propagate diffusively

$$F_{\text{drag}}(\boldsymbol{\rho}) = \boldsymbol{\nabla}_{\rho}\frac{\Theta}{4\pi\varkappa t}\exp\left(-\frac{\rho^2}{4\varkappa t}\right)$$

These surgeonfish (рыба-хирург) are shoaling (скопление). They are swimming somewhat independently, but in such a way that they stay connected.

en.wikipedia.org

Regimes of exciton propagation



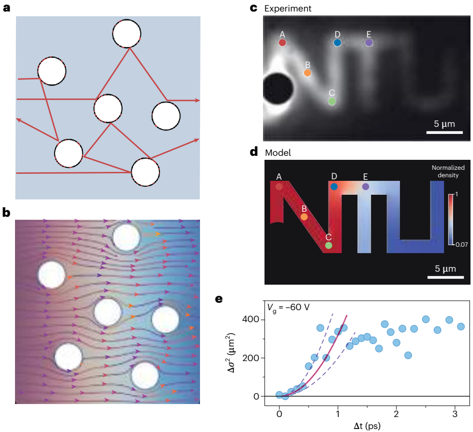
Ultrafast exciton propagation: experimental motivation

Usually exciton transport in 2D semiconductors is diffusive (large mass, efficient scattering)

Exciton diffusion in 2D van der Waals semiconductors, in 2D Excitonic Materials and Devices, ed. by P.B. Deotare and Z. Mi, Elsevier (2023).

Strong exciton-exciton interactions $\tau_{xx} \ll \tau_{x-ph}, \ldots \quad \Rightarrow \quad$ collective fluid-like behavior

del Águila, et al., Nat. Nano. (2023); MMG, Nat. Nano. (2023) + Butov's group experiments on TMDC (2021-23) + some other works

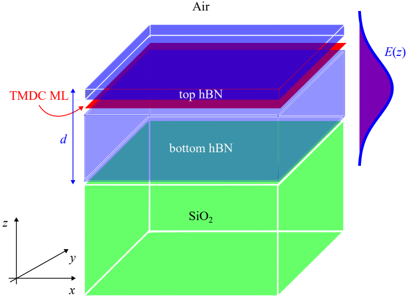


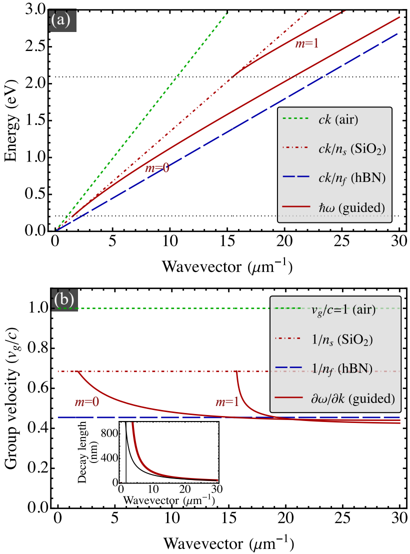
Ultimate fate at $n \gg M k_B T/\hbar^2$ is superfluidity, but $v_{\rm exp} \approx 0.07c$ is too high …

Gergel, Kazarinov, Suris, JETP ***27****, 159 (1968); Lozovik, Yudson, JETP Letters.* ***22****, 274 (1975); Fogler et al., Nat. Communs.* ***5****, 4555 (2014)*

Waveguide modes in hBN-based heterostructures

with R.A. Suris





Propagation process

incident photon

$\rightarrow$ scattering (roughnesses, excitons)

$\rightarrow$ photon in a waveguide mode

$\rightarrow$ exciton/scattering

$\rightarrow$ secondary photon

Wavguide modes with the group velocity $\sim 0.5c$ can be formed in hBN layers

In some cases, ultrafast propagation may be related with photon transport via these modes

cf. weak/strong coupling crossover in transport: *Osipov, Iorsh, Yulin, Shelykh, Phys. Rev. B* **108**, *104202 (2023)*

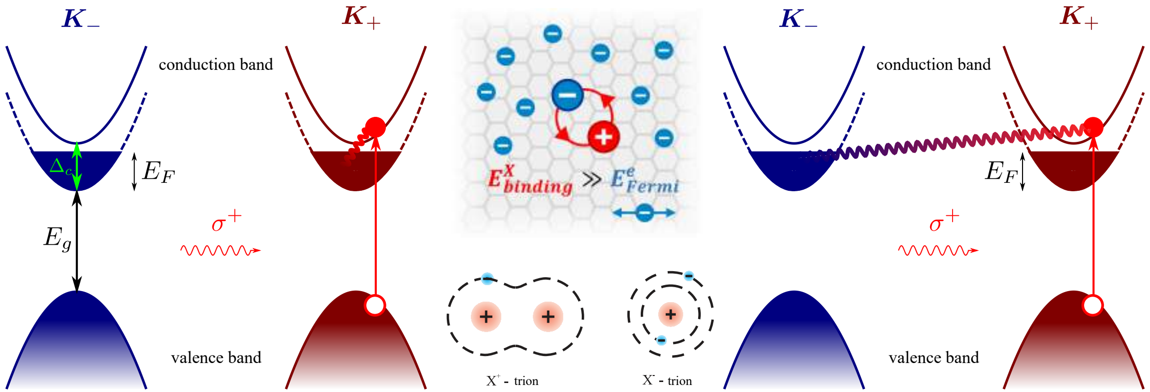
arXiv:2403.19571; JETP **166**, *20 (2024 (2024)*

Exciton floating in the Fermi sea of electrons: artistic view

Image #32073262 at VectorStock.com

Bose-Fermi mixtures of excitons and carriers: Trions & Fermi polarons

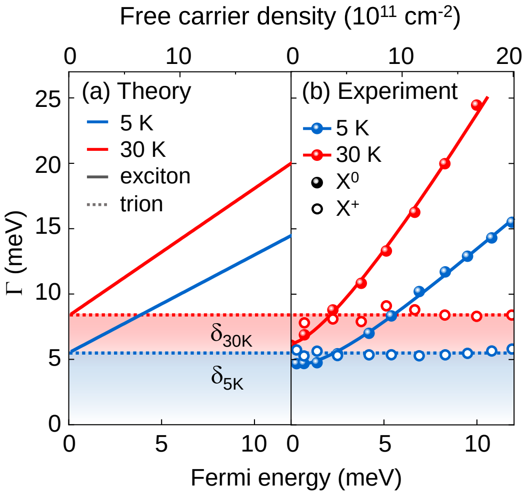
Exciton interacts with resident charge carriers



What are the diffusion mechanisms in Bose-Fermi mixtures?

details

Linewidths: experiment



Attractive polaron/trion

$$\Gamma_T = \delta$$

Repulsive polaron/exciton

$$\Gamma_X = \delta + E_F \frac{M_T}{M_X} \frac{\pi}{\ln^2[\delta/(2E_{b,T})] + \pi^2/4}$$

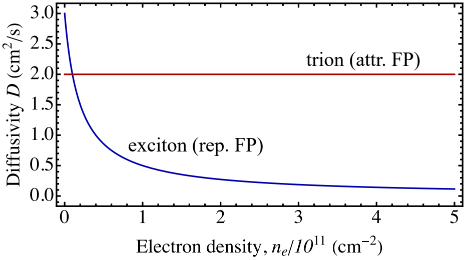
Attractive polaron (trion) linewidth is practically independent of free carrier density

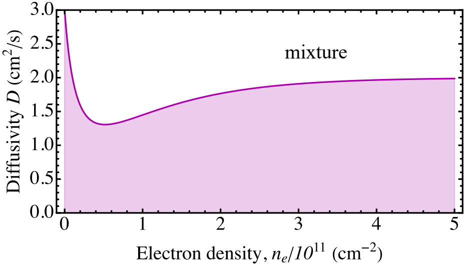
Repulsive polaron (exciton) linewidth increases with increasing the density

The increase rate depends on the temperature via $\delta(T)$

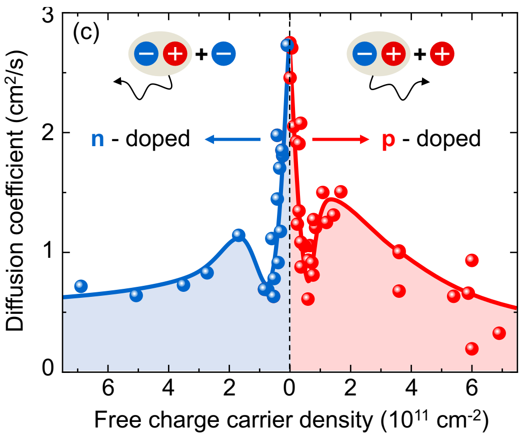
Nano Lett. **23**, *4708 (2023)*

Transport in Bose-Fermi mixtures: prediction & observation





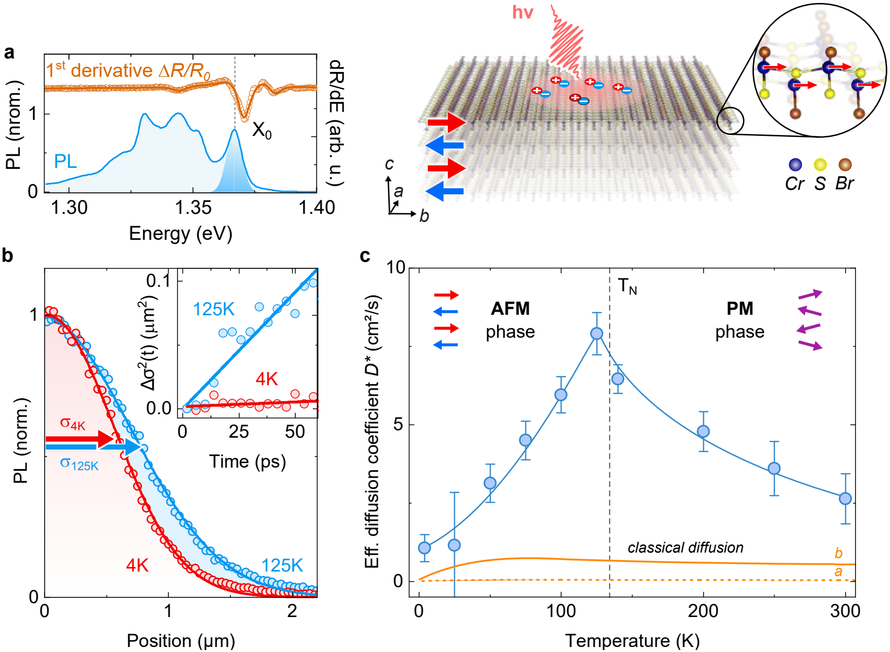
semiclassical model $D_{cl} = \frac{k_B T}{M}\tau_p, \quad \tau_p = \frac{\hbar}{\Gamma}$



Transition between the $X - e$ scattering to the trion (FP) formation with increasing density

Nano Lett. **23**, *4708 (2023)*

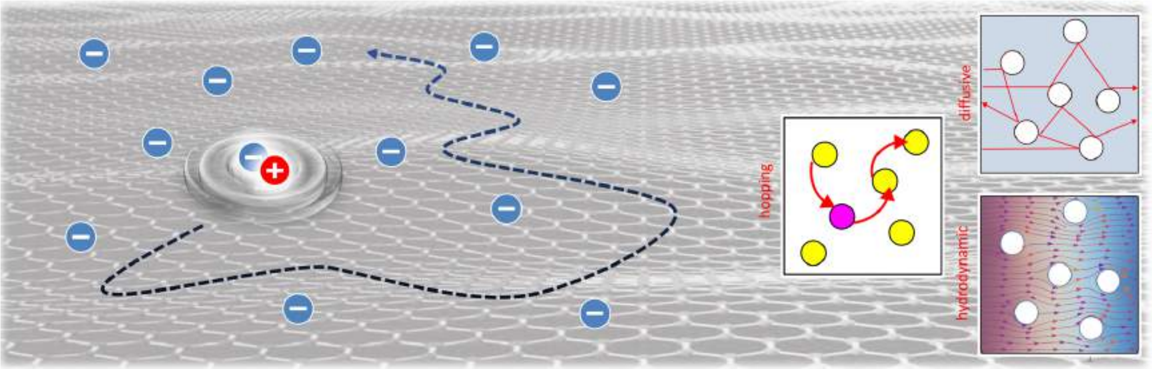
Exciton and magnon transport in CrSBr – van der Waals antiferromagnet



Ioffe + TU-Dresden + UAM Madrid + UCT Prague

F. Dirnberger, S. Terres, Z. A. Iakovlev, K. Mosina, Z. Sofer, A. Kamra, MMG, and A. Chernikov (2024)

Regimes of exciton propagation



weak (free propagation) $\Leftarrow$ **Disorder strength** $\Rightarrow$ strong (localization, hopping)

XX-interaction strength $\Rightarrow$ enhanced diffusivity $\Rightarrow$ hydrodynamics (superfluidity)

XE-interaction strength $\Rightarrow$ polaron formation

Итоги лекции 2

1. Режимы экситонного транспорта
   - Прыжковый транспорт
   - Полуклассическое распространение
   - Слабая локализация экситонов
2. Нелинейный транспорт экситонов
   - Экситон-экситонные столкновения
   - Экситонные жидкости
3. Диффузия экситонов в море Ферми электронов

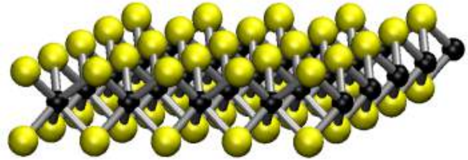
Открытые вопросы

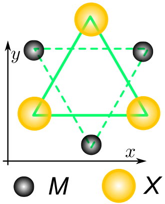
- Слабая локализация: расхождение теории и эксперимента
- Фотопроводимость: дрейф трионов/ферми-поляронов
- Гидродинамика экситонов и сверхбыстрый транспорт
- Новые системы – магнитный CrSBr:

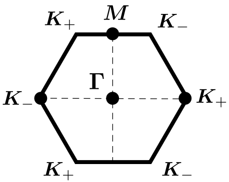
  взаимное увлечение экситонов и магнонов

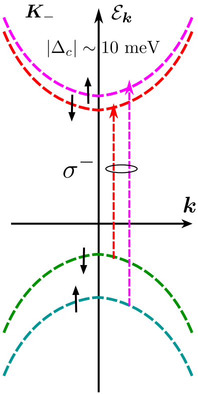
For review see: *Exciton diffusion in 2D van der Waals semiconductors* by Alexey Chernikov and Mikhail M. Glazov in *2D Excitonic Materials and Devices* (chap. 3, pp. 69-110), ed. by P.B. Deotare and Z. Mi Elsevier (2023)

Двумерные материалы – экситонные эффекты

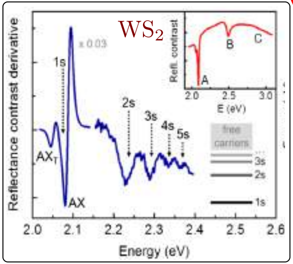






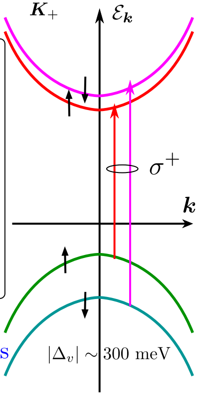


Excitons:



$E_{2B} \approx 200 \ldots 500$ meV

$a_B \approx 10 \ldots 30$ Å, $\tau_X \sim 1$ ps



- Прямозонные полупроводники $E_g \approx 2$ эВ
- Две долины K_+ и K_-
- Спин-орбитальное взаимодействие
- Киральные оптические правила отбора
- Кулоновские эффекты: экситоны

- Платформа для ван-дер-ваальсовых гетероструктур *Geim, Grigorieva (2013)*
- Транзисторы *Radisavljevic, …, Kis (2011)*
- Лазеры и однофотонные источники *Wu, …, Xu (2015); Koperski, …, Potemski (2015)*
- Сочетание необычных оптических и транспортных свойств

Коллеги и соавторы

- ФТИ им. А.Ф. Иоффе: З.А. Яковлев, М.В. Дурнев, Л.Е. Голуб, М.А. Семина, Е.Л. Ивченко
- МФТИ: А.М. Шенцев, К.К. Григорян, Д.С. Зограбян, Д.С. Эскоскин
- ФИАН им. П.Н. Лебедева: В.В. Белых, М.В. Кочиев
- Северо-восточный федеральный университет: П.В. Винокуров, С.А. Смагулова
- TU-Dresden/Uni-Regensburg: A. Chernikov, K. Lin, K. Wagner, E. Wietek, J. Ziegler
- Phillips-Marburg Uni./U. Chalmers: R. Rosati, R. Perea-Causín, E. Malic
- INSA-Toulouse: G. Wang, S. Shree, C. Robert, X. Marie, B. Urbaszek, T. Amand
- IOP Bejing: C.R. Zhu, B. Liu
- Weizmann Institute of Science: Sivan Refaely-Abramson

Экситоны в двумерных материалах

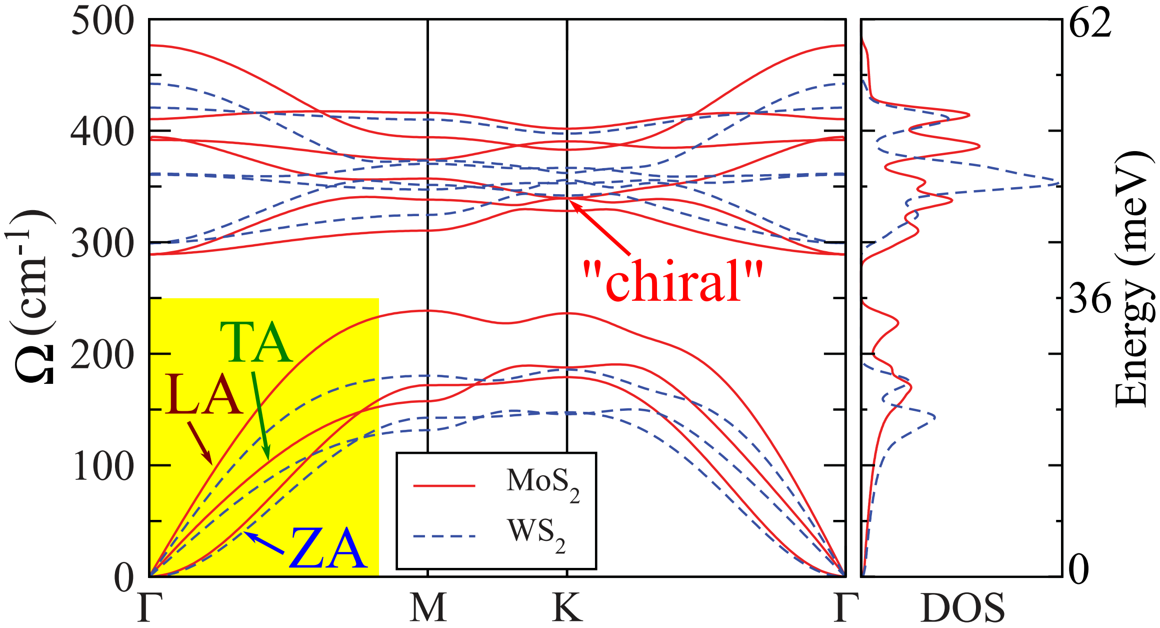
М.М. Глазов
ФТИ им. А.Ф. Иоффе, Санкт-Петербург

1. Двумерные дихалькогениды переходных металлов
2. Теория экситонов Ванье-Мотта
3. Особенности кулоновского взаимодействия и экситонной серии в 2D
4. Тонкая структура экситонных состояний
5. Взаимодействие экситонов и электронов: трионы и ферми-поляроны
6. Пара слов о том, как экситоны взаимодействуют друг с другом
7. Экситоны, фононы и упругие деформации
8. Экситонный транспорт: классические и квантовые эффекты

За кадром:

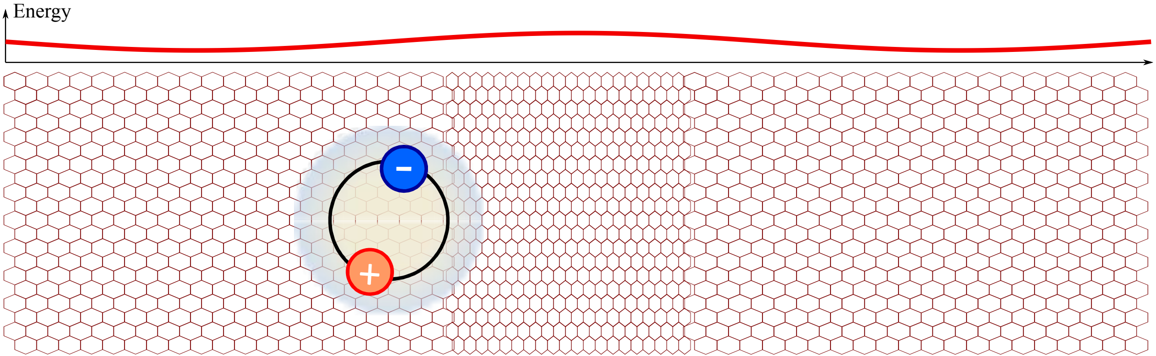
гетероструктуры, муар, коррелированные электронные и экситонные фазы, магнетизм

Vibration spectrum



Molina-Sánchez, Wirtz (2011)

Deformation potential



Lattice deformation ⇒ shift of the electron and hole energies ⇒ variation of exciton energy

$$\Delta E_x = (\Xi_c - \Xi_v)(\epsilon_{xx} + \epsilon_{yy}) \propto (\boldsymbol{q} \cdot \boldsymbol{u}_{\boldsymbol{q}})$$

Matrix element:

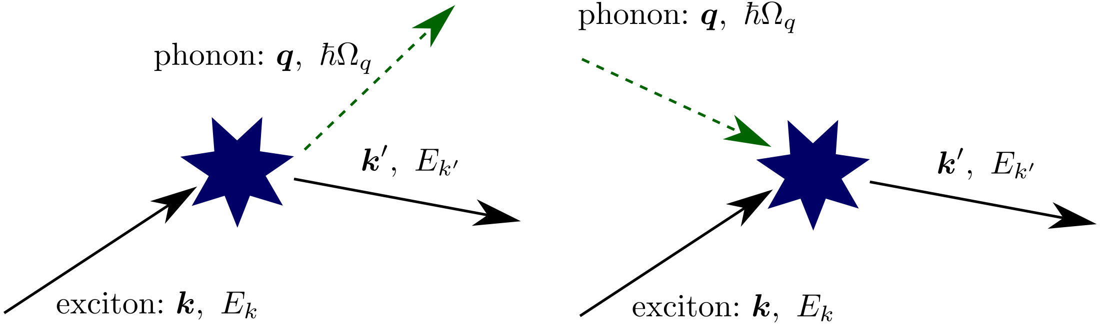
$$M_{\boldsymbol{k}'\boldsymbol{k}}^{\boldsymbol{q}} = \sqrt{\frac{\hbar}{2\rho\Omega_q \mathcal{S}}} q (D_c - D_v) \mathcal{F}(q), \quad \mathcal{F}(q) \approx \frac{1}{[1 + (q a_B/4)^2]^{3/2}} \approx 1$$

effective for LA mode; piezo interaction [$\propto \sqrt{q}/(1 + r_0 q)$ in 2D] is weak

Exciton-phonon scattering rates

Scattering rates

Emission Absorption



Quasi-elastic scattering

at $k_B T \gg Ms^2 \sim 100\ \mu\text{eV}$ the involved phonon energy $\hbar\Omega_q \ll k_B T$
(M is the exciton mass, s is the speed of sound)

$$\frac{1}{\tau_p} = \frac{2\pi}{\hbar} \sum_{\boldsymbol{q}} |M^q_{\boldsymbol{k}'\boldsymbol{k}}|^2 (1 - \cos\theta)(1 + 2n_q)\delta(E_k - E'_k) = c\frac{k_B T}{\hbar} \quad (c \sim 1)$$

Energy relaxation rate: $$\frac{1}{\tau_\epsilon} = \frac{2M^2(\Xi_c - \Xi_v)^2}{\hbar^3 \rho} \sim \frac{1}{10\ \text{ps}} \ll \frac{1}{\tau_p} \quad @\, T \gtrsim 2\ \text{K}$$

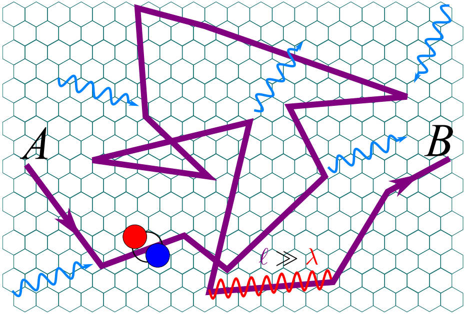
Semiclassics for TMDCs in a nutshell

$$D = \int_0^\infty \langle \hat{v}_x(t)\hat{v}_x(0)\rangle \, dt = \left\langle \frac{v^2 \tau_p}{2} \right\rangle$$

semiclassics: $\ell \gg \lambda \quad \Leftrightarrow \quad \frac{k_B T \tau_p}{\hbar} \gg 1$

diffusion coefficient from velocity correlator

$$\langle v_x(t) v_x(0)\rangle = v_x^2(0) e^{-t/\tau_p}, \qquad \frac{1}{\tau_p} = \sum_{\mathbf{k}'} W_{\mathbf{k}\mathbf{k}'}(1 - \cos\vartheta)$$



LA-phonon scattering in MX_2 MLs

$$\tau = \frac{Ms^2}{k_B T}\tau_0, \quad \tau_0^{-1} = \frac{M^2(\Xi_c - \Xi_v)^2}{\rho \hbar^3} \quad \Rightarrow \quad D = s^2 \tau_0 \sim 1\ldots3\ \text{cm}^2/\text{s} \sim \frac{\hbar}{M}$$

Here D is temperature independent; experiment at 4 K gives similar values (~ 2.5 cm^2/s).

Time scales

Quasielasticity:

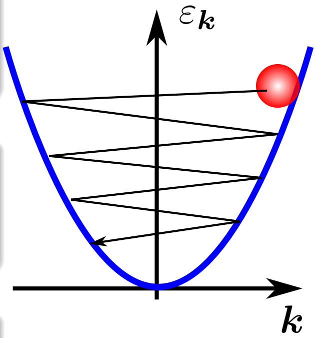
$$\Delta\epsilon \sim \sqrt{k_B T M s^2} \ll k_B T \quad \Rightarrow \quad \delta\epsilon^2(t) \sim (\Delta\epsilon)^2 \frac{t}{\tau}$$

Momentum relaxation time

$$\tau = \frac{Ms^2}{k_B T}\tau_0, \quad \tau_0^{-1} = \frac{M^2(\Xi_c - \Xi_v)^2}{\rho\hbar^3}$$

Energy relaxation time

$$\delta\epsilon(\tau_\epsilon) \sim k_B T \quad \Rightarrow \quad \tau_\epsilon = \frac{\tau_0}{2} \gg \tau$$



$$\tau_\epsilon \gg \tau_\phi \gg \tau$$

Phase relaxation time

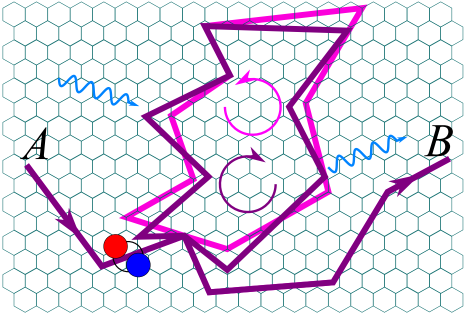
Altshuler, Aronov, Khmelnitsky (1981)

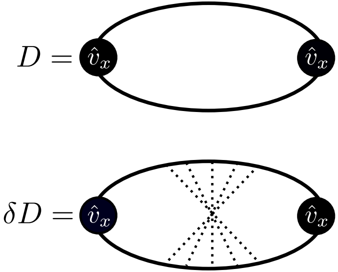
$$\delta\epsilon(\tau_\phi) \sim \frac{\hbar}{\tau_\phi} \quad \Rightarrow \quad \tau_\phi \sim \left[\frac{\hbar^2\tau_0}{(k_B T)^2}\right]^{1/3} \quad \Rightarrow \quad \frac{\tau_\phi}{\tau} \propto T^{1/3}$$

Acoustic phonon scattering: quantum effects get stronger with the temperature increase

Golubentsev (1984); Afonin, Galperin, Gurevich (1985); Adams, Paalanen (1987); Stephen (1987); Dyakonov, Kopelevich (1988)

Exciton weak localization: Dephasing





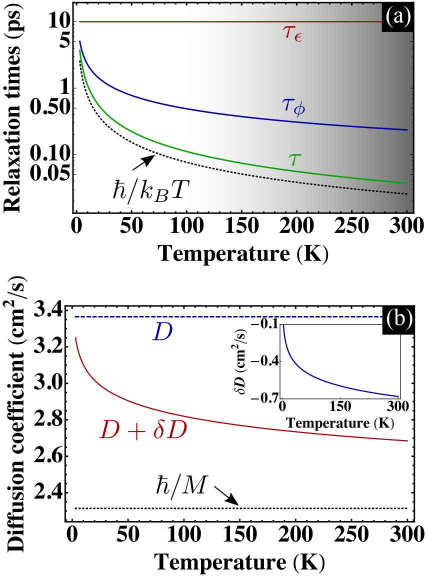
Interference amplitude (Cooperon):

$$C_\phi \sim \exp\left(-\frac{M\epsilon}{\hbar^2\tau}\int_{-t}^{t}\left[\boldsymbol{r}(t') - \boldsymbol{r}(t)\right]^2 dt'\right)$$

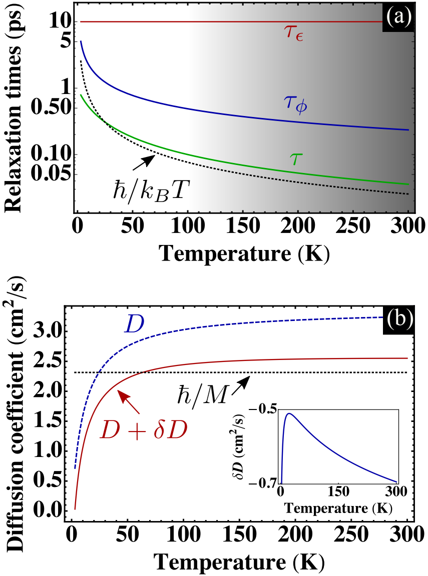
$$\frac{\delta D}{D} \sim -\frac{\hbar}{k_B T\tau}\ln\left(\frac{\tau_\phi}{\tau}\right), \quad \frac{\hbar}{\tau_\phi} \sim \delta\epsilon(\tau_\phi), \quad \frac{\tau_\phi}{\tau} = \sqrt[3]{\frac{\hbar^2}{Ms^2\tau^2 k_B T}} \propto T^{1/3}$$

Weak localization: results

Phonons only



Phonons and impurities



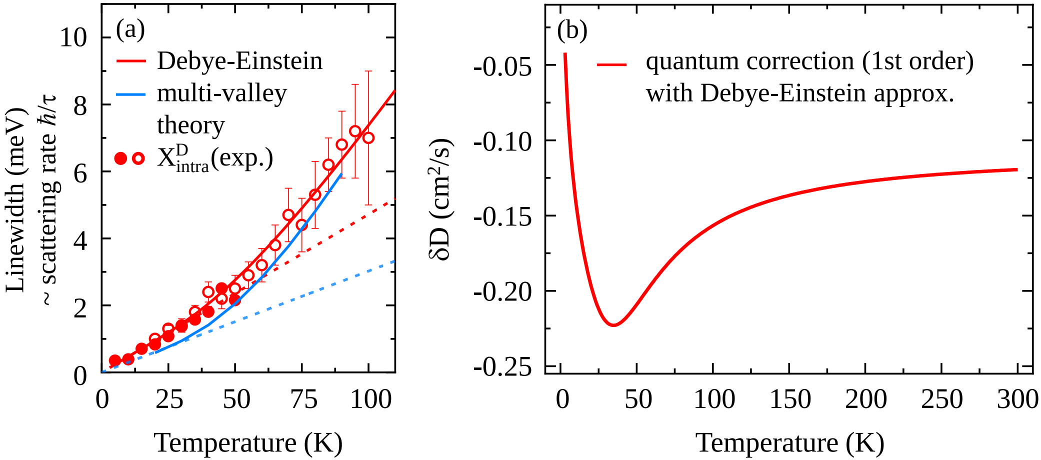
PRL **124**, 166802 (2020)

Various scattering mechanisms

Scattering mechanism	$D_{cl}(T)$	τ_ϕ/τ	$\delta D(T) < 0$
LA	T^0	$T^{1/3}$	$\downarrow$
LA + disorder	T	$T^{-2/3}$	$\uparrow$
flexural, 1ML	T^0	T^0	const
flexural, 2ML, $T < T_0$	T^0	$T^{2/3}$	$\downarrow$
flexural, 2ML, $T > T_0$	$T^{3/2}$	$T^{-5/6}$	$\uparrow$
LA, overdamped	T^0	$T^{1/2}$	$\downarrow$
LA + free e/h	T^0	T/n	$\downarrow$

Appl. Phys. Lett. **121**, *192106 (2022)*

Experiment: non-classical exciton propagation – II

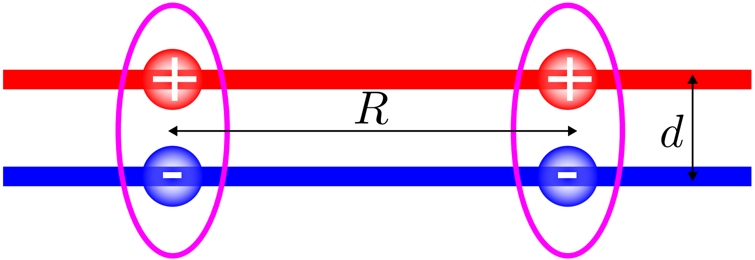


PRL **127**, 076801 (2021)

Dipolar repulsion of excitons in bilayers

Direct Coulomb interaction

$$U(R) \approx \frac{2e^2}{\varepsilon}\left(\frac{1}{R} - \frac{1}{\sqrt{R^2+d^2}}\right), \quad V = \int U(R) d^2R = \frac{4\pi e^2 d}{\varepsilon}$$



Interaction-induced blueshift $\Delta E(n) = Vn$ "plate capacitor model"

Butov, Shashkin, Dolgopolov, Campman, Gossard (1999)

Exciton-exciton correlations: $V \to \mathcal{K}(T)V, \quad \mathcal{K}(T) \approx 0.1\ldots1$

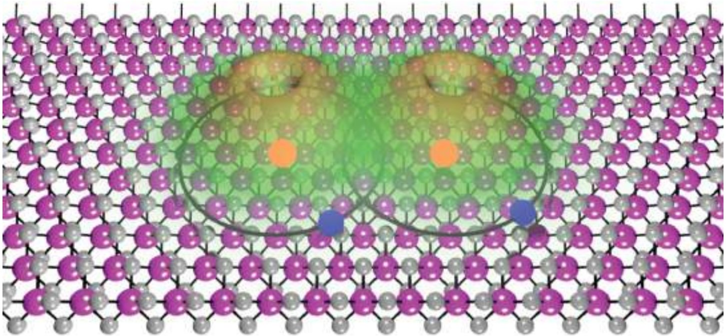
Zimmermann, Schindler (2007,2008); Laikhtman, Rapaport (2009)

$+$ band gap renormalization and screening …

2D semiconductors: *Erkensten, Brem, Perea-Causín, Malic (2022)*

Exchange interaction of excitons in monolayers

In monolayers direct Coulomb interaction is suppressed due to the charge neutrality



Exchange contribution ⇒ overlap of the wavefunctions:

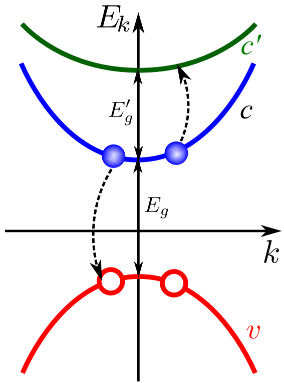
$$V_{\uparrow\uparrow} \sim E_B a_B^2, \quad V_{\uparrow\downarrow} \sim \frac{\hbar^2}{M} \ln\left(-\frac{E}{E_{bi}}\right)$$

Quantum wells: *Ciuti, Savona, Piermarocchi, Quattropani, Schwendimann (1998); Combescot, Betbeder-Matibet, Dubin (2008);*
Microcavities: *Tassone, Yamamoto (1999); MMG, Ouerdane, Pilozzi, Malpuech, Kavokin, D'Andrea (2009);*
2D semiconductors: *Shahnazaryan, Iorsh, Shelykh, Kyriienko (2017)*

Exciton-exciton annihilation: Auger-like process

(eV)	MoS_2	$MoSe_2$	WSe_2	$MoTe_2$
E_g	1.8	1.6	1.7	1.7
E'_g	1.2	1	1.4	1.3
E_B	0.2	0.18	0.16	0.16

Resonant interaction of excitons



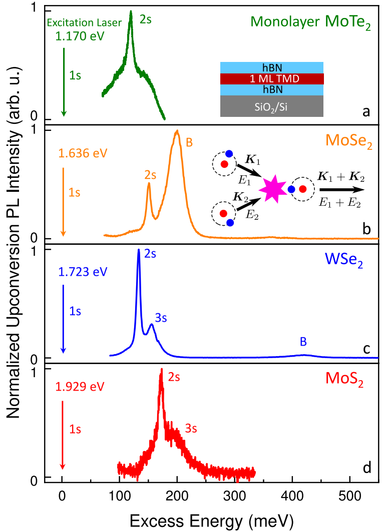
Conservation laws:

$$E_1 + E_2 = E_f, \quad \boldsymbol{K}_1 + \boldsymbol{K}_2 = \boldsymbol{K}_f$$

Resonance: $E'_g \approx E_g - E_B$

$$\frac{dn_{cv}}{dt} = -R_A n_{cv}^2 = -\frac{dn_{c'v}}{dt}$$

$$R_A \propto \frac{E_B^2}{k_B T} \left| \frac{p_{cv} p_{c'v}}{E_g E_{g'}} \right|^2 e^{-|\delta|/k_B T}$$



Resonant Auger process populates excited states (photoluminescence upconversion)

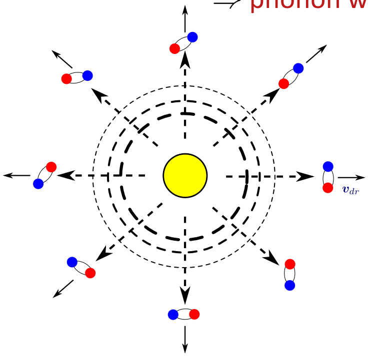
Nat. Commun. **8**, *14927 (2017); Phys. Rev. X* **8**, *031073 (2018); Nat. Commun.* **13**, *6980 (2022)*

Phonon wind effect

Low temperatures, ballistic phonons

- Pump pulse creates hot spot
- ballistic phonons
- momentum flux

$\Rightarrow$ phonon wind



Phonons drag excitons away

Keldysh (1976); Zinov'ev, Ivanov, Kozub, Yaroshetskii (1983)
Bulatov, Tikhodeev (1992)

Force field produced by phonons (2D)
can be found from the kinetic equation

Exciton distribution function f_k:

$$\frac{\partial f_k}{\partial t} + v_k \frac{\partial f_k}{\partial r} + \frac{f_k - \bar{f}_k}{\tau_p} = -\frac{f_k}{\tau_d} + g_k$$

$$+Q_{\text{exc}-\text{ph}}\{f_k\}$$

At $f_k \ll 1$, and high phonon occupancies $N_q \gg 1$

$$Q_{\text{exc}-\text{ph}}\{f_k\} = \frac{2\pi}{\hbar} \sum_q |M_q|^2 (f_{k+q} - f_k) \times$$

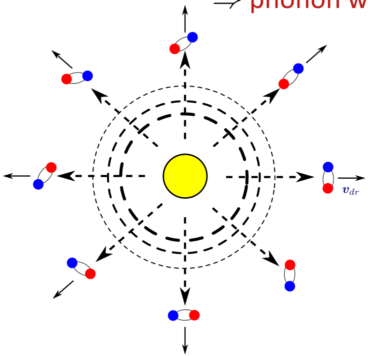
$$[N_q \delta(E_{k+q} - E_k - \hbar\Omega_q) + N_{-q}\delta(E_{k+q} - E_k + \hbar\Omega_q)]$$

Phonon wind effect

Low temperatures, ballistic phonons

- Pump pulse creates hot spot
- ballistic phonons
- momentum flux

$\Rightarrow$ phonon wind



Phonons drag excitons away

Keldysh (1976); Zinov'ev, Ivanov, Kozub, Yaroshetskii (1983)
Bulatov, Tikhodeev (1992)

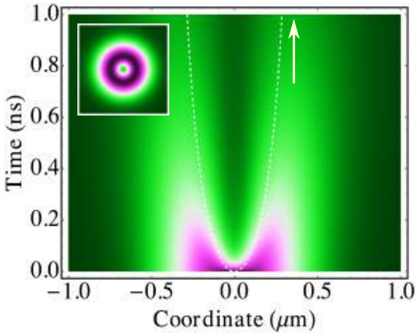
Force field produced by phonons (2D)

$$F_{\text{wind}}(\boldsymbol{\rho}) = \frac{U}{\rho}\frac{\boldsymbol{\rho}}{\rho}$$

Hot spot acts as a repulsive center

effective "Coulomb" repulsion

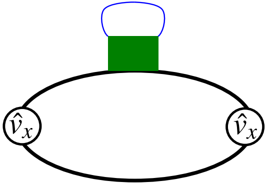
Cloud expansion $\rho(t) \approx \sqrt{Ut}$

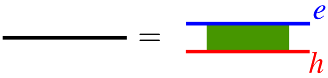


Semiclassical propagation

$$D = \int_0^\infty \langle\langle \hat{v}_x(t)\hat{v}_x(0)\rangle\rangle \, dt, \quad \langle v_x(t)v_x(0)\rangle = v_x^2 e^{-|t|/\tau}$$

$D_{cl} =$





Semiclassical approach (short-range scattering)

$$D = \left\langle \frac{v^2\tau}{2} \right\rangle, \quad \frac{1}{\tau} = \frac{2}{\hbar} \operatorname{Im} \Sigma(E_{\boldsymbol{k}}^x, \boldsymbol{k})$$

Scattering contribution to the linewidth

$$\Gamma = \frac{2}{\hbar} \operatorname{Im} \Sigma(0,0) \left(= \frac{1}{\tau} \quad \text{for many scattering mechanisms} \right)$$

low temperatures, phonons are suppressed: τ is the exciton-electron scattering time

Trion/Fermi polaron scattering

Chevy ansatz:

$$\Psi_{\boldsymbol{k}} = \varphi(\boldsymbol{k}) X^{\dagger}_{\boldsymbol{k}} |0\rangle + \sum_{\boldsymbol{p},\boldsymbol{q}} \underbrace{F_{\boldsymbol{p},\boldsymbol{q}}(\boldsymbol{k}) X^{\dagger}_{\boldsymbol{k}+\boldsymbol{q}-\boldsymbol{p}} e^{\dagger}_{\boldsymbol{p}}}_{\text{trion}} \underbrace{e_{\boldsymbol{q}}}_{\text{FS-hole}} |0\rangle$$

Short-range interaction model:

$$\varphi(\boldsymbol{k}) \approx \frac{V_0 N_e}{-E_{b,tr}} \mathcal{F}(\boldsymbol{k}), \quad F_{\boldsymbol{p},\boldsymbol{q}}(\boldsymbol{k}) \approx \frac{V_0}{E^{tr}_{\boldsymbol{k}} - E^{x}_{\boldsymbol{k}-\boldsymbol{p}} - E^{e}_{\boldsymbol{p}}} \mathcal{F}(\boldsymbol{k}), \quad |\mathcal{F}(\boldsymbol{k})|^2 \approx \frac{E_{b,tr}}{N_e \mathcal{D} V_0^2}.$$

Perturbation (external field)

$$\hat{V} = \sum_{\boldsymbol{k},\boldsymbol{k}'} V_x(\boldsymbol{k}' - \boldsymbol{k}) X^{\dagger}_{\boldsymbol{k}'} X_{\boldsymbol{k}} + \sum_{\boldsymbol{p}',\boldsymbol{p}} V_e(\boldsymbol{p}' - \boldsymbol{p}) e^{\dagger}_{\boldsymbol{p}'} e_{\boldsymbol{p}}; \quad V_x = V_e + V_h$$

Large momentum transfer $|\boldsymbol{k} - \boldsymbol{k}'| \gg k_F$

$$V_{\boldsymbol{k},\boldsymbol{k}'} \approx V_x + V_e$$

Small momentum transfer $|\boldsymbol{k} - \boldsymbol{k}'| \leqslant k_F$

$$V_{\boldsymbol{k},\boldsymbol{k}'} \approx V_x + V_e - V_{FS-h} \approx V_x.$$

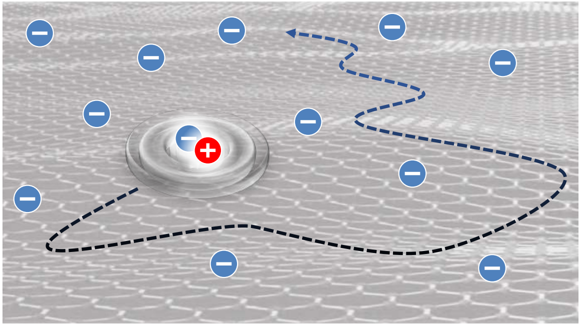
Repulsive (exciton) and attractive (trion) polaron scattering

Related problem: lifetimes of polaron quasiparticles *Petrov (2003); Yan (2019); Cotleţ et al. (2019); Adlong (2020)*

At low temperatures, the main scattering is due to the resident electrons

$$\mathrm{Im}\, T(\varepsilon, \boldsymbol{k}) = \left| \frac{V_0}{1 - V_0 S(\varepsilon, \boldsymbol{k})} \right|^2 \mathrm{Im}\, S(\varepsilon, \boldsymbol{k})$$

$$\mathrm{Im}\, S(\varepsilon, \boldsymbol{k}) = \pi \sum_{\boldsymbol{p}'} (1 - n_{\boldsymbol{p}'}) \delta(\varepsilon - E^e_{\boldsymbol{p}'} - E^x_{\boldsymbol{k}-\boldsymbol{p}'}) = \begin{cases} \pi D_{\mathrm{eff}}, & \varepsilon > 0 \quad \text{repulsive polaron/exciton} \\ 0, & \varepsilon < 0 \quad \text{attractive polaron/trion} \end{cases}$$



For repulsive polaron/exciton

$$\mathrm{Im}\, \Sigma(\varepsilon > 0) \neq 0 \quad \Rightarrow$$

effective scattering by electrons

For attractive polaron/trion

$$\mathrm{Im}\, \Sigma(\varepsilon < 0) = 0 \quad \Rightarrow$$

only higher-order (weaker) processes
and phonon + disorder scattering

Electron-exciton scattering

Clean limit (Fermi's golden rule):

$$\mathrm{Im}\,\Sigma_x(E^x_{\boldsymbol{k}},\boldsymbol{k}) = \pi \sum_{\boldsymbol{p},\boldsymbol{p}'} n_{\boldsymbol{p}}(1-n_{\boldsymbol{p}'})\delta(E^x_{\boldsymbol{k}} + E^e_{\boldsymbol{p}} - E^e_{\boldsymbol{p}'} - E^x_{\boldsymbol{k}+\boldsymbol{p}-\boldsymbol{p}'}) \left| \frac{V_0}{1 - V_0 S(\varepsilon,\boldsymbol{k})} \right|^2 \Rightarrow$$

$$\Gamma_x(k) \propto \frac{(k_B T)^{3/2}}{\sqrt{N_e}} \to 0 \,@\, k_B T \ll E_F$$ (Fermi liquid-like behavior)

Pauli blocking prevents exciton-electron scattering

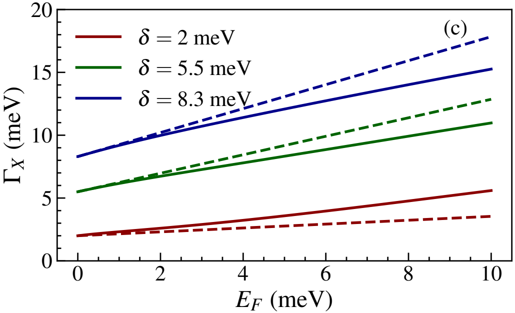
cf. Cotleţ et al. (2019); higher-order processes: Petrov (2003); Adlong (2020)

Dirty limit, $\delta \gg k_B T$ (acoustic phonon or impurity scattering is present)

$$T(\varepsilon,0) \approx \frac{1}{\mathcal{D}} \frac{1}{\ln\left(-\frac{E_{b,T}}{\varepsilon + i\delta/2}\right)}$$

$$\Gamma_X = \delta + E_F \frac{M_T}{M_X} \frac{\pi}{\ln^2[\delta/(2E_{b,T})] + \pi^2/4}$$

numerics: *Efimkin & MacDonald (2017); Rana et al. (2020); Katsch & Knorr (2022)*



Additional scattering breaks energy-momentum conservation
and enhances electron-exciton scattering